CSC 252/452: Computer Organization
Fall 2025: Lecture 2

Instructor: Yanan Guo
Department of Computer Science
University of Rochester

Action Items:

- **Get CSUG account**
- **Make sure you have VPN setup!!!!**

Announcement

- Make sure you can access CSUG machines!!!
- Programming assignment 1 will be posted today.
  - It is in C language. Seek help from TAs.
  - TAs are best positioned to answer your questions about programming assignments!!!
- Programming assignments do NOT repeat the lecture materials. They ask you to synthesize what you have learned from the lectures and work out something new.

Previously in 252…

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

Microarchitecture

Circuit

Previously in 252…

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

ISA is the contract between software and hardware.

Microarchitecture

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Previously in 252…

Problem

Algorithm

	Renting	Computing
Service provider	Landlord	Hardware
Service receiver	YOU	Software
Contract	Lease	ISA
Contract's language	Natural language (e.g., English)	Assembly programming language

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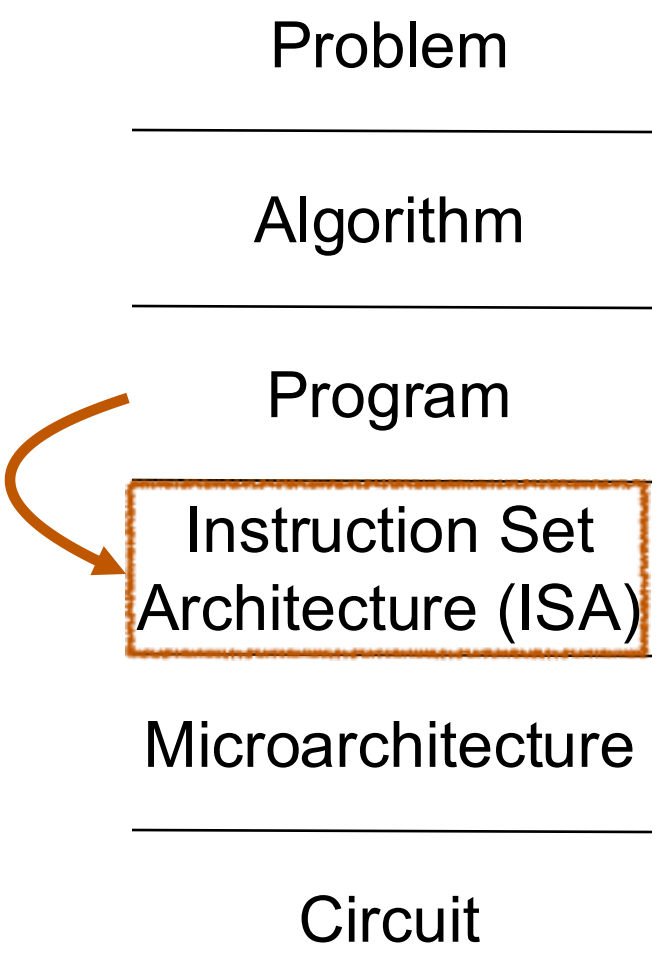
Circuit

Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?



ISA is the contract between software and hardware.

