CSC 252 Computer Organization

The Memory Hierarchy Part III

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Professor

Guest lecture
March 28, 2019

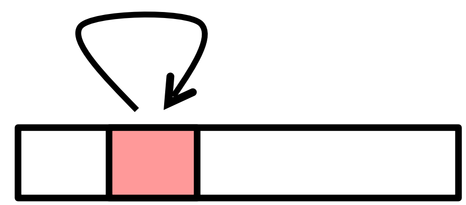
Required reading:

Section 6.5 — 6.7

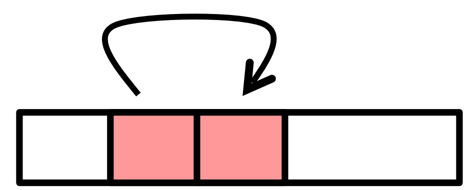
Locality

- Principle of Locality: Programs tend to use data and instructions with addresses near or equal to those they have used recently

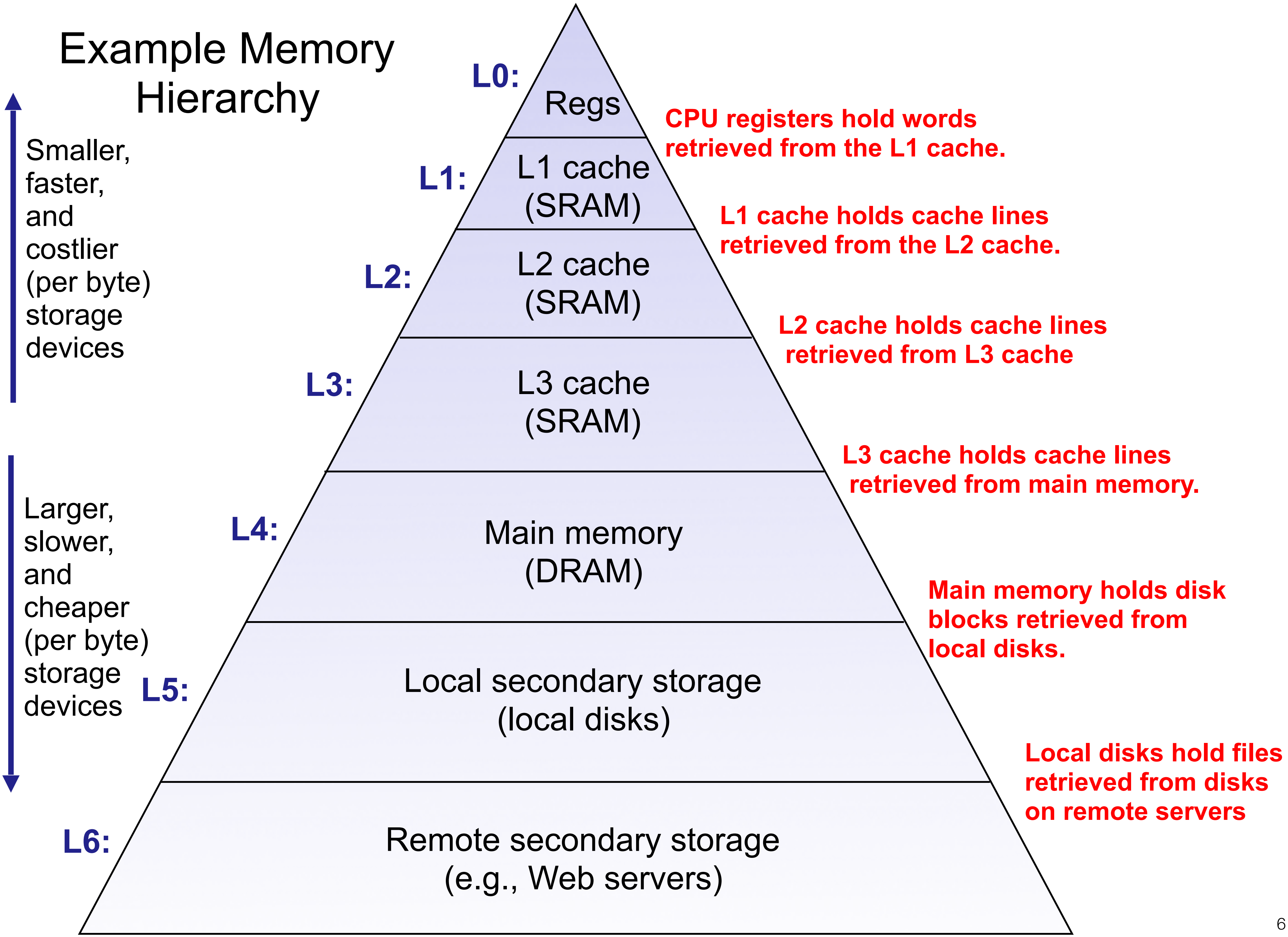
- Temporal locality:
  - Recently referenced items are likely to be referenced again in the near future



- Spatial locality:
  - Items with nearby addresses tend to be referenced close together in time



