Linear programming (part 3 of CSC 282),

http://www.cs.rochester.edu/~stefanko/Teaching/19CS282

IMPORTANT: **Write each problem on a separate sheet of paper. Write your name on each sheet.** (Why? To speed up the grading we will use parallelization—each problem will be graded by a different TA. At the beginning of the class (on the date when the homework is due) there will be a separate pile for each problem.)

Homework 4 due: Nov. 21 (Thursday); collected before class in Wegmans 1400.

Problem sessions before Homework 4 is due:
Nov. 18 (Monday) 6:15pm - 7:15pm in Hylan 202
Nov. 19 (Tuesday) 6:15pm - 7:15pm in Goergen 109
Nov. 20 (Wednesday) 6:15pm - 7:15pm in Gavett 310
Nov. 20 (Wednesday) 7:40pm - 8:40pm in Hutchinson 473

1 Homework - solve and turn in

4.1 (**due Nov. 21**) Krusty the Clown purchased a new super-efficient SUV with two engines. The first engine runs on a 2:1 mixture of gasoline and ethanol and gets 12 mpg. The second engine runs on a 1:2 mixture of gasoline and ethanol and gets 22 mpg. Krusty has 12 gallons of gasoline and 14 gallons of ethanol. How far can he get?

(a) Write a linear program for the problem. You are allowed to have only two variables in your linear program.

(b) Solve the linear program (you do not need to use the simplex method).

(c) Write down the dual of your linear program from part (a).

(d) Solve the dual program (you do not need to use the simplex method).

4.2 (**due Nov. 21**) This problem involves three chemicals A, B, C. You have a liters of chemical A, b liters of chemical B, and c liters of chemical C (for each chemical you have a pure solution). There are n potential customers. The i-th customer wants d_i liters of an $a_i : b_i : c_i$ mixture of chemicals A, B, C and is willing to pay p_i dollars per liter of the desired mixture. You need cash. You want to get as much money as possible by creating mixtures and selling them to the potential customers. The customers will purchase your mixture even if you only partially satisfy their demand (for example, if a customer wants 3 liters of a $7 : 8 : 9$ mixture and is willing to pay \$8 per liter and you supply 1.2 liters of a $7 : 8 : 9$ mixture they will pay you \$9.6). Give a linear program for the problem. **Clearly describe the variables in your linear program.**

4.3 (**due Nov. 21**) Mr. Hühnerbein and Ms. Hühnerbein live together. They own two cars—a Wartburg and a Trabant. Ms. Hühnerbein works in the Carl Zeiss factory and Mr. Hühnerbein works in the Fraunhofer Society. They use their cars only for commuting to/from work. They work every day. When it is raining in the morning Ms. Hühnerbein drives the Trabant and Mr. Hühnerbein drives the Wartburg. Otherwise Ms. Hühnerbein drives the Wartburg and Mr. Hühnerbein drives the Trabant. Every day in the morning Mr. Hühnerbein notes the odometer reading on both cars (as usual, the odometer shows the distance driven since the car was made rounded down to the nearest mile). He also keeps track of which days it was raining in the morning. Mr. Hühnerbein and Ms. Hühnerbein have an argument today. Mr. Hühnerbein claims that his commute is longer and hence he deserves an extra portion of chicken soup. Linear programming! To the rescue!

There are $n+1$ days numbered $0, \ldots, n$. The reading of Wartburg odometer on the morning of day $i \in \{0, \ldots, n\}$ is given by W_i (this is a part of the input). The reading of Trabant odometer on the morning of day $i \in \{0, \ldots, n\}$ is given by T_i (this is a part of the input). You are also given a sequence $r_0, \ldots, r_{n-1} \in \{R, S\}$ where $r_i = R$ if day

i if was raining in the morning (this is the last part of the input). Give a feasibility linear program that proves that Mr. Hühnerbein's commute is longer than Ms. Hühnerbein's commute (remember there is no objective in a feasibility linear program). The linear program should be non-feasible if and only if the odometer readings and the rainy day records provide irrefutable evidence that Mr. Hühnerbein's commute is longer than Ms. Hühnerbein's commute.[1] You are only allowed 4 variables in your linear program. Clearly state the meaning of the variables. Explain why non-feasibility of your linear program proves that Mr. Hühnerbein's commute is longer than Ms. Hühnerbein's commute.

2 Bonus Homework - solve and turn in

4.4 (due Nov. 21)
PART A:
We are given n pairs of real numbers $(x_1, y_1), \ldots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\max_{i \in \{1,\ldots,n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.
PART B:
$(x_1, y_1), \ldots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1,\ldots,n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.
PART C:
We are given n pairs of real numbers $(x_1, y_1), \ldots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1,\ldots,n\}} (ax_i + y_i - b)^2.$$

Find a formula for a and b (you do NOT need linear programming for this one).

[1] You are allowed strict inequalities in your linear program for this problem.