Previously in 252…

C Program

```
void add() {
  int a = 1;
  int b = 2;
  int c = a + b;
}
```

Assembly program

```
movl    $1, -4(%rbp)
movl    $2, -8(%rbp)
movl    -4(%rbp), %eax
addl    -8(%rbp), %eax
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Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Everything is bits

- Each bit is 0 or 1. Bits are how programs talk to the hardware
- Programs encode instructions in bits
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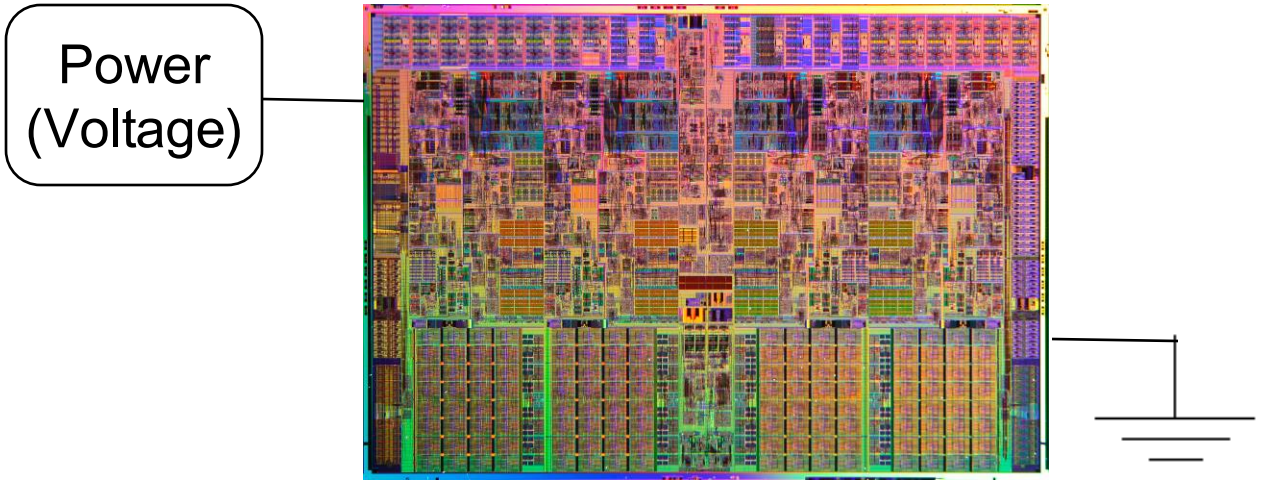