Example Memory Hierarchy

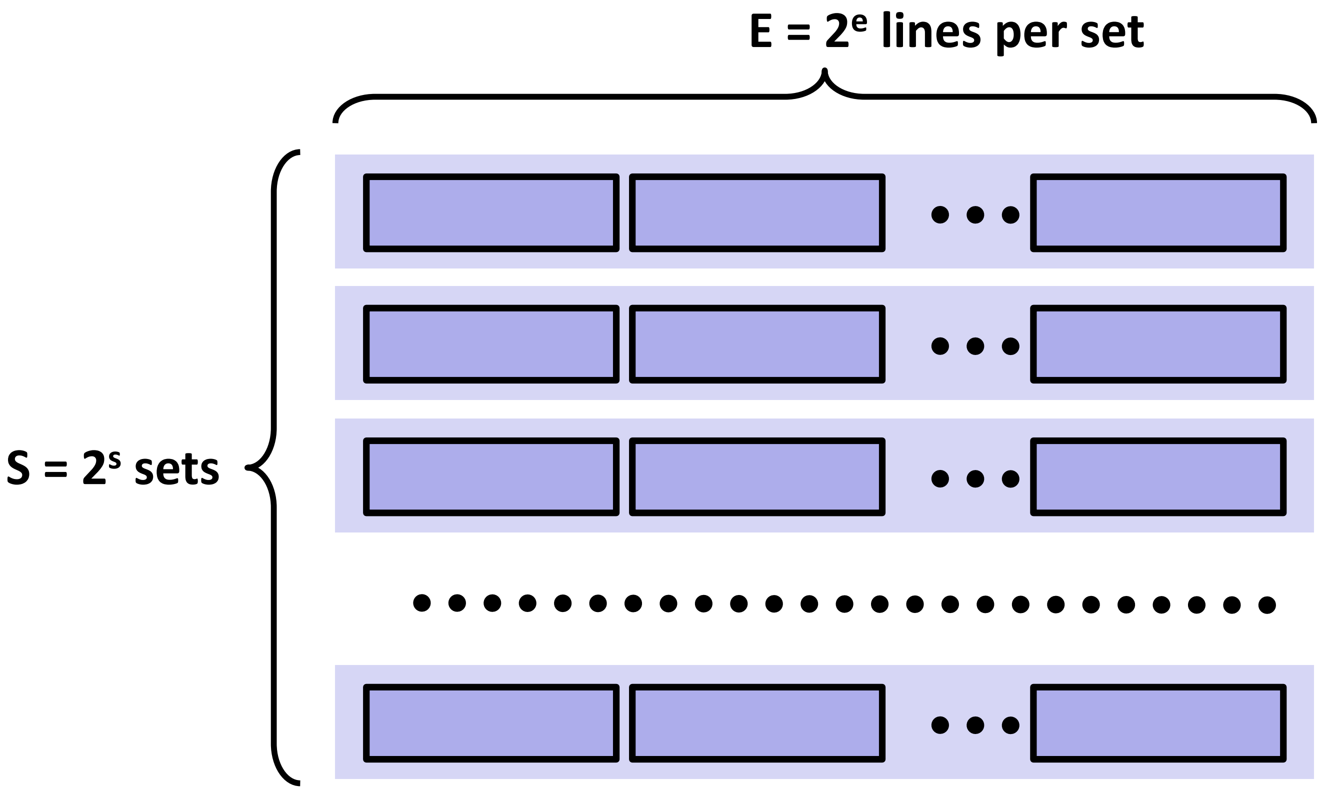


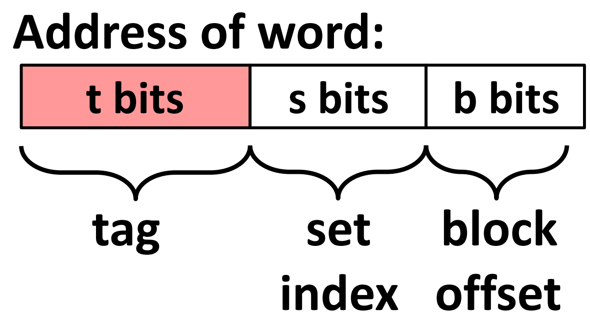
Today

- Review: Cache memory organization and operation
- Performance impact of caches
  - Analytical Model
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

Optimizing Irregular and dynamic applications [32]

