

1 Schedule

"Homework" problem session is in CSB 209, 7:30-8:30pm on Nov. 5 (Wednesday); led by Sean Esterkin.

Homework is **due Nov 6** (Thursday).

"Exam" problem session is in CSB 209, 6-7pm on Nov. 12 (Wednesday).

EXAM #3 will be on **Thursday, Nov. 13**.

2 Homework - solve and turn in

4.1 (due Nov 6) Compute the value of the zero-sum two person game given by the following payoff matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

4.2 (due Nov 6) Give a mathematical expression (sums are allowed; linear programs are NOT allowed) for the value of the two person zero-sum game with the following $n \times n$ payoff matrix

$$P = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & n \end{pmatrix}.$$

(More precisely, the matrix P is defined by $P_{ii} = i$ and $P_{ij} = 0$ for $i \neq j$).

4.3 (due Nov 6) We would like to find the largest axis-aligned¹ square inscribed in a convex polygon P . The polygon P is given as a intersection of n half-planes, that is, we are given $a_1, b_1, c_1, \dots, a_n, b_n, c_n$ which define P to be the set of points (x, y) such that $a_i x + b_i y \leq c_i$ for all $i \in \{1, \dots, n\}$. Give a linear program for the problem. Clearly explain the meaning of the variables and the constraints in your linear program.

4.4 (due Nov 6) Construct the linear program dual to the following linear program:

$$\begin{aligned} \max \quad & x_1 + 2x_2 + 3x_3 + 4x_4 \\ & x_2 + x_3 + x_4 \leq 1 \\ & x_1 + x_3 + x_4 \leq 2 \\ & x_1 + x_2 + x_4 \leq 3 \\ & x_1 + x_2 + x_3 \leq 4 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. \end{aligned}$$

Find the optimal solution of the primal and the dual problem. **Use a linear programming solver** to obtain the solutions (for example you can use freeware `lpsolve` or function `Maximize` in MATHEMATICA (installed in some labs)).

¹a square is axis-aligned if its sides are parallel with the x and y axis

4.5 (due Nov 6) Let a be a constant and x be a variable. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{array}{ll}\min & x \\ & x \geq a \\ & x \geq -a\end{array}$$

Let a, b be constants and x, y be variables. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{array}{ll}\min & x + y \\ & x \geq a \\ & x \geq -a \\ & y \geq b \\ & y \geq -b\end{array}$$

(If you don't know where to start—plug in concrete values for a and b (make sure to use negative numbers too), solve the resulting linear program (either by hand or using a solver) and formulate a conjecture based on the experiments.)

Let a, b be constants and x, y be variables. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{array}{ll}\min & x - y \\ & x \geq a \\ & x \geq -a \\ & y \geq b \\ & y \geq -b\end{array}$$

3 Bonus Homework - solve and turn in

4.6 (due Nov 6) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\max_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

4.7 (due Nov 6) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

4.8 (due Nov 6) We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1, \dots, n\}} (ax_i + y_i - b)^2.$$

Find a formula for a and b (you do NOT need linear programming for this one).

4 Additional problems from the book (do not turn in)

Try to solve the following problems. A few of them may be on the quiz. We will go over the ones that you choose in the problem sessions.

- 7.1, 7.2, 7.3, 7.4, 7.6, 7.7, 7.8, 7.9, 7.11, 7.12, 7.13, 7.14, 7.15, 7.19, 7.27, 7.28.