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  - Use high voltage to represent 1
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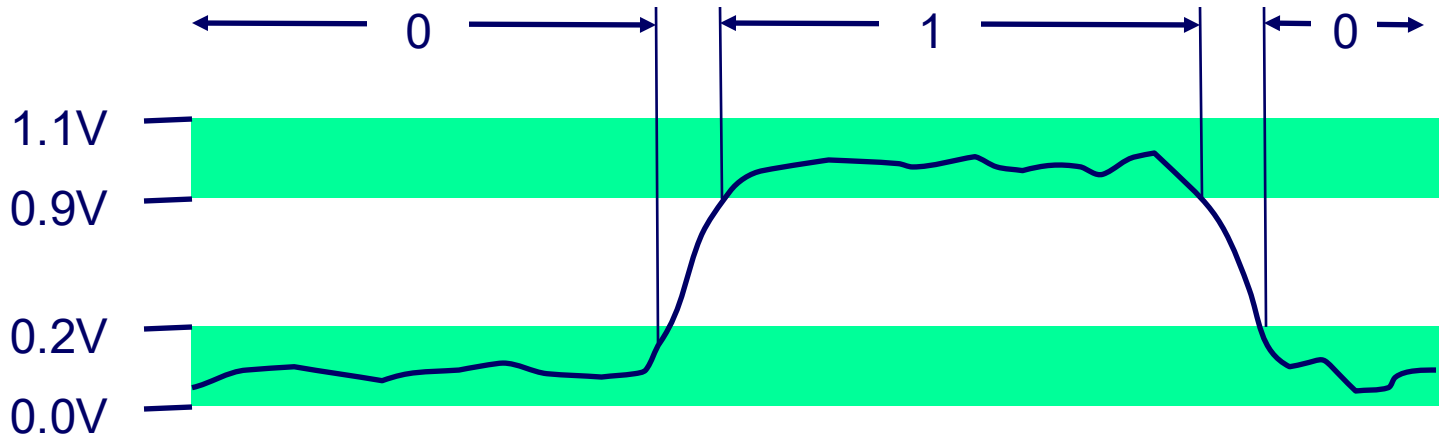

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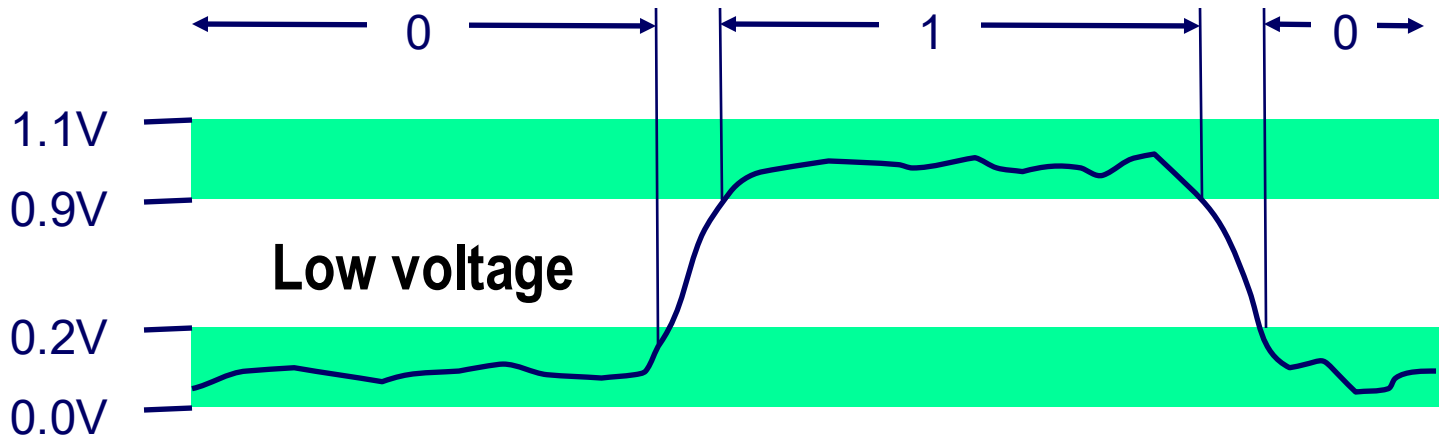

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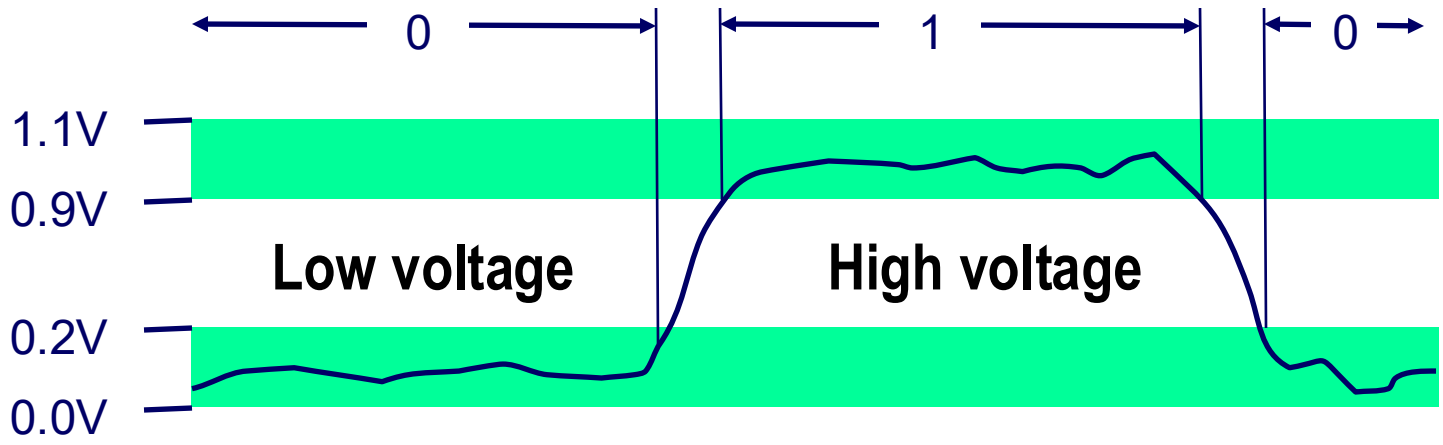
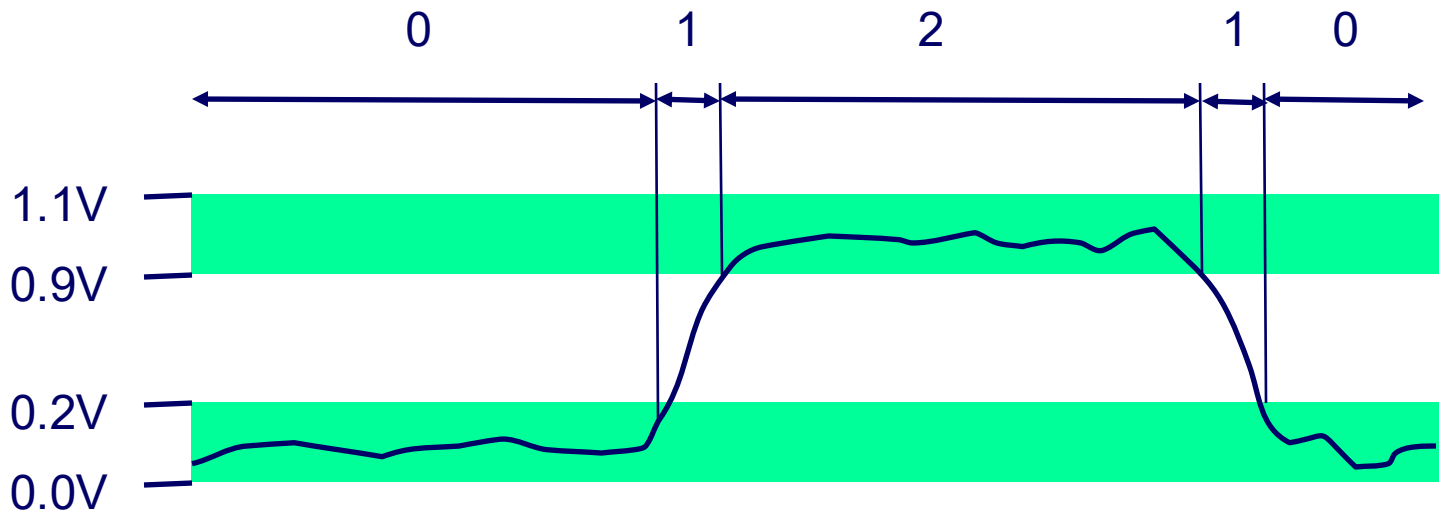


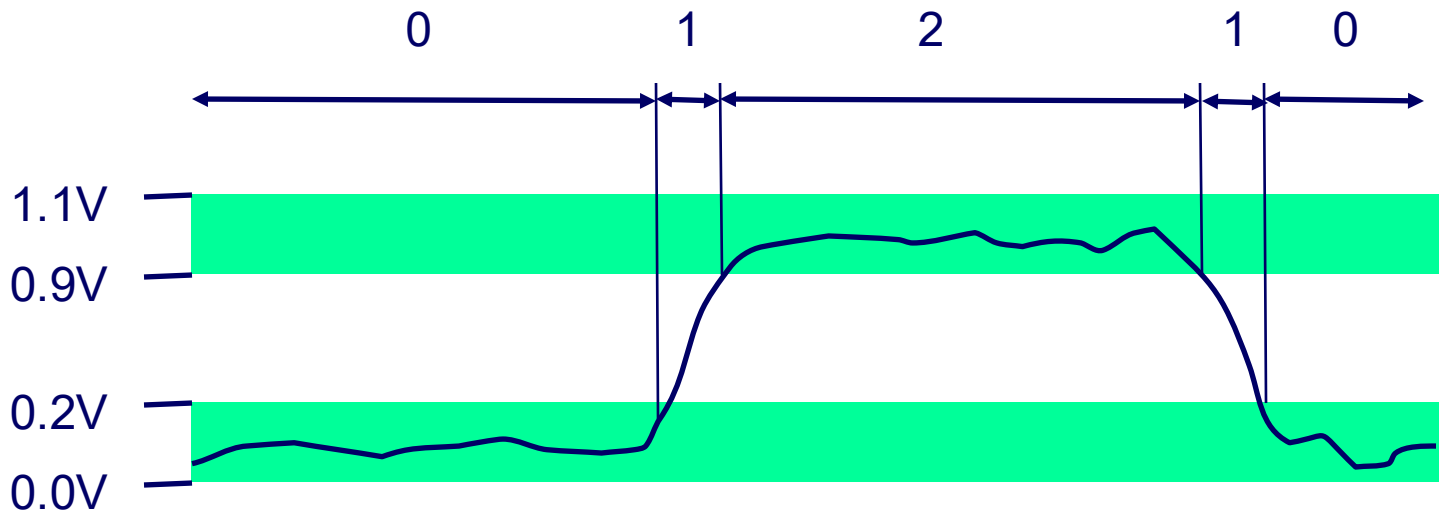
Why Limit Ourselves Only to Bits?

- Voltage is continuous. Why interpret it only as 0s and 1s?



Why Limit Ourselves Only to Bits?

- Voltage is continuous. Why interpret it only as 0s and 1s?
- Answer: Noise



Binary Notation

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- Base 2 Number Representation (Binary)

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  - Each bit has a weight depending on its position

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- Binary Arithmetic

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

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- Binary Arithmetic