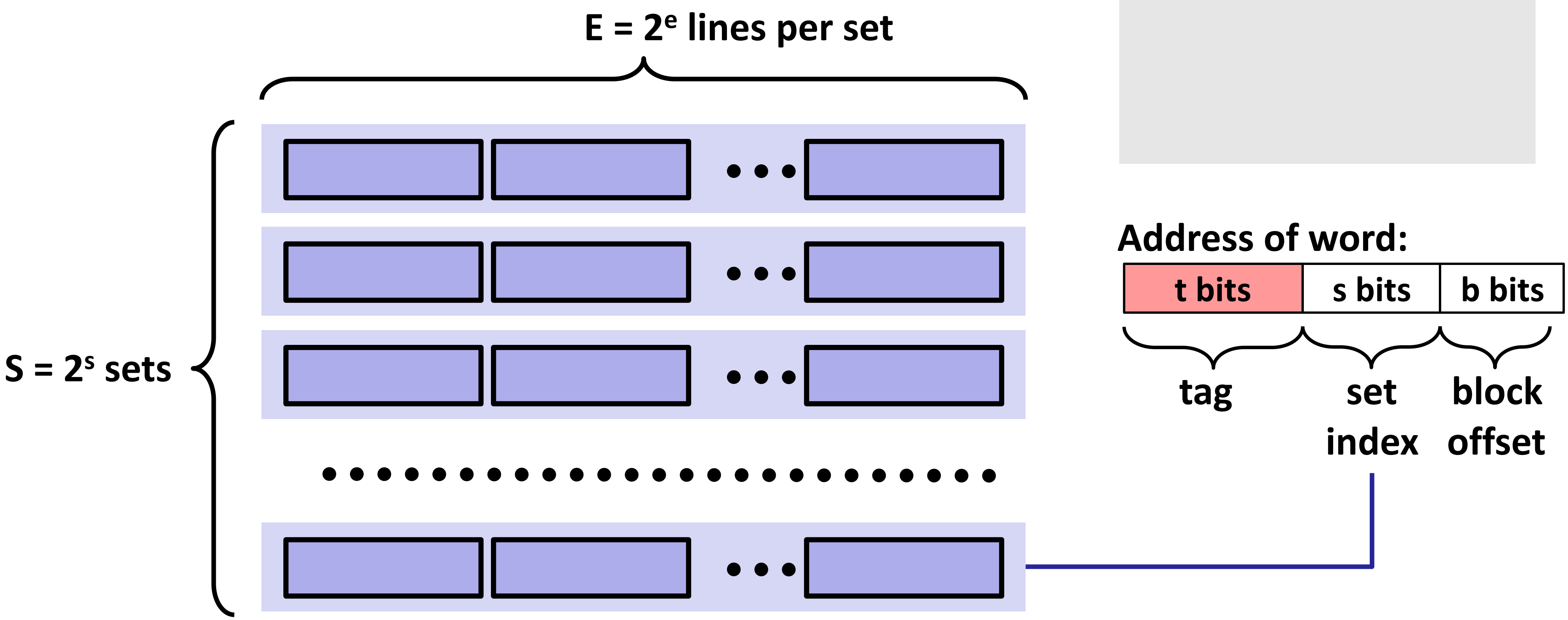
Cache Access





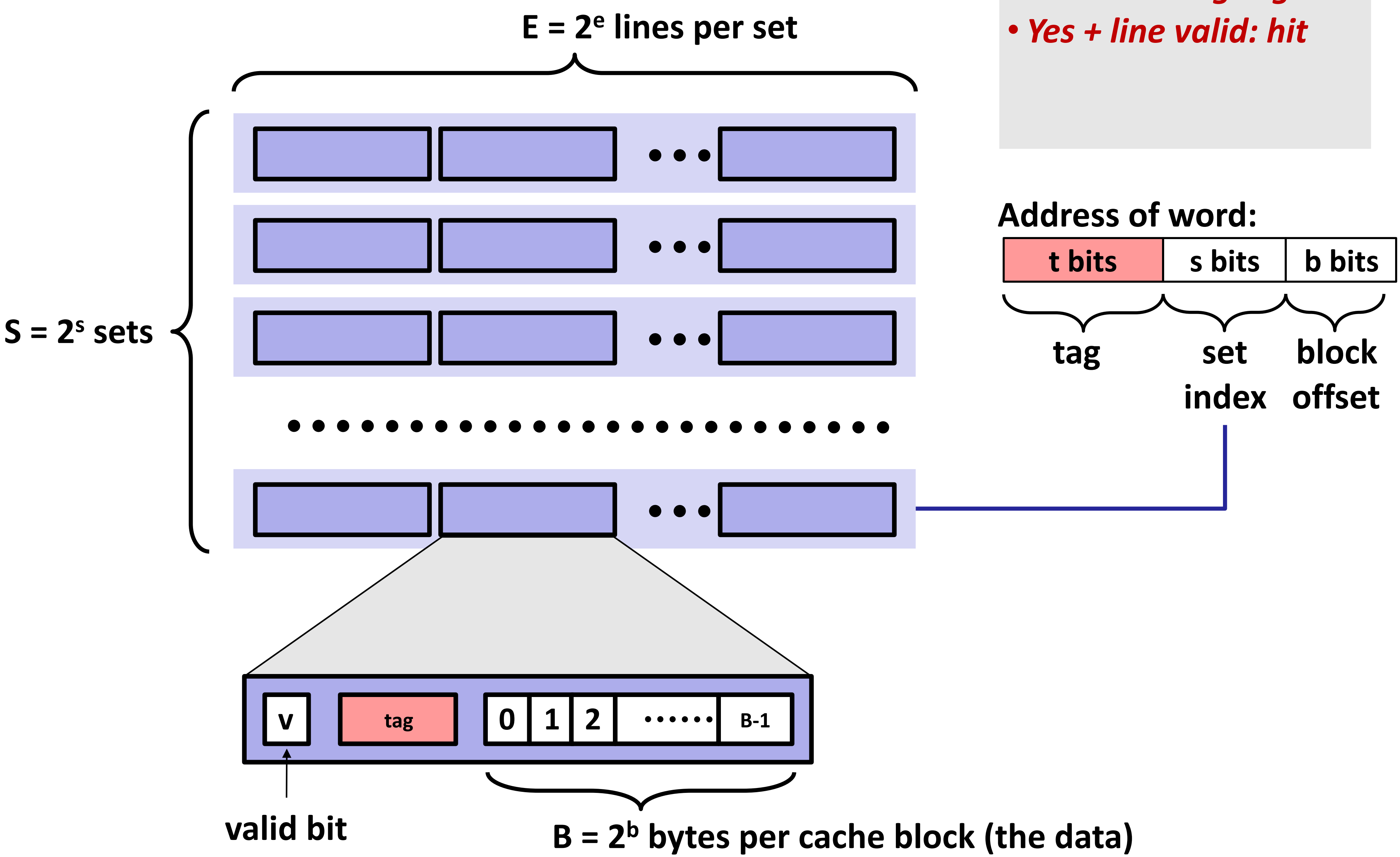
Cache Access

- ***Locate set***



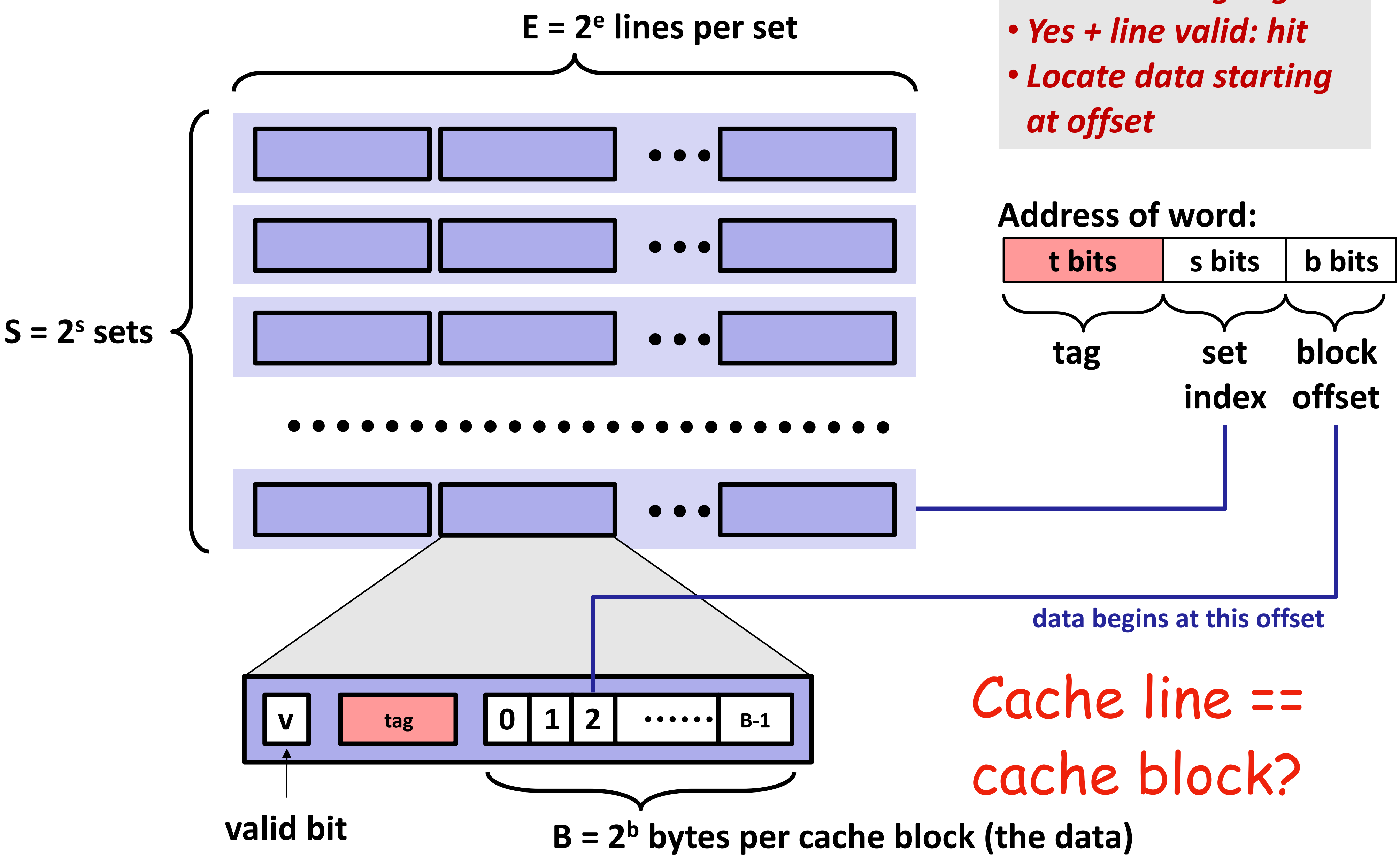
