2-D Simulator

- Robots
  - Good
  - Evil
  - Prey
  - Obstacles

- Lots of freedom in design of world

- Occupants

- Sensors

- Environment

- Complexity of behavior

- Start simple

- MATLAB (Saw?)

- Public presentation
  - Write up
Simulator Phase II

- Simpler logic
- More realistic physical modeling
- Model your Tutebot.
**EMF & Circuits**

Battery or other source of EMF maintains electric potential difference between two points (this p.d. moves electrons).

\[ \text{EMF } E = \frac{dW}{dq} = \text{Joule/coulomb} = \text{Volt} \]

\[ dq \text{ charge passes through here in time } dt, \text{ so EMF does } dW \text{ work to push them up to higher potential.} \]

\[ \text{positive charge carriers determine direction of current (convention)} \]
Opposing batteries: B wins ($E_B > E_A$)

Chemical Energy from B

Stored electric, magnetic field energy

- Work by $W$
- Heat in $R$
- Chemical energy stored in $A$ (charging)

"Reversible" reaction, unlike loss of energy through heat.
joule = 1 newton-meter; unit of work.

\( F = ma \) \quad \text{(Newton's Law)}

\( wt = \text{N} \cdot \text{m} = \text{kg} \cdot \frac{\text{m}}{\text{sec}^2} \)

Coulomb = charge through wire in 1 second if current is 1 ampere 

\( q = it \)

\( e = 1.60206 \times 10^{-19} \text{ coul} \)

the elementary charge.
Current Calculation

Joule heating: \( i^2 R \, dt \) is heat due to
caused by

Charge: \( i \, dt = dq \) is heat dt.

dq moves thru the circuit. So...
def. of emf

\[ dW = \varepsilon \, dq = \varepsilon \, i \, dt \]

this work = Joule heating by conservation of energy, so

\[ \varepsilon \, i \, dt = i^2 R \, dt \]

\[ \varepsilon \, i = i^2 R \]

\[ \varepsilon = iR \]

\[ i = \frac{\varepsilon}{R} \]
This relation \( i = \frac{e}{R} \) is a consequence of the fact that the algebraic sum of changes in potential encountered in complete traversal of a circuit must be zero.

\[
V = iR
\]

through \( R \), \(-iR\) (Ohm's law)

\( R \) measured in ohms.

Negative sign taken at higher pot.

Traversing \( R \) from bottom to top, increase \( \varepsilon \).

So if initial potential is \( V_a \),

\[
V_a - iR + \varepsilon = V_a
\]

\[-iR + \varepsilon = 0\]

Energy argument, potential argument.
Kirchhoff's 2nd Law: "Loop Theorem"

- Sum of potential changes around a circuit = 0.

\[ R = \frac{V}{I} \text{. Resistors turn current into voltage and vice versa.} \]

- Traverse in direction of current change in potential is \(-IR\). In opposite direction is \(+IR\).

- Traverse current seat in direction of current, change in potential is \(+E\). In opposite direction is \(-E\).
E.G. #1.

Batteries (and all cut seats) have internal resistance. In batteries, it grows with age.

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Example #2  Resistors in Series.

Clockwise from a:

\[-iR_1 - iR_2 - iR_3 + \varepsilon = 0\]

\[i = \frac{\varepsilon}{R_1 + R_2 + R_3}\]

So equivalent resistance

\[R = R_1 + R_2 + R_3\]

(they add!)
Potential Difference

How compute the pot. diff. of 2 points in a circuit?

what is $V_{ab}$

$V_{ab} = V_a - V_b$

for $E, i, v, R$?

$V_b + iR = V_a$

$V_{ab} = V_a - V_b = iR$

$a$ is more positive

potential diff. = $iR$.

we computed $i$ before

so

$$i = \frac{E}{R+v}$$

$$so \quad V_{ab} = E \frac{R}{R+v}$$

So pot. diff. is algebraic $E$ $\theta$

pot. changes from $a$ to $b$.

(partial loop theorem). All paths yield some $\phi$. \(\Box\)
MULTI-LOOP CIRCUITS

\[ \begin{align*}
\varepsilon_1 & \rightarrow \varepsilon_2 \\
R_1 & \rightarrow i_1 \rightarrow R_2 \rightarrow i_2
\end{align*} \]

b, c, d are junctions connected by 3 branches. (bad, bd, bcd).

given \( \varepsilon \)s and \( R \)s what are \( i \)s?

label \( i \) directions arbitrarily chosen.

(i3?!)

Charge does not accumulate or drain from d. So if \( i \)s cancel \( i \)s then

\[ i_1 + i_3 - i_2 = 0 \]  \( \Box \)
Junction theorem: Kirchhoff's 1st Law

\[ \sum \text{of currents at any junction} = 0 \]
(charge conservation)

- Apply this rule at b, get some eggs. So if we want to solve for b, we need 2 more eggs.