```
  0110
+ 0101
------
  1011
```

Decimal	Binary
0	0000
1	0001
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- Binary Arithmetic

```
  0110
+ 0101
------
  1011
```

```
  6
+ 5
---
 11
```

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
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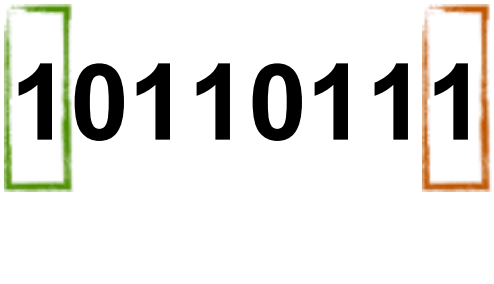
Hexdecimal (Hex) Notation

- Base 16 Number Representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Four bits per Hex digit
  - $11111110_2 = FE_{16}$
- Write $FA1D37B_{16}$ in C as
  - 0xFA1D37B
  - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit, Byte, Word

- Byte = 8 bits
  - Binary 00000000_2 to $11111111_{2;}$ Decimal: 0_{10} to 255_{10}; Hex: 00_{16} to FF_{16}
  - Least Significant Bit (LSb) vs. Most Significant Bit (MSb)



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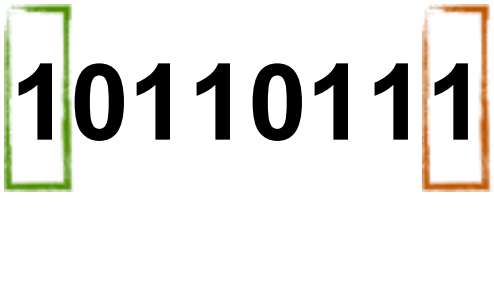

- Word = 4 Bytes (32-bit machine) / 8 Bytes (64-bit machine)
  - Least Significant Byte (LSB) vs. Most Significant Byte (MSB)

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Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Exclusive-Or (Xor)

- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

Bit Vector Operations

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
```

