

CSC 252/452: Computer Organization

Fall 2024: Lecture 2

Instructor: Yanan Guo
Department of Computer Science
University of Rochester

Action Items:

- **Get CSUG account**
- **Make sure you have VPN setup!!!!**

Announcement

- Make sure you can access CSUG machines!!!
- Programming assignment 1 will be posted today.
 - It is in C language. Seek help from TAs.
 - TAs are best positioned to answer your questions about programming assignments!!!
- Programming assignments do NOT repeat the lecture materials. They ask you to synthesize what you have learned from the lectures and work out something new.

Previously in 252...

Problem

Algorithm

Program

Instruction Set
Architecture (ISA)

Microarchitecture

Circuit

Previously in 252...

Problem

Algorithm

Program

Instruction Set
Architecture (ISA)

ISA is the contract
between software and
hardware.

Microarchitecture

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Previously in 252...

Problem

Algorithm

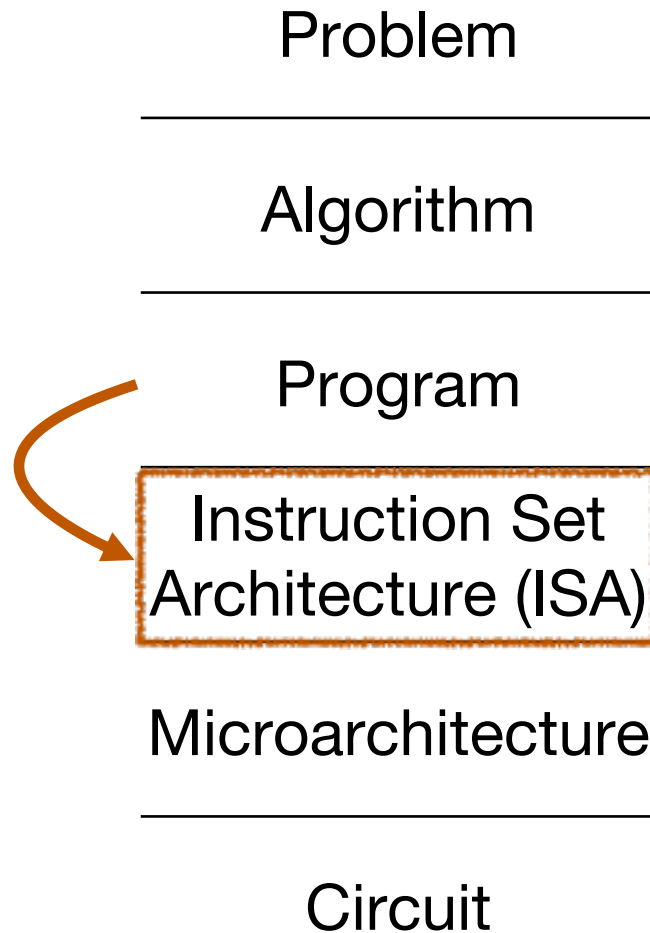
	Renting	Computing
Service provider	Landlord	Hardware
Service receiver	YOU	Software
Contract	Lease	ISA
Contract's language	Natural language (e.g., English)	Assembly programming language

Circuit

ct
e and

Previously in 252...

- How is a human-readable program translated to a representation that computers can understand?



ISA is the contract between software and hardware.

Previously in 252...

C Program

```
void add() {  
    int a = 1;  
    int b = 2;  
    int c = a + b;  
}
```