- We have several loops with \pm currents, so we can use loop theorem:

  \text{left: } \ v_1 - i_1 R_1 + i_3 R_3 = 0 \quad (2)

  \text{right: } -i_3 R_3 - i_2 R_2 - v_2 = 0 \quad (3)

  \text{another: no new relationships}

  So solve (1), (2), (3).

E.g. \( i_1 = \frac{v_1 (R_2 + R_3) - v_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \)

\[ i_2 = \frac{\text{always same}}{R_3} \]
E.g.

Resistors in parallel

\[ \text{What is equivalent resistance of } R_1, R_2, R_3? \]
\( R \)

Let \( V \) be the potential diff. between \( a, b \). Then

\[ i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad i_3 = \frac{V}{R_3} \]

Junction theorem at \( a \):

1. \( i = i_1 + i_2 + i_3 = V \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \)
2. \( i = \frac{V}{R} \), or \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \)

[\( \frac{12}{12} \)]
**CAPACITANCE**

**CHARGED CONDUCTING SPHERE** has a potential:

\[ V_0 = \frac{1}{4\pi \varepsilon_0} \frac{q}{R} \]

Twice the potential difference

"spheres get bigger, closer... At 0 distance no difference"
\[ C = \frac{q}{V} \]

\[ C : \text{farad} = \frac{\text{coul}}{\text{volt}} \]

\[ \text{uf} \]

\[ \text{uf} \text{ is practice} \]

Generally:

A capacitor.

Practically, one type is foil:

- Insulator
- Positive
- Negative
CAPACITORS in Practice

\[ C = \frac{Q}{V} \quad Q = CV \text{ or } \frac{dQ}{dt} = C \frac{dV}{dt} = i \]

E. G. Capacitors in Parallel

\[ \text{pot. difference same for all: all upper plates connected to } a, \text{ lower to } b. \]

\[ q_1 = C_1 V \quad q_2 = C_2 V \quad q_3 = C_3 V \]

\[ q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3) V \]

\[ C = \frac{q}{V} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{(like series resistors)} \]
E.G. Capacitors in Series

\[ V_1, \quad V_2, \quad V_3 \]

\[ C_1, \quad C_2, \quad C_3 \]

\[ a \quad \rightarrow \quad V \quad \rightarrow \quad b \]

\( q \) must be same on all plates. Initially no charge, battery just moves it around. Charge can't leave the area.

\[ V = \frac{q}{C_1} \quad V_2 = \frac{q}{C_2} \quad V_3 = \frac{q}{C_3} \]

\[ V = V_1 + V_2 + V_3 \quad \text{(series)} \]

\[ = q \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \]

\[ C = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \]

or\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]
RC circuit

Capacitors ⇒ time-varying currents!
(physics is energy storage in electric field).

![Diagram of an RC circuit with a switch S, resistor R, and capacitor C, along with equations for energy and charge.]  

Throw S to a: then what?  
Work is Joule heat + changing C.

$$\epsilon \frac{dq}{dt} = i^2 R + \frac{1}{C} \frac{d\epsilon}{dt}$$

$$\epsilon \frac{dq}{dt} = i^2 R + \frac{1}{C} \frac{d\epsilon}{dt}$$

$$\epsilon = iR + \frac{q}{C}$$  

(by or from Joule's theorem)
\[ E = iR + \frac{q}{C} \quad ; \quad i = \frac{dq}{dt} \]

So

\[ E = R \frac{dq}{dt} + \frac{q}{C} \quad \text{diff. eq.} \]

Just a slight generalisation of

\[ x + ax = 0 \]

\[ x = -ax \]

\[-ae^{-at} = -ae^{-at} \]

if \( x = e^{-at} \)

Solutions to 1st order linear ordinary differential equations of form \( e^{-kt} \)

\[ \begin{array}{c|c}
1 & 1 \\
\hline 
t & + \\
\end{array} \]
*$\text{ CHECK}\star$

\[\varepsilon = R \frac{dQ}{dt} + \frac{q}{C}\]

claim \( q = C \varepsilon (1 - e^{-t/RC}) \)

\[\frac{dQ}{dt} = i = -\frac{C \varepsilon}{RC} e^{-t/RC}\]

recall \( \varepsilon = R \frac{dQ}{dt} + \frac{q}{C}\)

\[\varepsilon = R \frac{C \varepsilon}{RC} e^{-t/RC} + \frac{C \varepsilon (1 - e^{-t/RC})}{C}\]

\[\varepsilon = C e^{-t/RC} + \varepsilon (1 - e^{-t/RC}) = \varepsilon\]

\[\square\]
So the general solution (can be guessed) is

\[ q = CE(1 - e^{-t/RC}) \]

(can be checked).

What happens when we throw the switch?

\( RC \) has dimensions of time - the capacitance time constant.

Time it takes to charge to \((1 - e^{-1}) = 63\% \).
Now throw switch to b. after \( t \gg RC \)

\[
\text{Loop theorem: } \quad iR + \frac{q}{C} = 0
\]

\[
i = \frac{dq}{dt}, \text{ so diff. eq. is } R \frac{dq}{dt} + \frac{q}{C} = 0
\]

so \( q = q_0 e^{-\frac{t}{RC}} \) with \( q_0 \) the initial charge = \( CE \).

