

Homework problems:

4.1 (due Oct 5, 2006) Consider the coin change problem with coin values 1, 3, 5. Does the greedy algorithm always find an optimal solution? If the answer is no, provide a counterexample. If the answer is yes, give a proof.

4.2 (due Oct 5, 2006) Consider the coin change problem with coin values 1, 4, 6. Does the greedy algorithm always find an optimal solution? If the answer is no, provide a counterexample. If the answer is yes, give a proof.

4.3 (due Oct 5, 2006) Consider the coin change problem with coin values 1, 4, 7. Does the greedy algorithm always find an optimal solution? If the answer is no, provide a counterexample. If the answer is yes, give a proof.

4.4 (due Oct 5, 2006) An alphabet contains 8 letters. The frequencies of the letters are:

$$1/30, 1/30, 1/15, 1/15, 1/10, 3/20, 13/60, 1/3.$$

Construct an optimal Huffman code for the alphabet. Compute the expected number of bits per character.

4.5 (due Oct 5, 2006) We are given N coins, one of which is counterfeit. The counterfeit coin has a different weight than the genuine coins, however we do not know if it is lighter or heavier. We would like to use a balance to determine which coin is counterfeit and whether it is lighter or heavier. A measurement on the balance results in one of the following outcomes: left-heavy, right-heavy, equal.

- Show that if $N = 12$ then we can solve the problem using 3 measurements.
- Show that if $N = 14$ then 3 measurements do not suffice.

Bonus problems:

4.6 (due Oct 5, 2006) Give an algorithm which decides whether the greedy algorithm for the coin change problem with coin values 1, A , B always gives an optimal solution. The input to your algorithm is a pair of integers A , B . Your algorithm should output:

- “YES”, if the greedy algorithm always gives an optimal solution (for coin values 1, A , B).
- “NO, P is a counterexample”, if the greedy algorithm does not always give an optimal solution (for coin values 1, A , B). In this case, the algorithm should also output a counterexample P (P is a counterexample to the optimality of the greedy algorithm).

4.7 (due Oct 5, 2006) Do 3 measurements suffice if $N = 13$ in problem 4.5? If the answer is yes, give an algorithm. If the answer is no, give a proof that no algorithm exists.