Bit Vector Operations

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  01101001     01101001     01101001
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  --------     --------     --------     --------
  01000001
```

Bit Vector Operations

- Operate on Bit Vectors
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```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------
  01000001     01111101
```

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  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
----------   ----------   ----------   ----------
  01000001     01111101     00111100     10101010
```

Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any "integral" data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - ~0x41 → 0xBE
    - $\sim 01000001_2 \rightarrow 10111110_2$
  - ~0x00 → 0xFF
    - $\sim 00000000_2 \rightarrow 11111111_2$
  - 0x69 & 0x55 → 0x41
    - $01101001_2 \;\&\; 01010101_2 \rightarrow 01000001_2$
  - 0x69 | 0x55 → 0x7D
    - $01101001_2 \mid 01010101_2 \rightarrow 01111101_2$

Contrast: Logic Operations in C

- Contrast to Logical Operators
  - &&, ||, !
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination (e.g., 0 && 1 && 1)
- Examples (char data type)
  - !0x41 → 0x00
  - !0x00 → 0x01
  - !!0x41 → 0x01

  - 0x69 && 0x55 → 0x01
  - 0x69 || 0x55 → 0x01
  - p && *p (avoids null pointer access)

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
    - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
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  - Arithmetic shift
    - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount ≥ total amount of bits

Argument x	01100010
<< 3	
Log. >> 2	
Arith. >> 2	

Argument x	10100010
<< 3	
Log. >> 2	
Arith. >> 2	

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Arith. >> 2	

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Arith. >> 2	

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Log. >> 2	
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Arith. >> 2	101000

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Arith. >> 2	*00*011000

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Arith. >> 2	*11*101000

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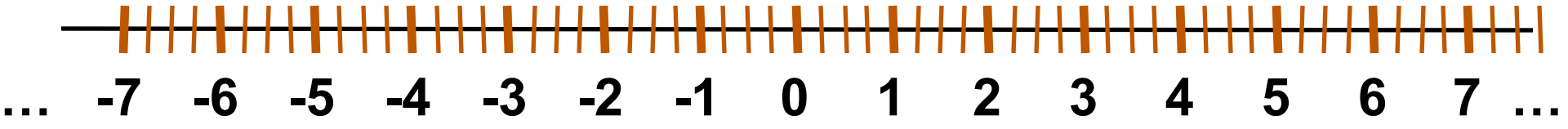
Representing Numbers in Binary

- Different types of number
  - Integer (Negative and Non-negative)
  - Fractions



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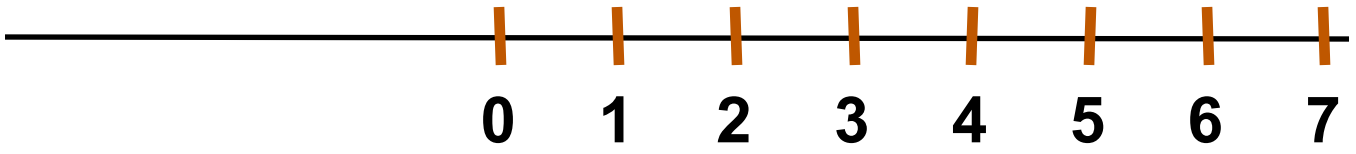

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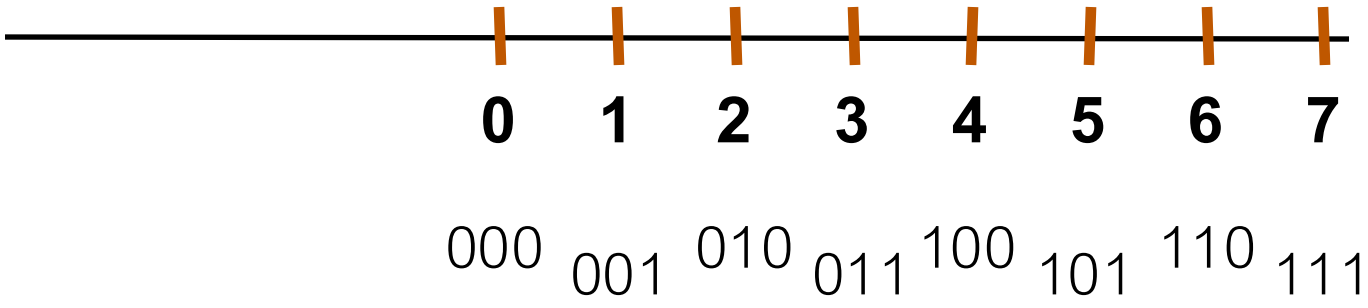

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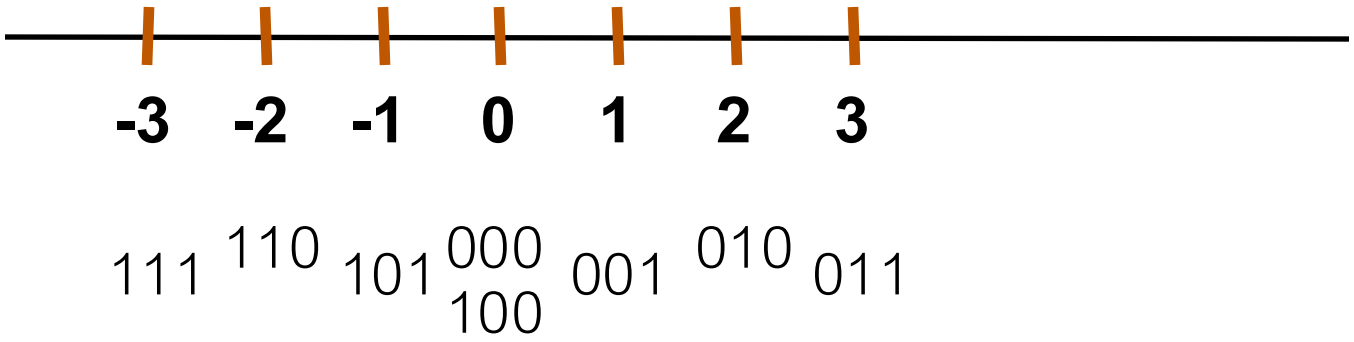

Sign-Magnitude Implications

- Bits have different semantics
  - Two zeros…
  - Normal arithmetic doesn't work
  - Make hardware design harder

Signed Value	Binary
0	000
1	001
2	010
