1 Schedule

IMPORTANT: Homework is due Dec 8, 1pm.
Bring it to CSB 620 or 621 anytime between 9 am and 1 pm (the TA is travelling—late homework will not be graded).

There is no QUIZ for this section.

2 List of algorithms covered in the class

(B-basic, I-intermediate, A-advanced):

I: Vertex cover (p. 278, DSV).
I: Clustering (p. 280, DSV).
I: TSP (p. 282, DSV).
A: Knapsack (p. 283, DSV).
A: Set-cover.

3 Basic material

Important concepts, problems, theorems, and algorithms:

- NP, NP-hard, NP-complete, reduction.
- Basic NP-complete problems (SAT, 3SAT, vertex cover, clique, k-coloring, integer linear programming)

Homework):

6.1 (due Dec 8, 1pm) Give a reduction from 3-COLORING to integer linear programming.

6.2 (due Dec 8, 1pm) Suppose that you have a black-box which for any input graph $G$ answers whether $G$ is 3-colorable or not. Assume that the black-box takes time $O(1)$ to answer the queries.

Give an efficient algorithm which, for any 3-colorable input graph $G$, finds a 3-coloring of $G$. At the beginning of your solution clearly state:

- the (asymptotic) running time of your algorithm, and
- the (asymptotic) number of queries made to the black-box.

6.3 (due Dec 8, 1pm) A vertex cover of a graph $G = (V,E)$ is a subset $S$ of vertices such that for each edge $e \in E$ at least one of its endpoints is in $S$. Consider the following optimization problem:

**VERTEX-COVER**

INSTANCE: A graph $G$.

SOLUTION: A vertex cover $S$ of $G$.

OBJECTIVE: Minimize the size of $S$. 

1
Consider the following reduction from **Vertex-Cover** to **Integer Linear Programming**. For each vertex $i \in V = \{1, \ldots, n\}$ we will have a variable $x_i$ and constraints $0 \leq x_i$ and $x_i \leq 1$. For each edge $\{i, j\} \in E$ we will have a constraint $x_i + x_j \geq 1$. Finally, the objective is to minimize $x_1 + \cdots + x_n$. Thus our integer linear program is

\[
\begin{align*}
\min \sum_{i \in V} x_i \\
x_i \leq 1, & \quad \text{for } i \in V, \\
x_i \geq 0, & \quad \text{for } i \in V, \\
x_i + x_j \geq 1, & \quad \text{for } \{i, j\} \in E, \\
x_i \in \mathbb{Z}, & \quad \text{for } i \in V \quad \text{(integrality)}. 
\end{align*}
\]

Let $O$ be the optimum of (1).

Now view (1) as a linear program (i.e., drop the integrality constraint). Let $R$ be the optimum of this linear program.

a) Which of the following two is always true?

\[
R \leq O \quad \quad \quad O \leq R
\]

b) Find a graph $G$ for which the values of $O$ and $R$ are different.

6.4 (due Dec 8, 1pm) **BONUS PROBLEM** Prove that if the graph $G$ in Problem 6.3 is bipartite, then $O = R$.

4 Additional problems from the book (do NOT turn in)

Solve the following problems from the book.

- 8.1, 8.3, 8.4, 8.6, 8.14, 8.18, 9.7, 9.9.