Cache Access

- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*



Cache Access

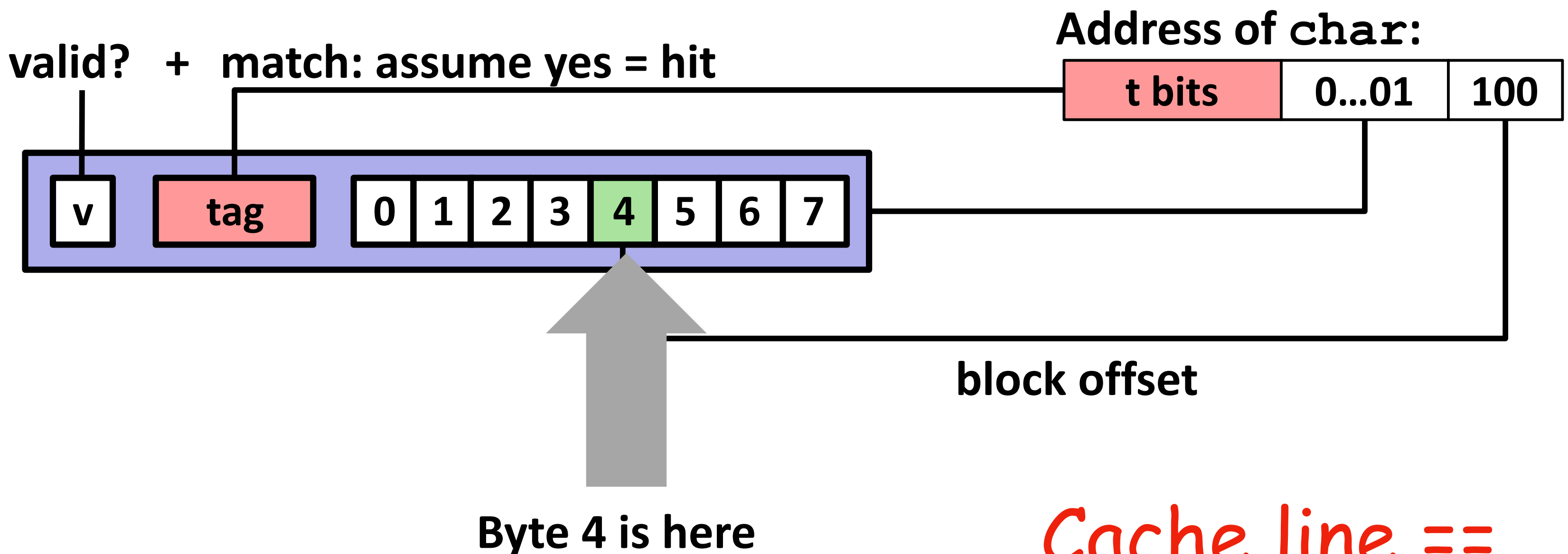
- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*