Assembly program

```
movl    $1, -4(%rbp)  
movl    $2, -8(%rbp)  
movl    -4(%rbp), %eax  
addl    -8(%rbp), %eax
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Today: Representing Information in Binary

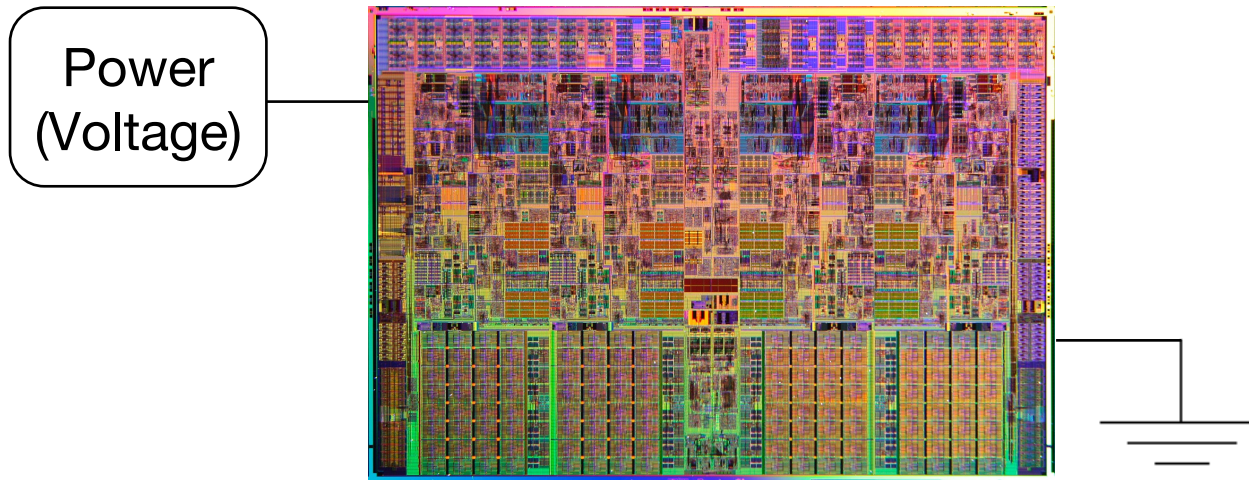
- Why Binary (bits)?
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- Representations in memory, pointers, strings

Everything is bits

- Each bit is 0 or 1. Bits are how programs talk to the hardware
- Programs encode instructions in bits
- Hardware then interprets the bits

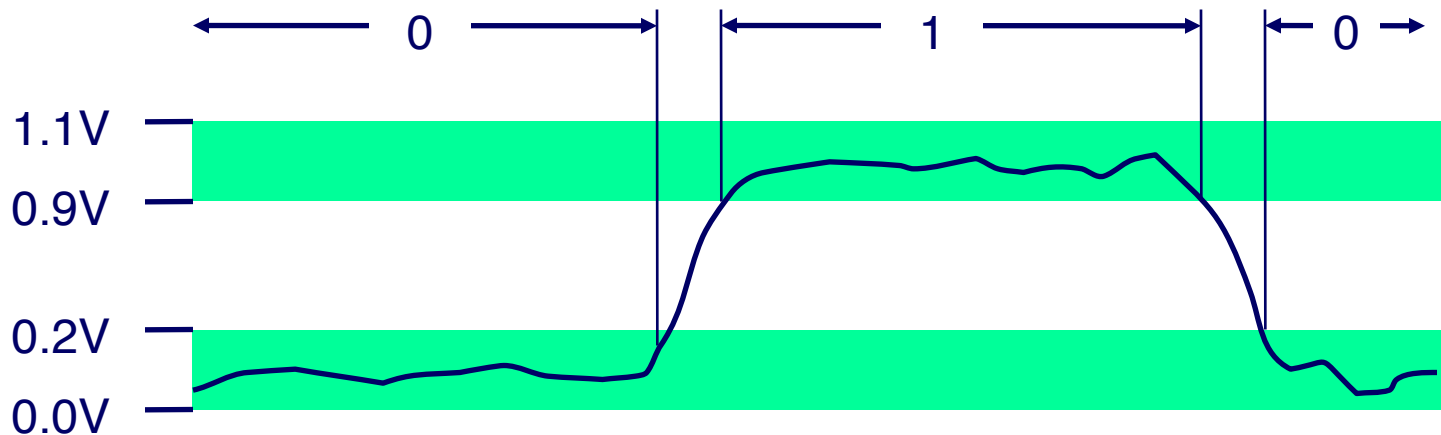
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- Why bits? Electronic Implementation



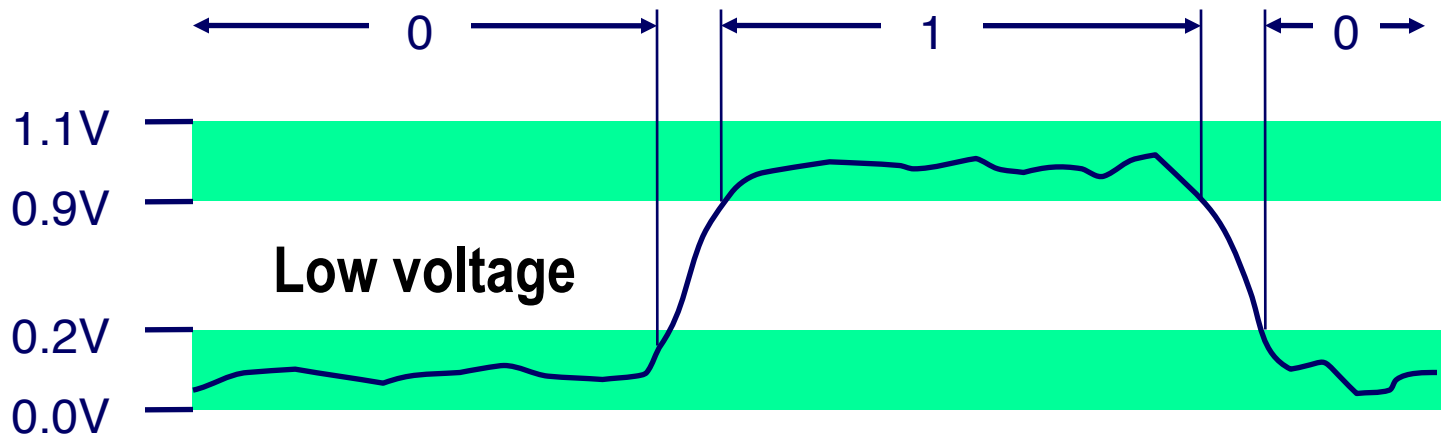
Why Bits?

- Each bit is 0 or 1. Bits are how programs talk to the hardware
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- Why bits? Electronic Implementation
 - Use high voltage to represent 1
 - Use low voltage to represent 0



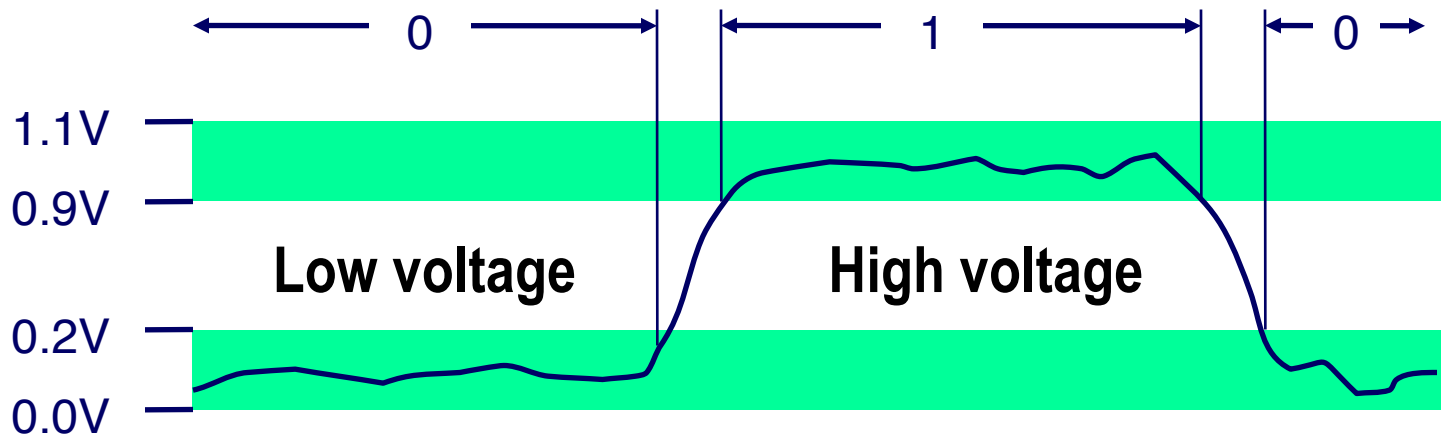
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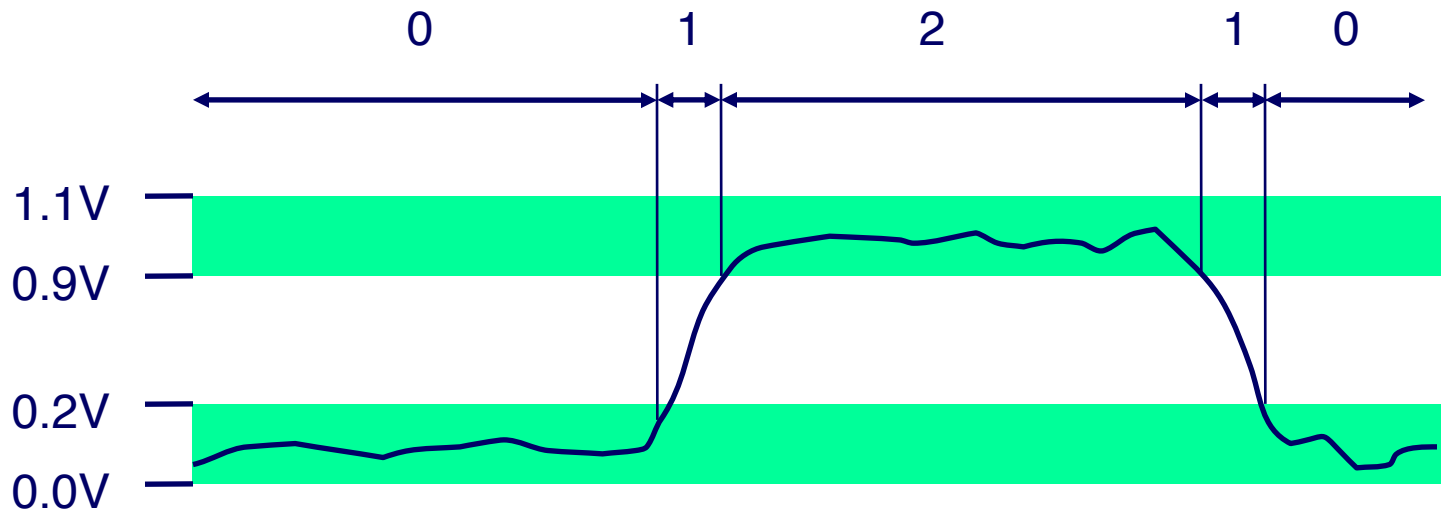
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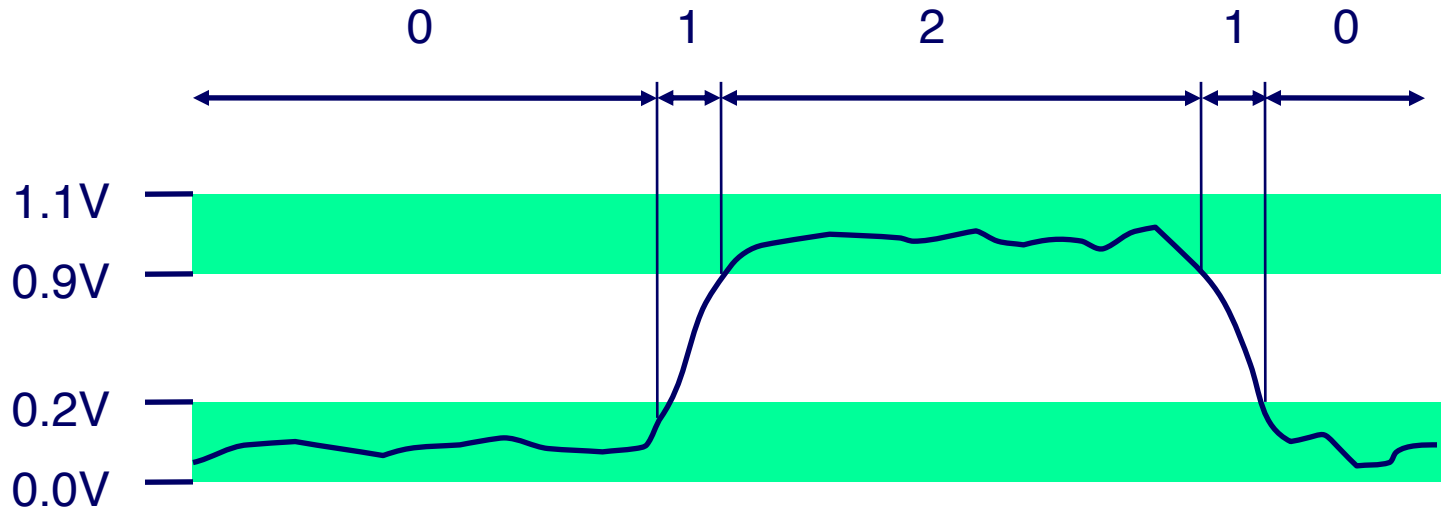
Why Limit Ourselves Only to Bits?

- Voltage is continuous. Why interpret it only as 0s and 1s?



Why Limit Ourselves Only to Bits?

- Voltage is continuous. Why interpret it only as 0s and 1s?
- Answer: Noise



Binary Notation

Binary Notation

- Base 2 Number Representation (Binary)

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- C.f., Base 10 number representation (Decimal)

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 - Each bit has a weight depending on its position