Current during discharge \( i = \frac{dq}{dt} = \)

\[
= -\frac{q_0}{RC} e^{-\frac{t}{RC}}
\]

so current is reversed from direction in picture. Since \( q_0 = CE \)

\[
i = -\frac{E}{R} e^{-\frac{t}{RC}}
\]

so initially current is \( -\frac{E}{R} \) which makes sense since capacitor is charged to full pot. diff. eq.
E.G. A Differentiator

Voltage across $C$ is $V_{in} - V$.

$I = C \frac{dV}{dt}$ (capacitance), definition

$i = C \frac{d\phi}{dt}$

$I = C \frac{d}{dt}(V_{in} - V)$

$I = \frac{V}{R}$ also

If choose $\frac{dV}{dt} \ll \frac{dV_{in}}{dt}$

$C \frac{dV_{in}}{dt} \approx \frac{V}{R}$

$V(t) = RC \frac{dV_{in}}{dt}$

Make $RC$ small.

If $R$ too small you load the input.

$V_{in}$

$V$

$\text{Ans!}$
E.G. An integrator

\[ V_{\text{in}} \xrightarrow{R} \frac{1}{C} \xrightarrow{0} V \]

Voltage across \( R = V_{\text{in}} - V \), so

\[ I = \frac{V_{\text{in}} - V}{R} \quad \text{if} \quad V_{CC} < V_{\text{in}} \text{(with large RC)} \]

\[ C \frac{dV}{dt} \approx \frac{V_{\text{in}}}{R} \approx I \]

\[ V(t) = \frac{1}{RC} \int_{0}^{t} V_{\text{in}}(t) \, dt + \text{const.} \]

Note: \( V_{CC} V_{\text{in}} \approx I R V_{\text{in}} \). We'd have perfect integrator if we had a current source. A large voltage across a large \( R \) approximates a current source.

Actually, picks off the start of exp. charging curve.
Integrator (cont.)

\[
\frac{v_{in}}{t}
\]

\[
\frac{v}{t}
\]

Ramp Generator: change a capacitor with constant current.

\[
I = C \frac{dv}{dt} \Rightarrow v(t) = \left( \frac{I}{C} \right) t
\]
Diodes

R, C are linear circuit elements. (Doubly signify doubles the response.) Also they are 2-terminal, passive (no power).

Diode is non-linear, passive, 2-terminal.

Breakdown
Diodes

- Anode (arrow) points in dir. of forward current flow.
- Approximates 1-way conductor.
- Voltage drop $0.5 - 0.8\, V$.
- But no resistance in terms of Ohm's Law.

E.G. Rectification
$(AC \rightarrow DC)$

$V$ \[ \sin \] $t$

$V$ \[ \text{---} \]

I ideally

Half-wave:

$\text{Half-wave} \Rightarrow$
Full Wave Rectifier:

2 Diodes in series = twice the voltage drop. Bad for low-voltage supplies.

\[ V \]
\[ t \]

Mega-Cap: Filtering, or... how smooth this out?

\[ R_{\text{load}} C \gg \frac{1}{f} \]
**E.G. Diode Gate**

Pass higher 12 voltages

- [Diagram of a diode gate]

- DC Power Supply
- 12 V Battery
- 12-15V electric clock

---

**E.G. Diode Clamp**

Limit voltage in a signal to ~5.6 V, with no effect on lesser (including negative) voltages. (Protect MOS from static)

- $\approx 10k$

- IN

- OUT

- 5V
Damped Harmonic Oscillator
- 2nd-order system
- Very basic approach to all systems.

\[ F = m a \]
\[ -kx - b \frac{dx}{dt} = m \frac{d^2x}{dt^2} \]
\[ m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0 \]

\[ x = A e^{-\frac{b}{2m} t} \cos(\omega' t + \phi) \]
\[ \omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \]
Inductance

\[ i(t) \Rightarrow E(t) \]

Self-induction

\[ L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = 0 \]

\[ m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = 0 \]

\[ \Rightarrow \text{Analog computers, controllers, ...} \]
Microcomputers 101 (SF 84.3)

IC hides many realities. It is a multi-tasking OS. But not a bad idea to learn bare-bones ideas concept (CSC 254?).

Microcontrollers come with useful capabilities for interfacing: serial lines, I/O, timers, pulse counters. Replaces multi-card system.

CPU → Memory → I/O ...

Bus

Compare to RW's MC684611

p. 51
CPU clock:

XTAL

Instruction timing: 2 - 40 cycles (1 - 20.5 μsec.)

Instruction fetch
delay
fetch, opcodes

Bus

Bus is 16 bits wide so $2^{15}$ 16K locations. (8-bit bytes).

Fig. 3.5 shows how bus is configured for use as address & data line on it.

Ports look like memory to CPU (locations).

Outside, ports are lines to take signals.
Memory: RAM, ROM, PROM, EEPROM

Microcontroller versions exist with all memory types...