Cache line == cache block?

Example: Direct Mapped Cache

Direct mapped: One line per set
Assume: cache block size 8 bytes



Cache line ==
cache block?

If tag doesn't match: **old line is evicted and replaced**

Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

4-bit address space, i.e., Memory = 16 bytes
B=2 bytes/line, S=4 sets, E=1 line/set

Address trace (reads, one byte per read):

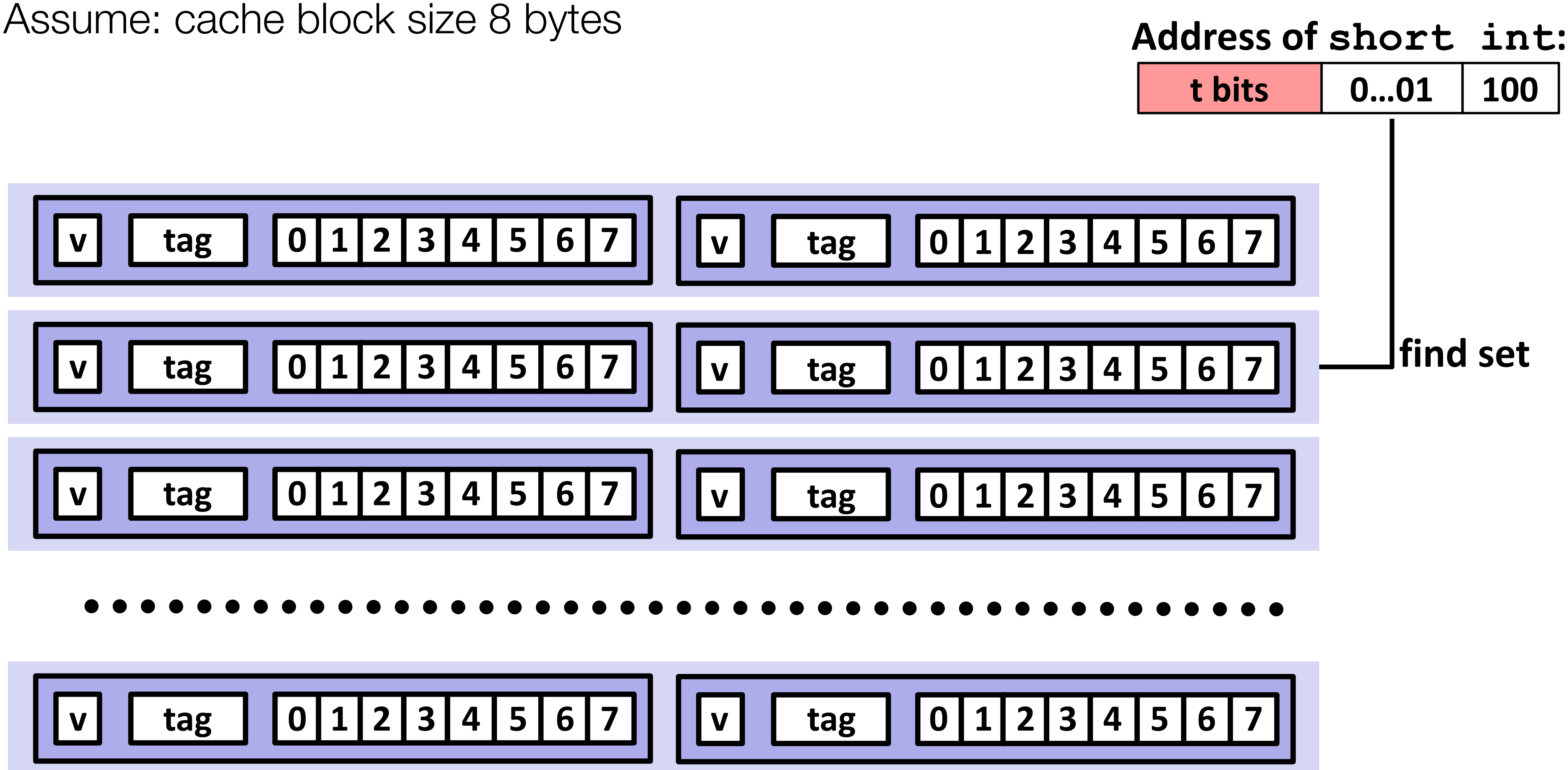
0	**$[0\underline{00}0_2]$,**	miss
1	**$[0\underline{00}1_2]$,**	hit
7	**$[0\underline{11}1_2]$,**	miss
8	**$[1\underline{00}0_2]$,**	miss
0	**$[0\underline{00}0_2]$**	miss

	v	Tag	Line
Set 0	1	0	M[0-1]
Set 1			
Set 2			
Set 3	1	0	M[6-7]