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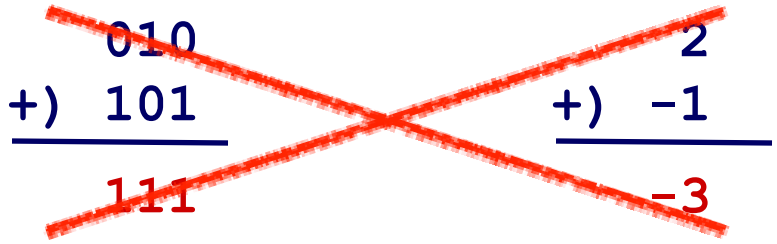

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- Solution 2: Two's Complement

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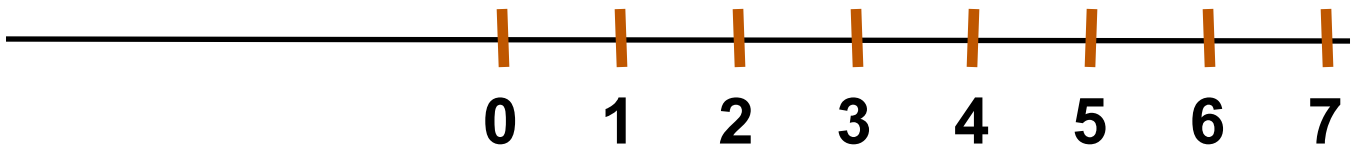

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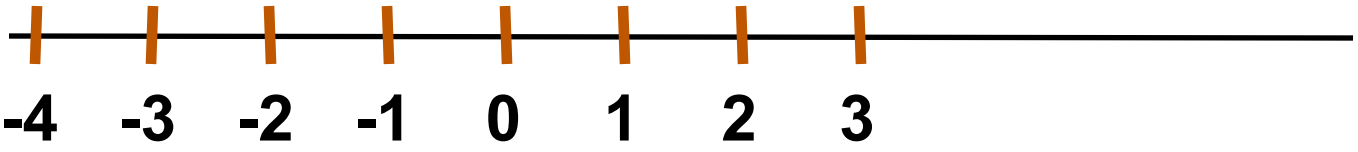

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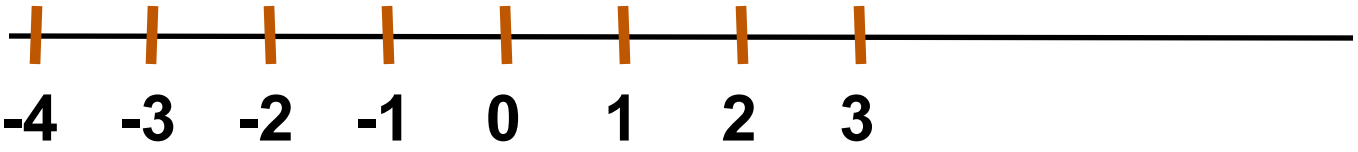

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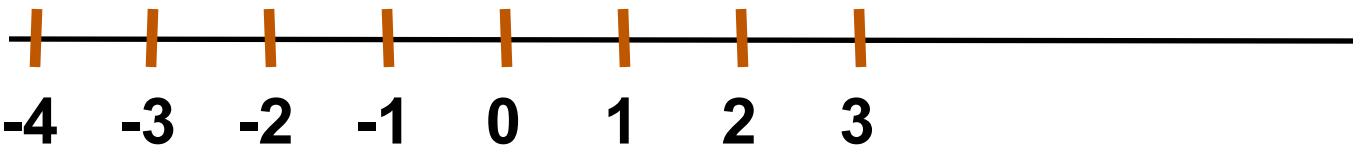

Signed Weight	Unsigned Weight	Bit Position
2^0	2^0	0
2^1	2^1	1
-2^2	2^2	2

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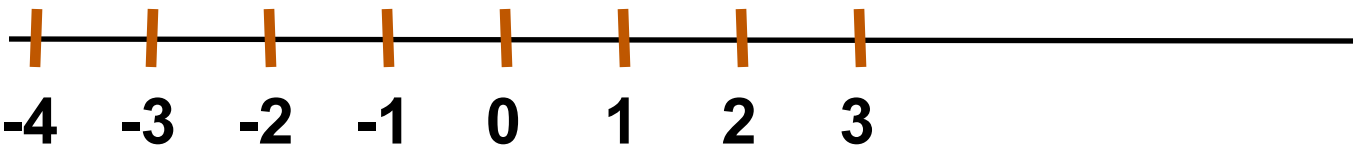

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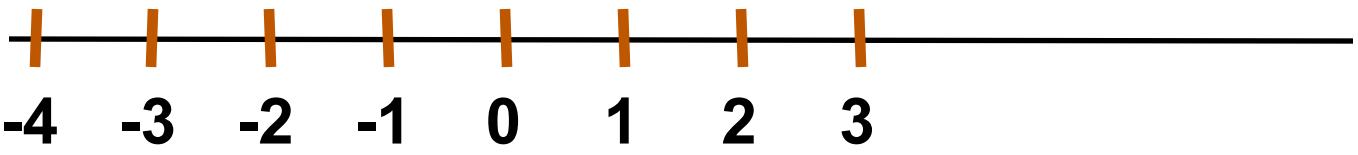

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$$101_2 = 1*2^0 + 0*2^1 - 1*2^2 = -3_{10}$$

Two-Complement Encoding Example

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Two-Complement Implications

- Only 1 zero
- Usual arithmetic still works
- There is a bit that represents sign!
- Most widely used in today's machines

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-----        ----
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```

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- Unsigned Values
  - *UMin* = 0

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Values for *W* = 16

	Decimal	Hex	Binary
UMax	**65535**	**FF FF**	**11111111 11111111**
TMax	**32767**	**7F FF**	**01111111 11111111**
TMin	**-32768**	**80 00**	**10000000 00000000**
-1	**-1**	**FF FF**	**11111111 11111111**
0	**0**	**00 00**	**00000000 00000000**

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    011…1
- Other Values
  - Minus 1
    111…1

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0	**0**	**00 00**	**00000000 00000000**

Data Representations in C (in Bytes)

- By default variables are signed
- Unless explicitly declared as unsigned (e.g., `unsigned int`)
- Signed variables use two-complement encoding

C Data Type	32-bit	64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8

Data Representations in C (in Bytes)

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
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- C Language
  - `#include <limits.h>`
  - Declares constants, e.g.,
    - `ULONG_MAX`
    - `LONG_MAX`
    - `LONG_MIN`
  - Values platform specific

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

One Bit Sequence, Two Interpretations

- A sequence of bits can be interpreted as either a signed integer or an unsigned integer

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Signed vs. Unsigned Conversion in C

- What happens when we convert between signed and unsigned numbers?
- Casting (In C terminology)
  - Explicit casting between signed & unsigned