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- Binary Arithmetic

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Binary Notation

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$$\begin{array}{r} 0110 \\ + 0101 \\ \hline 1011 \end{array}$$

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
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- $b_3b_2b_1b_0 = b_0*2^0 + b_1*2^1 + b_2*2^2 + b_3*2^3$
- Binary Arithmetic

$$\begin{array}{r} 0110 \\ + 0101 \\ \hline 1011 \end{array}$$

$$\begin{array}{r} 6 \\ + 5 \\ \hline 11 \end{array}$$

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
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Hexadecimal (Hex) Notation

- **Base 16** Number Representation
 - Use characters '0' to '9' and 'A' to 'F'
 - Four bits per Hex digit
 - $11111110_2 = FE_{16}$
- Write $FA1D37B_{16}$ in C as
 - `0xFA1D37B`
 - `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

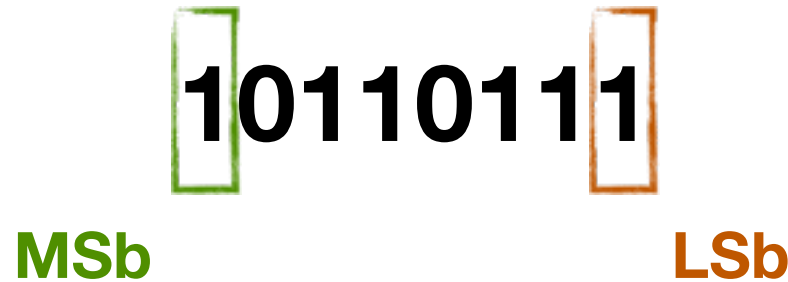
Bit, Byte, Word

- Byte = 8 bits

- Binary 00000000_2 to 11111111_2 ; Decimal: 0_{10} to 255_{10} ; Hex: 00_{16} to FF_{16}
- Least Significant Bit (LSb) vs. Most Significant Bit (MSb)

10110111

MSb **LSb**

The diagram shows the binary sequence '10110111'. The first bit '1' is enclosed in a green rectangular box, and the last bit '1' is enclosed in an orange rectangular box. Below the green box is the label 'MSb' in green text, and below the orange box is the label 'LSb' in orange text.

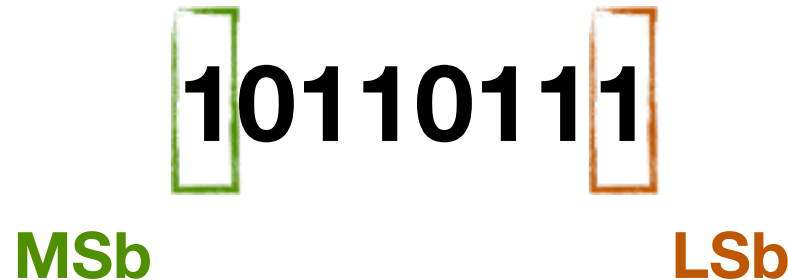
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- Word = 4 Bytes (32-bit machine) / 8 Bytes (64-bit machine)
- Least Significant Byte (LSB) vs. Most Significant Byte (MSB)