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

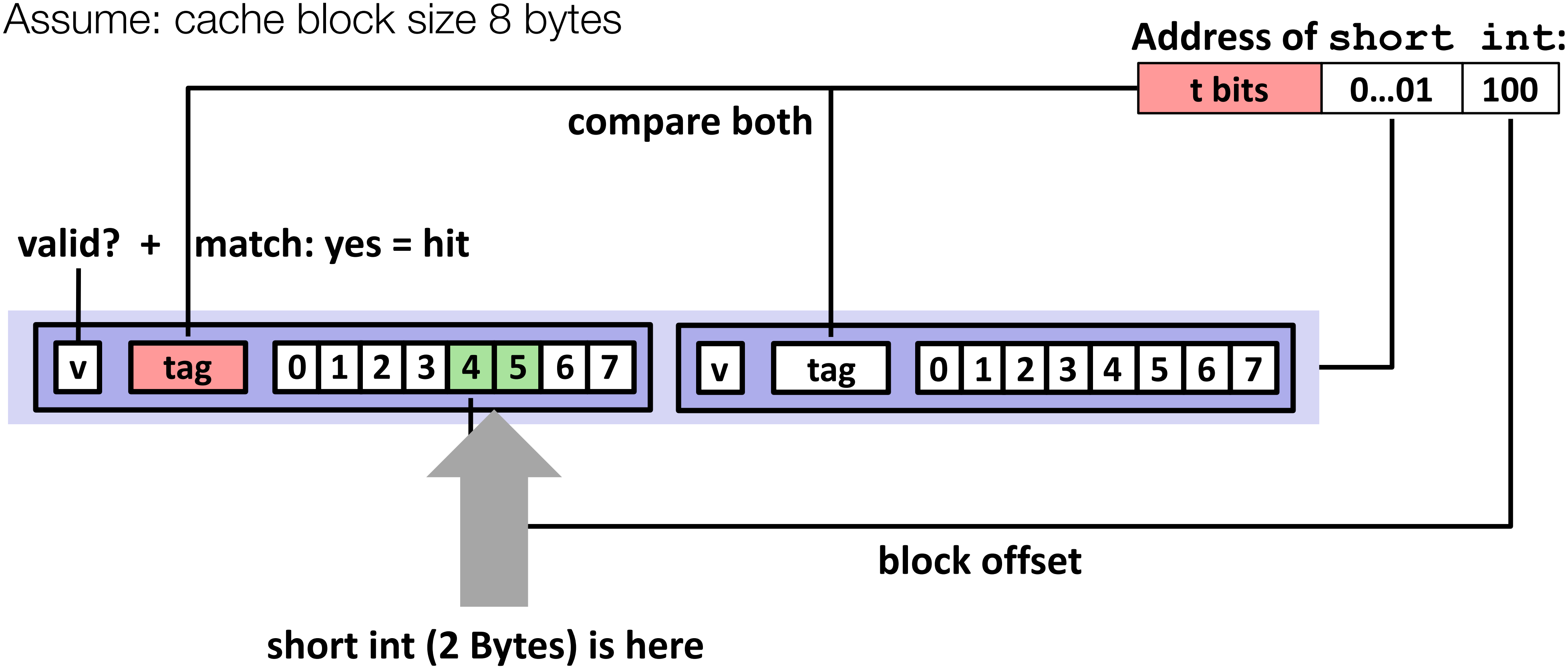
Assume: cache block size 8 bytes



E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes



No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), …

2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

4-bit address space, i.e., Memory = 16 bytes
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

0	**$[00\underline{0}0_2]$,**
1	**$[00\underline{0}1_2]$,**
7	**$[01\underline{1}1_2]$,**
8	**$[10\underline{0}0_2]$,**
0	**$[00\underline{0}0_2]$**

	v	**Tag**	**Block**
Set 0	0	?	?
	0		
Set 1	0		
	0		

2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

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1	**$[00\underline{0}1_2]$,**	
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0	**$[00\underline{0}0_2]$**	

	v	**Tag**	**Block**
Set 0	0	?	?
	0		
Set 1	0		
	0		

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xx	x	x

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1	**$[00\underline{0}1_2]$,**	
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8	**$[10\underline{0}0_2]$,**	
0	**$[00\underline{0}0_2]$**	

	v	**Tag**	**Block**
Set 0	1	00	M[0-1]
	0		
Set 1	0		
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2-Way Set Associative Cache Simulation

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xx	x	x

4-bit address space, i.e., Memory = 16 bytes
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

0	**[00<u>0</u>0$_2$],**	miss
1	**[00<u>0</u>1$_2$],**	hit
7	**[01<u>1</u>1$_2$],**	
8	**[10<u>0</u>0$_2$],**	
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Set 0	1	00	M[0-1]
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	v	**Tag**	**Block**
Set 0	1	00	M[0-1]
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Set 1	1	01	M[6-7]
	0		

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Set 0	1	00	M[0-1]
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Address trace (reads, one byte per read):

0	**[00<u>0</u>0$_2$],**	miss
1	**[00<u>0</u>1$_2$],**	hit
7	**[01<u>1</u>1$_2$],**	miss
8	**[10<u>0</u>0$_2$],**	miss
0	**[00<u>0</u>0$_2$]**	

	v	**Tag**	**Block**
Set 0	1	00	M[0-1]
	1	10	M[8-9]
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2-Way Set Associative Cache Simulation

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0	$[00\underline{0}0_2]$	hit

	v	**Tag**	**Block**
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

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- Review: Cache memory organization and operation
- Performance impact of caches
    - Analytical Model
    - Rearranging loops to improve spatial locality
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Optimizing Irregular and dynamic applications [32]

Cache Performance Metrics

- Miss Rate
  - Fraction of memory references not found in cache (misses / accesses)
    = 1 – hit rate
  - Typical numbers (in percentages):
    - 3-10% for L1
    - can be quite small (e.g., < 1%) for L2, depending on size, etc.