```
int tx, ty = -3;
unsigned ux = 7, uy;
tx = (int) ux; // U2T
uy = (unsigned) ty; // T2U
```

  - Implicit casting
    - e.g., assignments, function calls

```
tx = ux;
uy = ty;
```

Mapping Between Signed & Unsigned

- Mappings between unsigned and two's complement numbers: Keep bit representations and reinterpret

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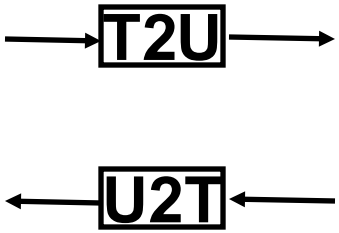

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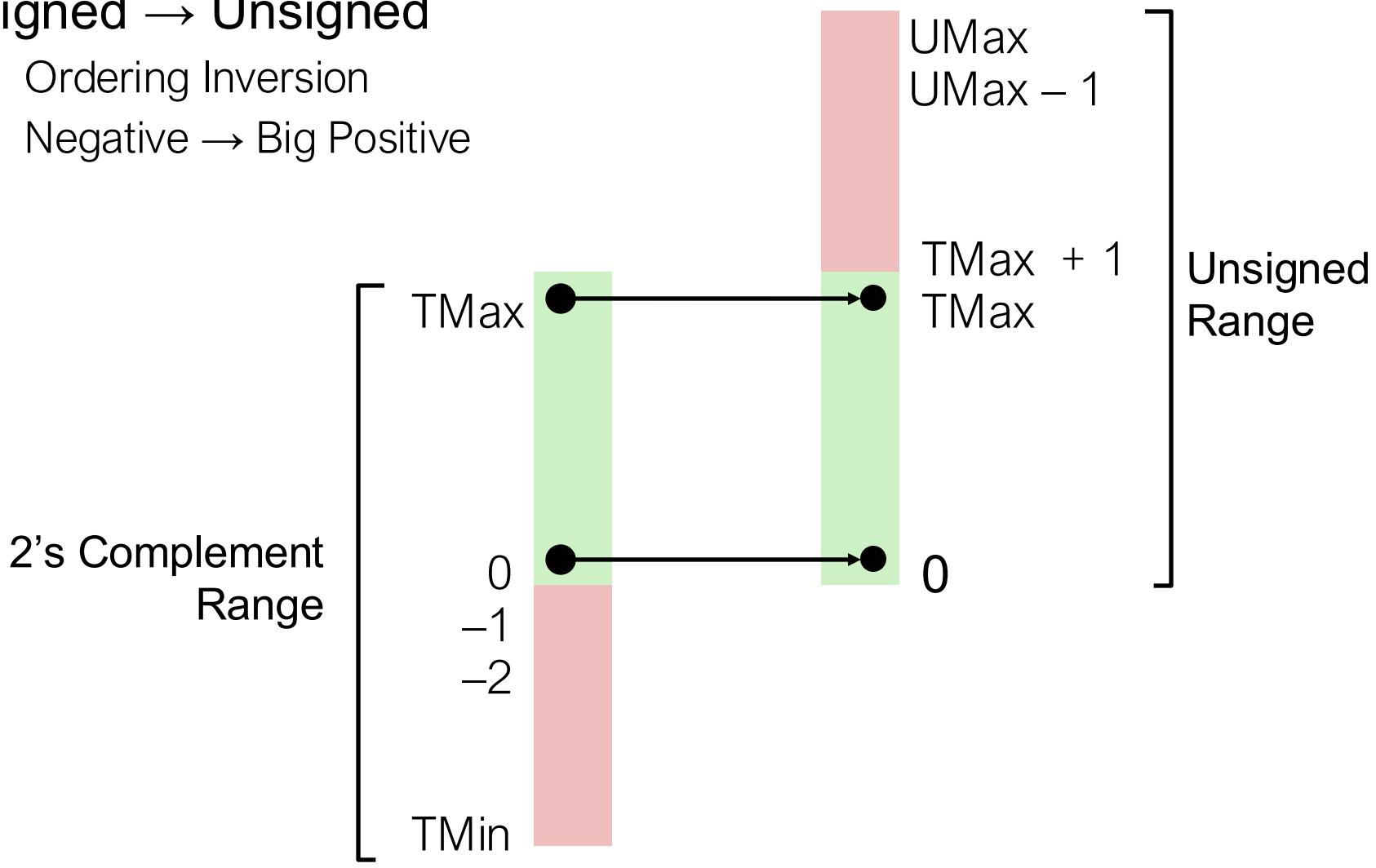

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