Today: Representing Information in Binary

- Why Binary (bits)?
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Boolean Algebra

- Developed by George Boole in 19th Century
 - Algebraic representation of logic
 - Encode “True” as 1 and “False” as 0

And

- $A \& B = 1$ when both $A=1$ and $B=1$

$\&$	0	1
0	0	0
1	0	1

Or

- $A | B = 1$ when either $A=1$ or $B=1$

$ $	0	1
0	0	1
1	1	1

Not

- $\sim A = 1$ when $A=0$

$\sim$	
0	1
1	0

Exclusive-Or (Xor)

- $A \wedge B = 1$ when either $A=1$ or $B=1$, but not both

$\wedge$	0	1
0	0	1
1	1	0

Bit Vector Operations

- Operate on Bit Vectors
 - Operations applied bitwise

01101001	01101001	01101001	
& 01010101	01010101	^ 01010101	~ 01010101

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<u> </u>	<u> </u>	<u> </u>	<u> </u>
01000001			

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01000001	01111101		

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01000001	01111101	00111100	10101010

Bit-Level Operations in C

- Operations `&`, `|`, `~`, `^` Available in C
 - Apply to any “integral” data type
 - `long`, `int`, `short`, `char`, `unsigned`
 - View arguments as bit vectors
 - Arguments applied bit-wise
- Examples (Char data type)
 - `~0x41 → 0xBE`
 - `~010000012 → 101111102`
 - `~0x00 → 0xFF`
 - `~000000002 → 111111112`
 - `0x69 & 0x55 → 0x41`
 - `011010012 & 010101012 → 010000012`
 - `0x69 | 0x55 → 0x7D`
 - `011010012 | 010101012 → 011111012`

Contrast: Logic Operations in C

- Contrast to Logical Operators
 - `&&`, `||`, `!`
 - View 0 as “False”
 - Anything nonzero as “True”
 - Always return 0 or 1
 - Early termination (e.g., `0 && 1 && 1`)
- Examples (char data type)
 - `!0x41` → `0x00`
 - `!0x00` → `0x01`
 - `!!0x41` → `0x01`