Cache Performance Metrics

- Hit Time
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typical numbers:
    - 1~4 clock cycle for L1
    - 5~10 clock cycles for L2

Cache Performance Metrics

- Miss Penalty
  - Additional time required because of a miss
    - Typically 50-200 cycles for main memory
    - Trend: increasing!

Let’s think about those numbers

- Huge difference between a hit and a miss
  - Could be 100x, if just L1 and main memory
- Compare 97% hit rate with 99% hit rate
  - Assume:
    cache hit time of 1 cycle
    miss penalty of 100 cycles
  - Average access time:

    97% hit rate: 1 cycle + 0.03 * 100 cycles = 4 cycles

    99% hit rate: 1 cycle + 0.01 * 100 cycles = 2 cycles
- **Think of it as reducing the miss rate from 3% to 1% (3X improvement) rather than improving hit rate**
- Improving hit rate by even a little bit helps overall speed a lot

Writing Cache Friendly Code

- Make the common case go fast
  - Inner loops get executed most often. So focus on those

- Minimize the misses in the inner loops
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

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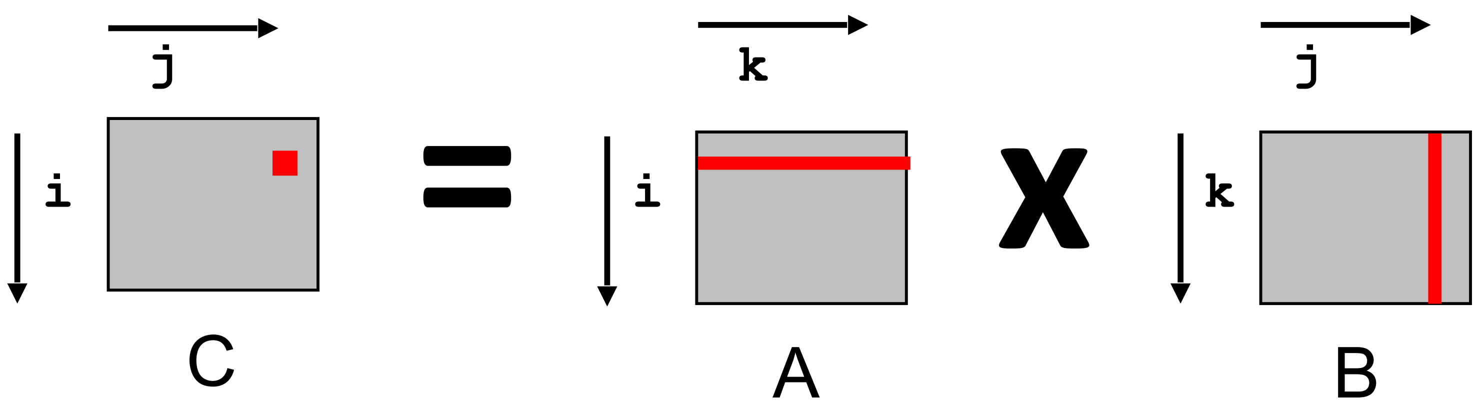
Optimizing Irregular and dynamic applications [32]

Matrix Multiplication Example

- Multiply N x N matrices
- Matrix elements are doubles (8 bytes)
- O(N^3) total operations

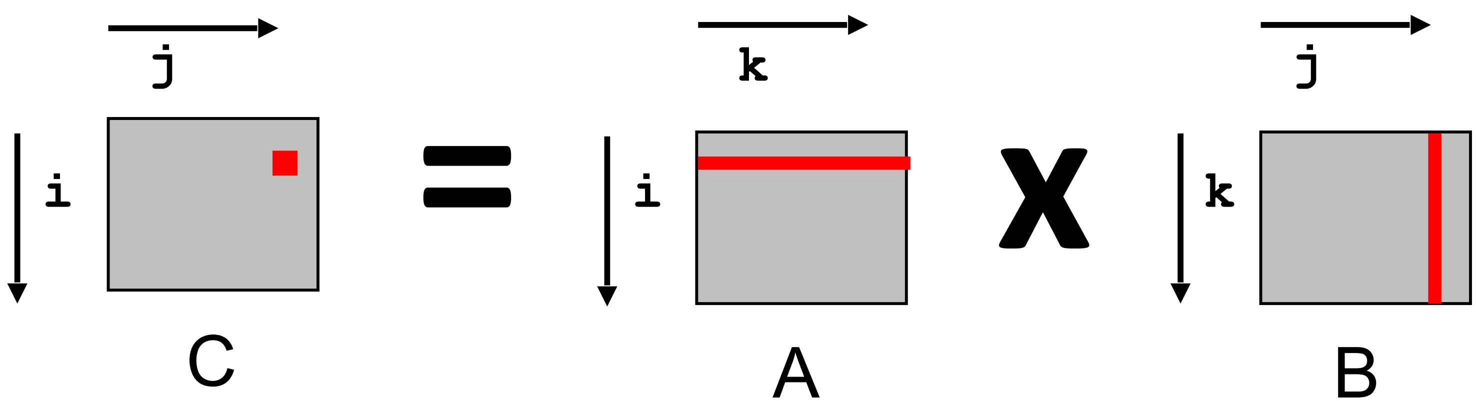
```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Variable ***sum*** held in register



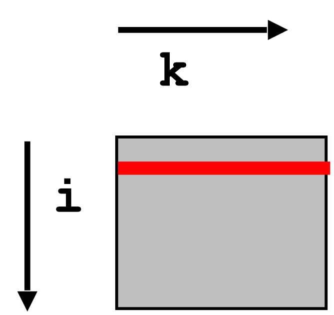
Miss Rate Analysis for Matrix Multiply

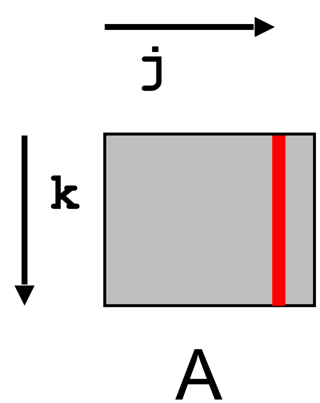
- Assume:
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations
- Stepping through columns in one row:
  - ```
    for (i = 0; i < N; i++)
       sum += a[0][i];
    ```
  - accesses successive elements
  - cache line size (32) > size of an element (8 bytes), exploiting spatial locality!
    - miss rate = 8 / 32 = 25%

