1 Schedule

Problem sessions:
Wed, Dec. 5, 8:45pm-9:45pm
Mon, Dec. 10, 7pm-8pm
Wed, Dec. 12, 8:45pm-9:45pm

The quiz will be on Thursday, Dec. 13.

2 List of algorithms covered in the class

(B-basic, I-intermediate, A-advanced):

I: Vertex cover (p. 278, DSV).
I: Clustering (p. 280, DSV).
I: TSP (p. 282, DSV).
A: Knapsack (p. 283, DSV).
A: Set-cover.

3 Basic material

Important concepts, problems, theorems, and algorithms:

- NP, NP-hard, NP-complete, reduction.
- Basic NP-complete problems (SAT, 3SAT, vertex cover, clique, k-coloring, integer linear programming)

Testing method:

- Solve a small instance of the basic NP-complete problems.
- Give a (simple) reduction between a pair basic NP-complete problems.

Example problems (homework):

8.1 (due Dec 6, 2007) Let $V = \{0, 1, 2, 3\} \times \{0, 1, 2, 3\}$ and $E = \{(a, b), (c, d)\} \in \binom{V}{2} | a = c \text{ or } b = d\}$. Find the largest clique in $G = (V, E)$.

8.2 (due Dec 6, 2007) Let $V = \{0, 1, 2, 3\} \times \{0, 1, 2, 3\}$ and $E = \{(a, b), (c, d)\} \in \binom{V}{2} | a = c \text{ or } b = d\}$. Find the smallest vertex cover in $G = (V, E)$.

8.3 (due Dec 6, 2007) Give a reduction from 3-SAT to integer linear programming.

4 Additional homework

8.4 (due Dec 11, 2007) [Problem rating: I.] A vertex cover of a graph $G = (V, E)$ is a subset $S$ of vertices such that for each edge $e \in E$ at least one of its endpoints is in $S$. Consider the following optimization problem:

**VERTEX-COVER**
INSTANCE: A graph $G$. 
SOLUTION: A vertex cover $S$ of size $G$.

OBJECTIVE: Minimize the size of $S$.

Consider the following reduction from Vertex-Cover to Integer Linear Programming. For each vertex $i \in V = \{1, \ldots, n\}$ we will have a variable $x_i$ and constraints $0 \leq x_i$ and $x_i \leq 1$. For each edge $\{i, j\} \in E$ we will have a constraint $x_i + x_j \geq 1$. Finally, the objective is to minimize $x_1 + \cdots + x_n$. Thus our integer linear program is

$$\min \sum_{i \in V} x_i$$
$$x_i \leq 1, \text{ for } i \in V,$$
$$x_i \geq 0, \text{ for } i \in V,$$
$$x_i + x_j \geq 1, \text{ for } \{i, j\} \in E,$$
$$x_i \in \mathbb{Z}, \text{ for } i \in V \quad \text{(integrality)}.$$ 

Let $O$ be the optimum of (1).

Now view (1) as a linear program (i.e., drop the integrality constraint). Let $R$ be the optimum of this linear program.

a) Which of the following two is always true?

$$R \leq O \quad \text{ or } \quad O \leq R$$

b) Find a graph $G$ for which the values of $O$ and $R$ are different.

8.5 (due Dec 11, 2007) [Problem rating: A.] Prove that if the graph $G$ in Problem 8.4 is bipartite, then $O = R$.

5 Additional problems from the book (do not turn in)

Try to solve the following problems. A few of them will be on the quiz. We will go over the ones that you choose in the problem sessions.

• 8.1, 8.3, 8.4, 8.6, 8.14, 8.18, 9.7, 9.9.