 - `0x69 && 0x55` → `0x01`
 - `0x69 || 0x55` → `0x01`
 - `p && *p` (avoids null pointer access)

Shift Operations

- Left Shift: $x \ll y$
 - Shift bit-vector x left y positions
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 - Replicate most significant bit on left
- Undefined Behavior
 - Shift amount $\geq$ total amount of bits

Argument x	01100010
$\ll$ 3	
Log. $\gg$ 2	
Arith. $\gg$ 2	

Argument x	10100010
$\ll$ 3	
Log. $\gg$ 2	
Arith. $\gg$ 2	

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$\ll$ 3	00010
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Argument x	01100010
<< 3	00010000
Log. >> 2	
Arith. >> 2	

Argument x	10100010
<< 3	
Log. >> 2	
Arith. >> 2	

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Arith. >> 2	

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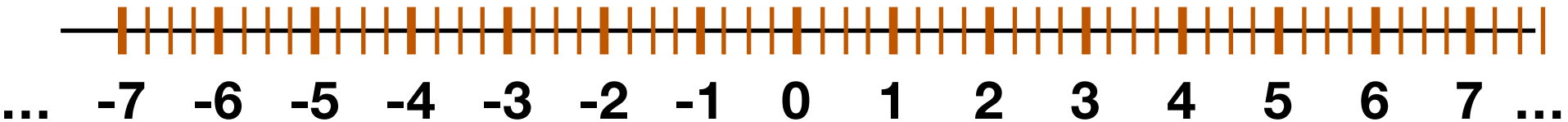
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Representing Numbers in Binary

- Different types of number
 - Integer (Negative and Non-negative)
 - Fractions



Encoding Negative Numbers

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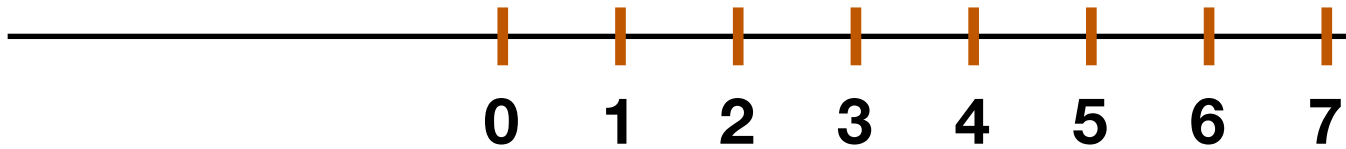
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- Solution 1: Sign-magnitude
 - First bit represents sign; 0 for positive; 1 for negative
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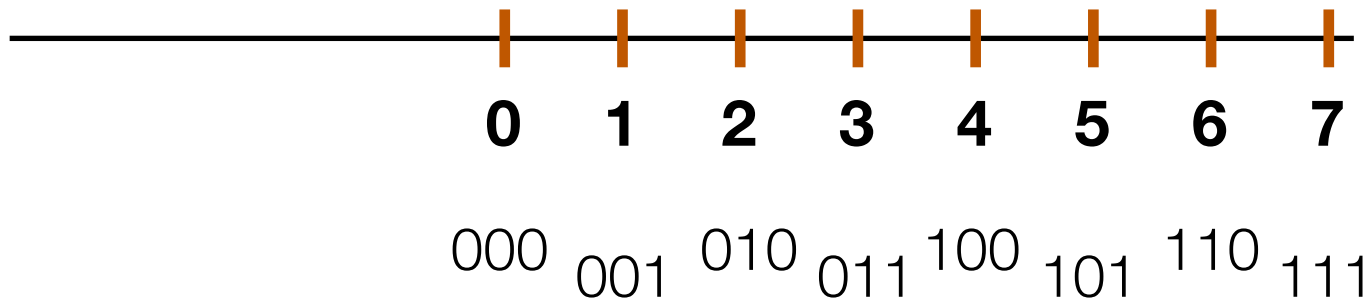
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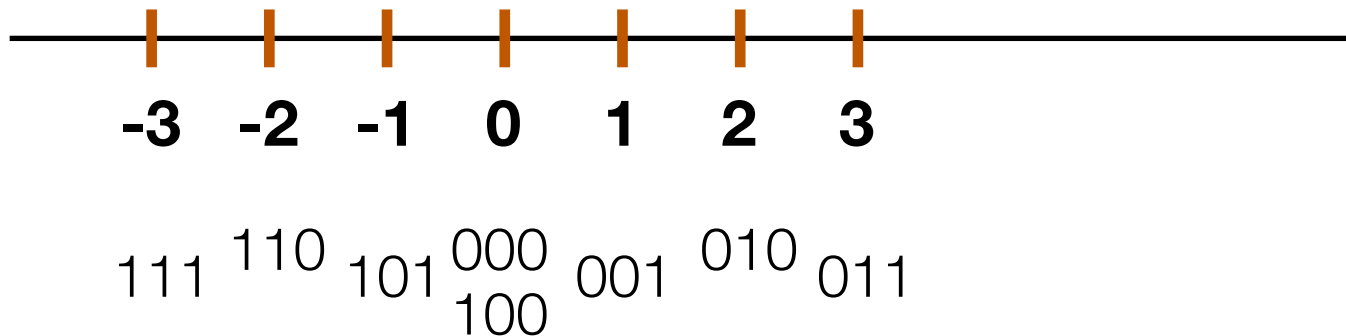
Encoding Negative Numbers

- So far we have been discussing non-negative numbers: so called **unsigned**. How about negative numbers?
- Solution 1: Sign-magnitude
 - First bit represents sign; 0 for positive; 1 for negative
