

**IMPORTANT: Write each problem on a separate sheet of paper. Write your name on each sheet.** (Why? To speed up the grading we will use parallelization—each problem will be graded by a different TA. At the beginning of the class (on the date when the homework is due) there will be a separate pile for each problem.)

**Homework 4 due: Nov. 10 (Thursday);** collected before class in B&L 109.

Problem sessions before Homework 4 is due:

Nov. 7 (Monday) 6:15pm - 7:15pm in Hylan 202

Nov. 8 (Tuesday) 6:15pm - 7:15pm in Goergen 109

Nov. 9 (Wednesday) 6:15pm - 7:15pm in Gavett 310

Nov. 9 (Wednesday) 7:40pm - 8:40pm in Hutchinson 473

**Exam 3 is on: Nov. 17 (Thursday).**

Problem sessions before Exam 3:

Nov. 14 (Monday) 6:15pm - 7:15pm in Hylan 202

Nov. 15 (Tuesday) 6:15pm - 7:15pm in Goergen 109

Nov. 16 (Wednesday) 6:15pm - 7:15pm in Gavett 310

Nov. 16 (Wednesday) 7:40pm - 8:40pm in Hutchinson 473

## 1 Homework - solve and turn in

**4.1 (due Nov. 10)** Krusty the Clown purchased a new super-efficient SUV with two engines. The first engine runs on a 2:1 mixture of gasoline and ethanol and gets 12 mpg. The second engine runs on a 1:2 mixture of gasoline and ethanol and gets 22 mpg. Krusty has 12 gallons of gasoline and 14 gallons of ethanol. How far can he get?

- (a) Write a linear program for the problem. You are allowed to have only two variables in your linear program.
- (b) Solve the linear program (you do not need to use the simplex method).
- (c) Write down the dual of your linear program from part (a).
- (d) Solve the dual program (you do not need to use the simplex method).

**4.2 (due Nov. 10)** Give a mathematical expression (sums are allowed; linear programs are NOT allowed) for the value of the two person zero-sum game with the following  $n \times n$  payoff matrix

$$P = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & n \end{pmatrix}.$$

(More precisely, the matrix  $P$  is defined by  $P_{ii} = i$  and  $P_{ij} = 0$  for  $i \neq j$ ).

(Hint: Solve for small  $n$  first and then generalize. For example, for  $n = 2$  solve the following linear program:

min  $z$

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} z \\ z \end{pmatrix}$$

$$\begin{aligned}x_1 + x_2 &= 1 \\x_1 \geq 0, x_2 &\geq 0;\end{aligned}$$

we are using the convention that the entries in the matrix are the payoff for the row player.)

**4.3 (due Nov. 10)** Let  $a$  be a constant and  $x$  be a variable. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{aligned}\min x \\x &\geq a \\x &\geq -a\end{aligned}$$

Let  $a, b$  be constants and  $x, y$  be variables. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{aligned}\min x + y \\x &\geq a \\x &\geq -a \\y &\geq b \\y &\geq -b\end{aligned}$$

(If you don't know where to start—plug in concrete values for  $a$  and  $b$  (make sure to use negative numbers too), solve the resulting linear program (either by hand or using a solver) and formulate a conjecture based on the experiments.)

Let  $a, b$  be constants and  $x, y$  be variables. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{aligned}\min x - y \\x &\geq a \\x &\geq -a \\y &\geq b \\y &\geq -b\end{aligned}$$

## 2 Bonus Homework - solve and turn in

4.4 (due Nov. 10)

**PART A:**

We are given  $n$  pairs of real numbers  $(x_1, y_1), \dots, (x_n, y_n)$ . We want to find real numbers  $a, b$  which minimize

$$\max_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

**PART B:**

$(x_1, y_1), \dots, (x_n, y_n)$ . We want to find real numbers  $a, b$  which minimize

$$\sum_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

**PART C:**

We are given  $n$  pairs of real numbers  $(x_1, y_1), \dots, (x_n, y_n)$ . We want to find real numbers  $a, b$  which minimize

$$\sum_{i \in \{1, \dots, n\}} (ax_i + y_i - b)^2.$$

Find a formula for  $a$  and  $b$  (you do NOT need linear programming for this one).