Homework problems:

9.1 (due Nov 9, 2006) Suppose that you have a black-box which solves the Max-Weight Matching problem on any weighed graph $G = (V,E,w)$ in time $O(E + V)$. Show how you can use the black-box to solve the Max-Weight Perfect Matching problem in time $O(E + V)$.

9.2 (due Nov 9, 2006) Suppose that you have a black-box which solves the Max-Weight Perfect Matching problem on any weighed graph $G = (V,E,w)$ in time $O(E + V)$. Show how you can use the black-box to solve the Max-Weight Matching problem in time $O(E + V)$.

9.3 (due Nov 9, 2006) Suppose that you have a black-box which for any graph $G$ answers whether $G$ is 3-colorable or not. The black-box takes time $O(1)$ to answer the queries.

Give an efficient algorithm which, for any 3-colorable input graph $G$, finds a 3-coloring of $G$. At the beginning of your solution clearly state:

- the (asymptotic) running time of your algorithm, and
- the (asymptotic) number of queries made to the black-box.

Bonus problems:

9.4 (due Nov 9, 2006)
Let CLIQUE be the following problem:

INSTANCE: a graph $G$ (given by an adjacency list), and a number $k$

QUESTION: does the graph $G$ have a clique of size $\geq k$?

Suppose that you have a black-box that solves the CLIQUE problem in $O(1)$-time. Give an efficient algorithm which, for any input graph $G$, finds the maximum clique in $G$.

At the beginning of your solution clearly state:

- the (asymptotic) running time of your algorithm, and
- the (asymptotic) number of queries made to the black-box.