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Sign-Magnitude Implications

- Bits have different semantics
 - Two zeros...
 - Normal arithmetic doesn't work
 - Make hardware design harder

Signed Value	Binary
0	000
1	001
2	010
3	011
-0	100
-1	101
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$$\begin{array}{r} 010 \\ +) 101 \\ \hline 111 \end{array}$$

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The diagram shows two arithmetic operations, each crossed out with a large red 'X'.
Left operation:
$$\begin{array}{r} 010 \\ +) 101 \\ \hline 111 \end{array}$$

Right operation:
$$\begin{array}{r} 2 \\ +) -1 \\ \hline -3 \end{array}$$

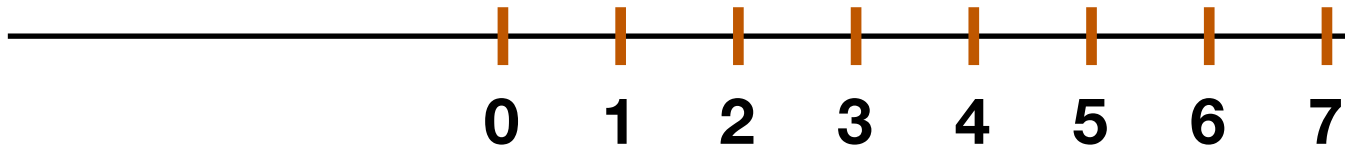
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Encoding Negative Numbers

- Solution 2: Two's Complement

Encoding Negative Numbers

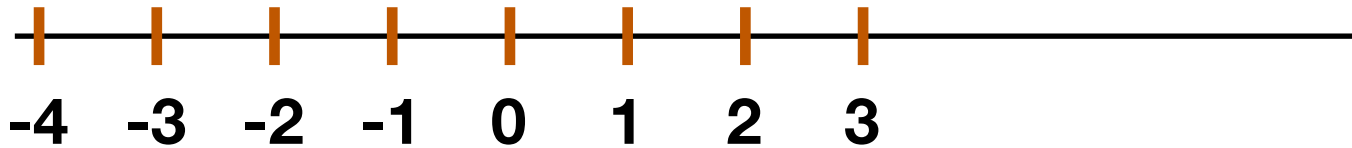
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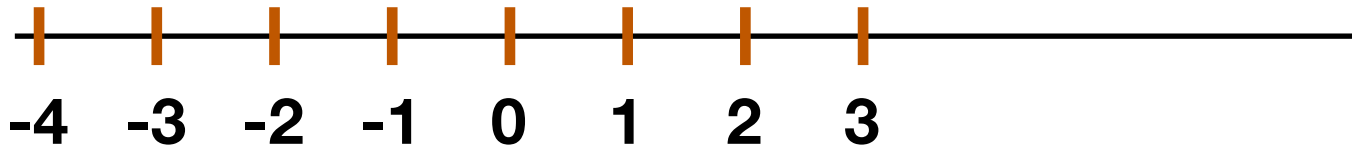
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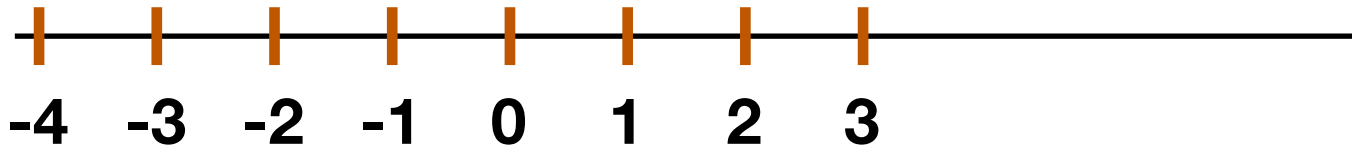
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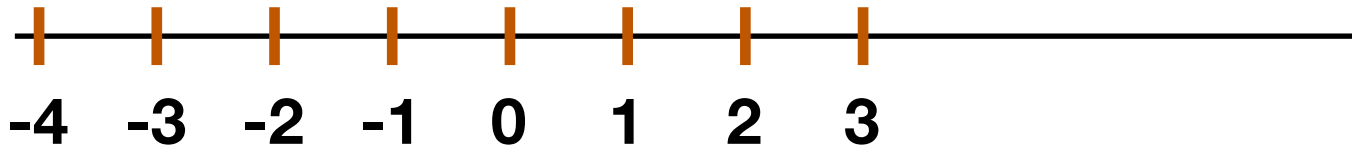


Signed Weight	Unsigned Weight	Bit Position
2^0	2^0	0
2^1	2^1	1
-2^2	2^2	2

Signed	Unsigned	Binary
0	0	000
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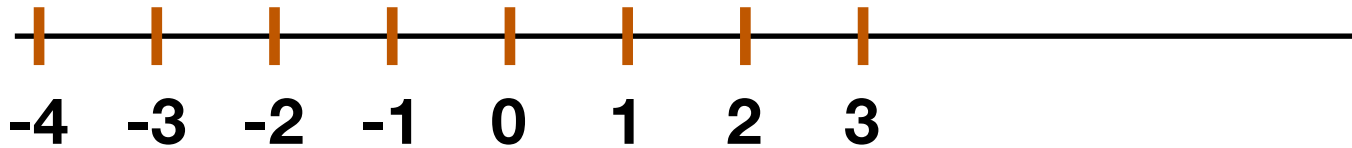


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$$101_2 = 1 \cdot 2^0 + 0 \cdot 2^1 - 1 \cdot 2^2 = -3_{10}$$

Two-Complement Encoding Example

x = 15213: 00111011 01101101
y = -15213: 11000100 10010011

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum	15213		-15213	

Two-Complement Implications

- Only 1 zero
- Usual arithmetic still works
- There is a bit that represents sign!
- Most widely used in today's machines

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- $UMin = 0$
000...0

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111...1

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Values for $W = 16$

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

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- Other Values

- Minus 1
111...1

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Data Representations in C (in Bytes)

- By default variables are signed
- Unless explicitly declared as unsigned (e.g., `unsigned int`)
- Signed variables use two-complement encoding

C Data Type	32-bit	64-bit
<code>char</code>	1	1
<code>short</code>	2	2
<code>int</code>	4	4
<code>long</code>	4	8

Data Representations in C (in Bytes)

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

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- C Language
 - `#include <limits.h>`
 - Declares constants, e.g.,
 - `ULONG_MAX`
 - `LONG_MAX`
 - `LONG_MIN`
 - Values platform specific

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- **Integers**
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- Representations in memory, pointers, strings

One Bit Sequence, Two Interpretations

- A sequence of bits can be interpreted as either a signed integer or an unsigned integer

Signed	Unsigned	Binary
0	0	000
1	1	001
2	2	010
3	3	011
-4	4	100
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Signed vs. Unsigned Conversion in C

- What happens when we convert between signed and unsigned numbers?
- Casting (In C terminology)
 - Explicit casting between signed & unsigned

