

## 1 Schedule

Problem sessions:

Part 1 of the homework is **due Dec 4, 2008**.  
Part 2 of the homework is **due Dec 9, 2008**.  
The **QUIZ** will be on **Thursday, Dec. 11**.

## 2 List of algorithms covered in the class

(B-basic, I-intermediate, A-advanced):

- I: Vertex cover (p. 278, DSV).
- I: Clustering (p. 280, DSV).
- I: TSP (p. 282, DSV).
- A: Knapsack (p. 283, DSV).
- A: Set-cover.

## 3 Basic material

**Important concepts, problems, theorems, and algorithms:**

- NP, NP-hard, NP-complete, reduction.
- Basic NP-complete problems (SAT, 3SAT, vertex cover, clique,  $k$ -coloring, integer linear programming)

**Testing method:**

- Solve a small instance of the basic NP-complete problems.
- Give a (simple) reduction between a pair basic NP-complete problems.

**Example problems (homework):**

**6.1 (due Dec 4, 2008)** Let  $V = \{0, 1, 2, 3\} \times \{0, 1, 2, 3\}$  and  $E = \{(a, b), (c, d) \in \binom{V}{2} \mid a = c \text{ or } b = d\}$ . Find the largest clique in  $G = (V, E)$ .

**6.2 (due Dec 4, 2008)** Let  $V = \{0, 1, 2, 3\} \times \{0, 1, 2, 3\}$  and  $E = \{(a, b), (c, d) \in \binom{V}{2} \mid a = c \text{ or } b = d\}$ . Find the smallest vertex cover in  $G = (V, E)$ .

**6.3 (due Dec 4, 2008)** Give a reduction from 3-SAT to integer linear programming.

**6.4 (due Dec 4, 2008)** Suppose that you have a black-box which if you any graph  $G$  answers whether  $G$  is 3-colorable or not. The black-box takes time  $O(1)$  to answer the queries.

Give an efficient algorithm which, for any 3-colorable input graph  $G$ , finds a 3-coloring of  $G$ . At the beginning of your solution clearly state:

- the (asymptotic) running time of your algorithm, and
- the (asymptotic) number of queries made to the black-box.

## 4 Additional homework

**6.5 (due Dec 9, 2008)** A vertex cover of a graph  $G = (V, E)$  is a subset  $S$  of vertices such that for each edge  $e \in E$  at least one of its endpoints is in  $S$ . Consider the following optimization problem:

VERTEX-COVER

INSTANCE: A graph  $G$ .

SOLUTION: A vertex cover  $S$  of size  $G$ .

OBJECTIVE: Minimize the size of  $S$ .

Consider the following reduction from VERTEX-COVER to INTEGER LINEAR PROGRAMMING. For each vertex  $i \in V = \{1, \dots, n\}$  we will have a variable  $x_i$  and constraints  $0 \leq x_i$  and  $x_i \leq 1$ . For each edge  $\{i, j\} \in E$  we will have a constraint  $x_i + x_j \geq 1$ . Finally, the objective is to minimize  $x_1 + \dots + x_n$ . Thus our integer linear program is

$$\begin{aligned} \min \quad & \sum_{i \in V} x_i \\ & x_i \leq 1, \text{ for } i \in V, \\ & x_i \geq 0, \text{ for } i \in V, \\ & x_i + x_j \geq 1, \text{ for } \{i, j\} \in E, \\ & x_i \in \mathbb{Z}, \text{ for } i \in V \quad (\text{integrality}). \end{aligned} \tag{1}$$

Let  $O$  be the optimum of (1).

Now view (1) as a linear program (i. e., drop the integrality constraint). Let  $R$  be the optimum of this linear program.

a) Which of the following two is always true?

$$R \leq O \qquad O \leq R$$

b) Find a graph  $G$  for which the values of  $O$  and  $R$  are different.

**6.6 (due Dec 9, 2008)[BONUS PROBLEM]** Prove that if the graph  $G$  in Problem 6.5 is bipartite, then  $O = R$ .

## 5 Additional problems from the book (do not turn in)

Try to solve the following problems. A few of them will be on the quiz. We will go over the ones that you choose in the problem sessions.

- 8.1, 8.3, 8.4, 8.6, 8.14, 8.18, 9.7, 9.9.