Homework problems:

1.1 (due Sep 14, 2006) We are given an array $A$ with $n$ elements. We would like to cyclically rotate the array left by $k$ positions (for example if $A = [1, 2, 3, 4, 5, 6, 7, 8]$ and $k = 3$, the result of the rotation will be $A = [4, 5, 6, 7, 8, 1, 2, 3]$). We would like to have an algorithm which uses very little extra space (few extra variables are ok, but anything larger, such as, an auxiliary array is not allowed). Write an algorithm which runs in time $O(n)$.

1.2 (due Sep 14, 2006)
Let $\{a_n\}, \{b_n\}$ be sequences of positive numbers. Prove

1. $\max\{a_n, b_n\} = O(a_n + b_n)$.
2. $a_n + b_n = O(\max\{a_n, b_n\})$.

1.3 (due Sep 14, 2006)
We are given an array $A$ with $n$ elements and a number $C$. Assume that the sum of the elements in $A$ is larger than $C$. We would like to compute the smallest number of elements from $A$ whose sum is at least $C$. (For example, if $A = [8, 3, 9, 2, 7, 1, 5]$ and $C = 18$ then the answer is 3.) Give a linear-time algorithm for this problem.

Bonus problems:

1.4 (due Sep 14, 2006)
Find two sequences $\{a_n\}$ and $\{b_n\}$ of positive numbers such that neither $a_n = O(b_n)$ nor $b_n = O(a_n)$ holds.

1.5 (due Sep 14, 2006)
Let $\{a_n\}$ and $\{b_n\}$ be sequences of positive numbers such that $2^{a_n} = O(2^{b_n})$.

1. Does $a_n = O(b_n)$ hold?
2. Does $3^{a_n} = O(3^{b_n})$ hold?

Prove your answers.