```
int tx, ty = -3;  
unsigned ux = 7, uy;  
tx = (int) ux; // U2T  
uy = (unsigned) ty; // T2U
```

- Implicit casting
 - e.g., assignments, function calls

```
tx = ux;  
uy = ty;
```

Mapping Between Signed & Unsigned

- Mappings between unsigned and two's complement numbers: **Keep bit representations and reinterpret**

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Mapping Signed $\leftrightarrow$ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

Mapping Signed $\leftrightarrow$ Unsigned

Bits	Signed		Unsigned
0000	0	→ T2U →	0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
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0011	3		3
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0101	5		5
0110	6		6
0111	7		7
1000	-8	$\leftarrow$ U2T $\leftarrow$	8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

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1101	-3	13
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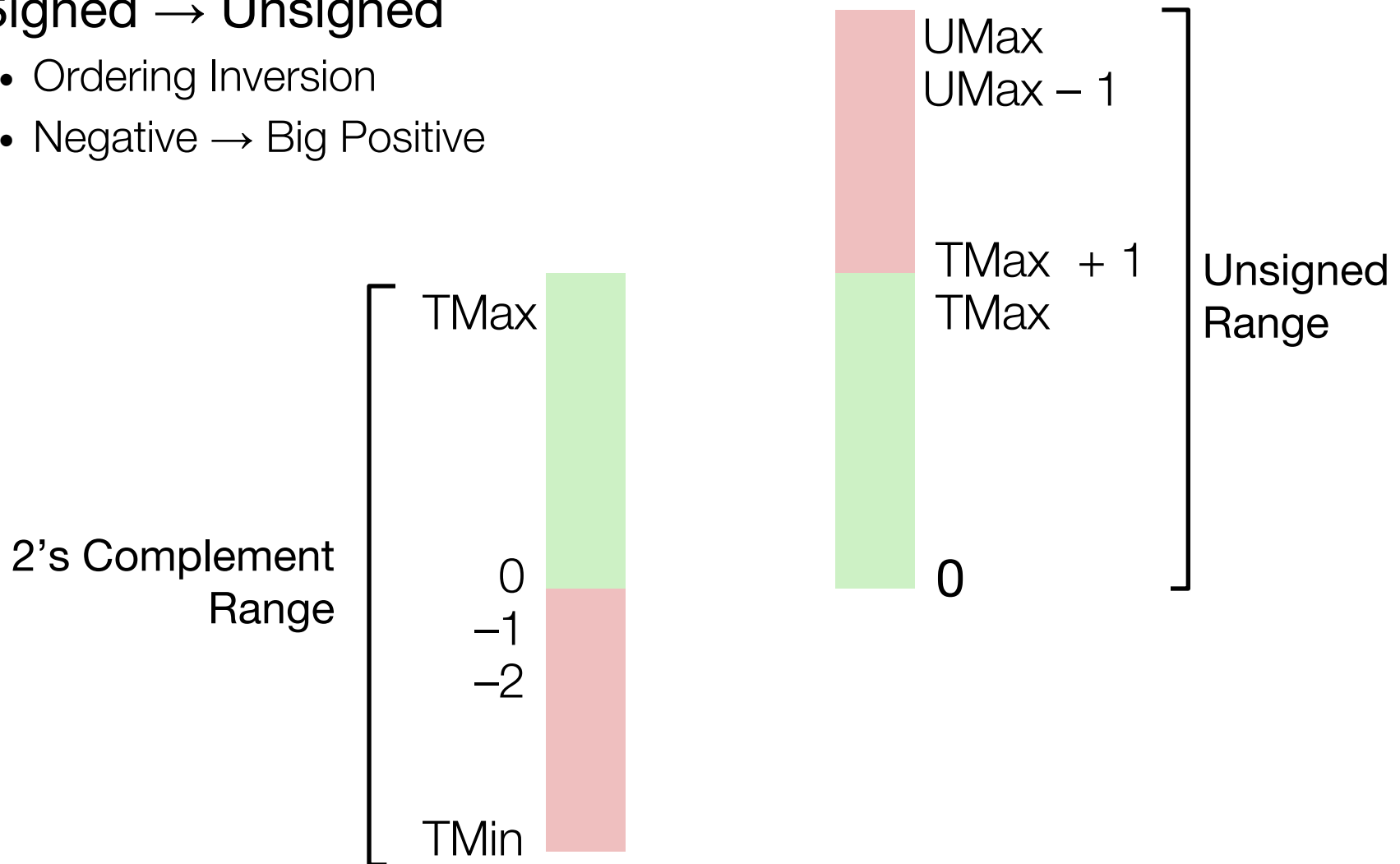


Mapping Signed $\leftrightarrow$ Unsigned

Bits	Signed		Unsigned
0000	0	$\longleftrightarrow$ = $\longleftrightarrow$	0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8	$\longleftrightarrow$ +/- 16 $\longleftrightarrow$	8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Conversion Visualized

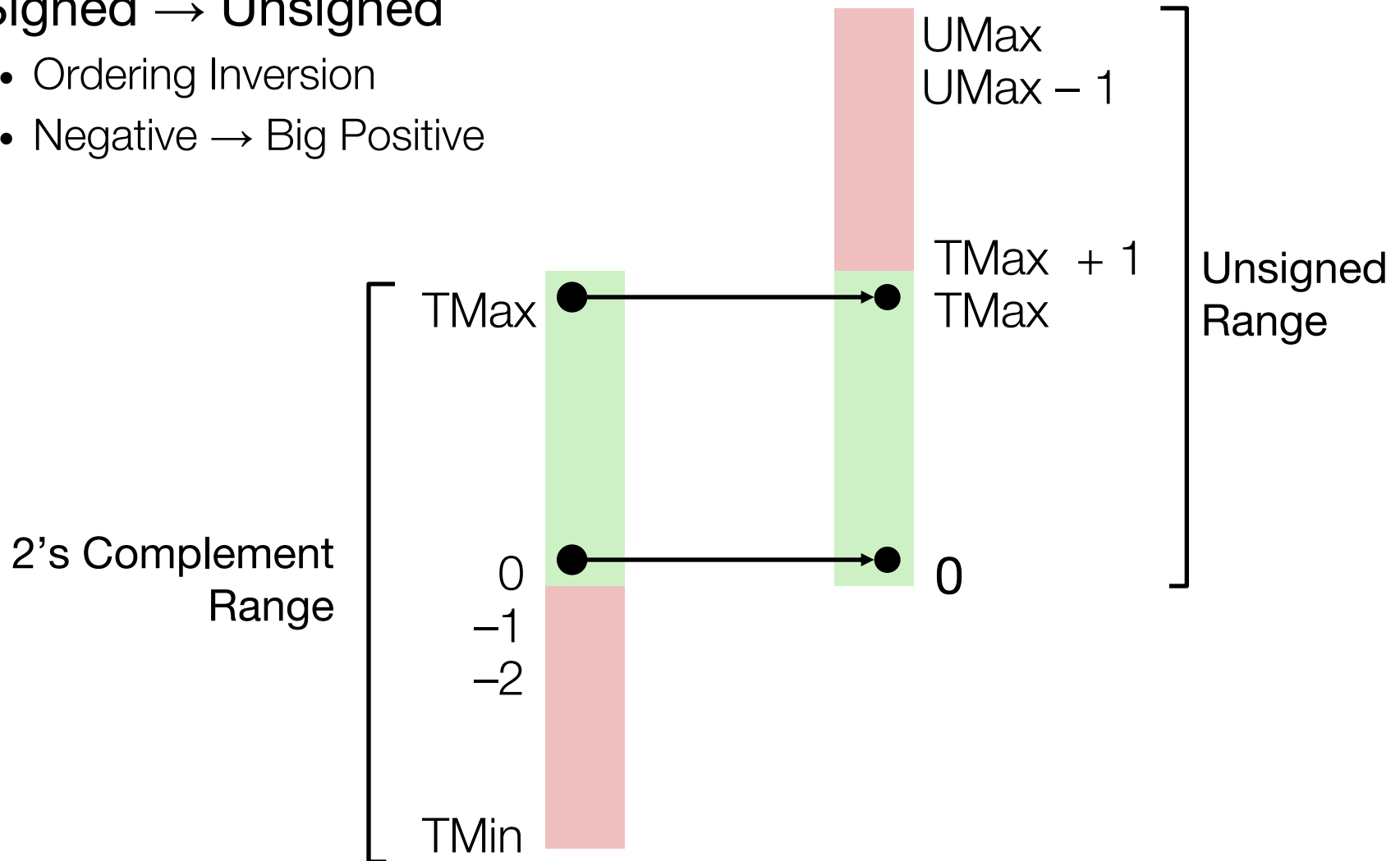
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