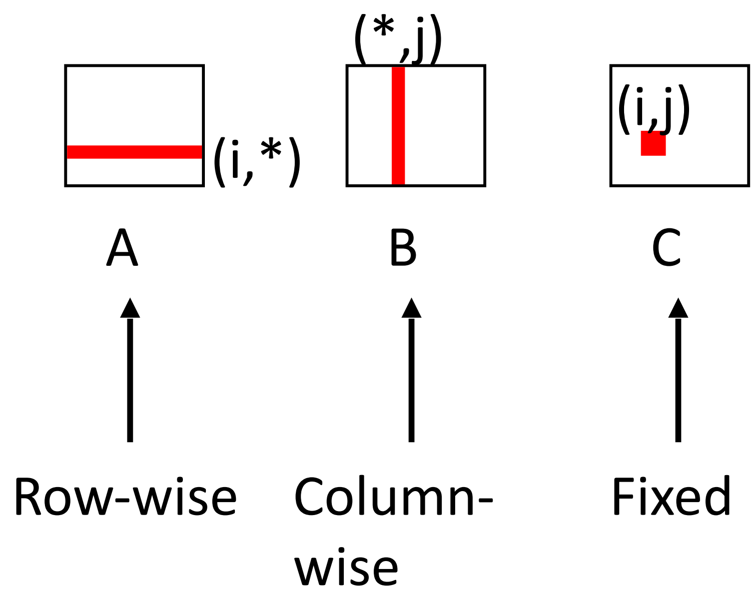


A
    ```

Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations
- Stepping through rows in one column:
  - ```
    for (i = 0; j < n; j++)
       sum += a[i][0];
    ```
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)


    ```

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                        matmult/mm.c
```

Inner loop:



Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
                      matmult/mm.c
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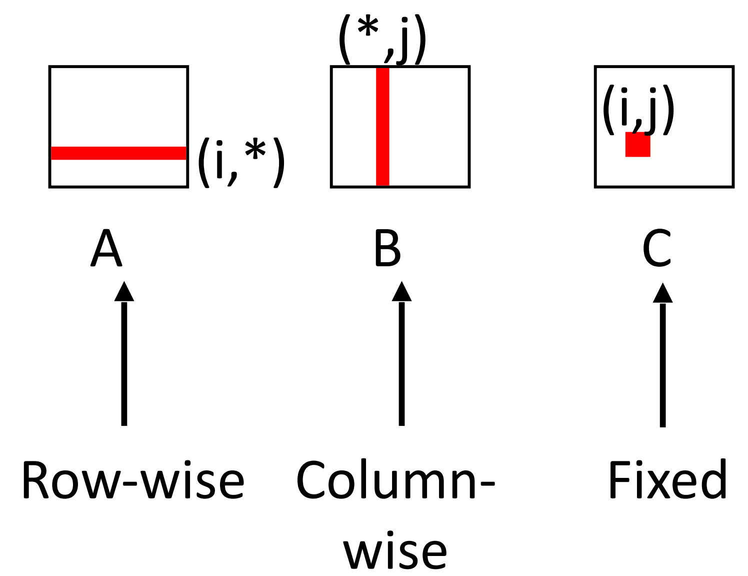
Inner loop:



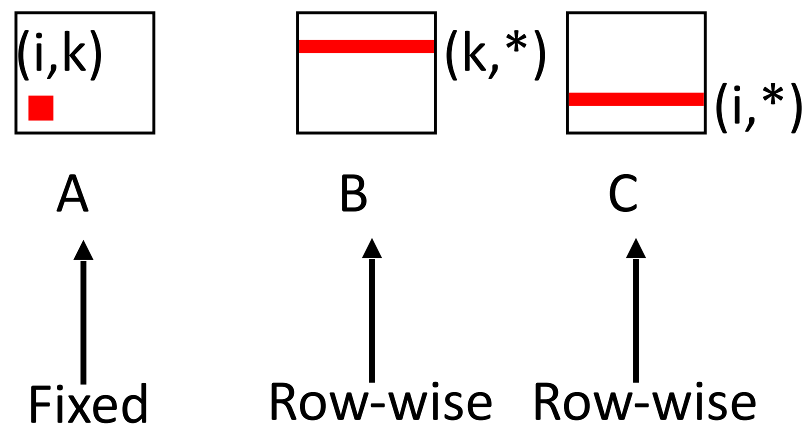
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```
matmult/mm.c

Inner loop:



Misses per inner loop iteration:

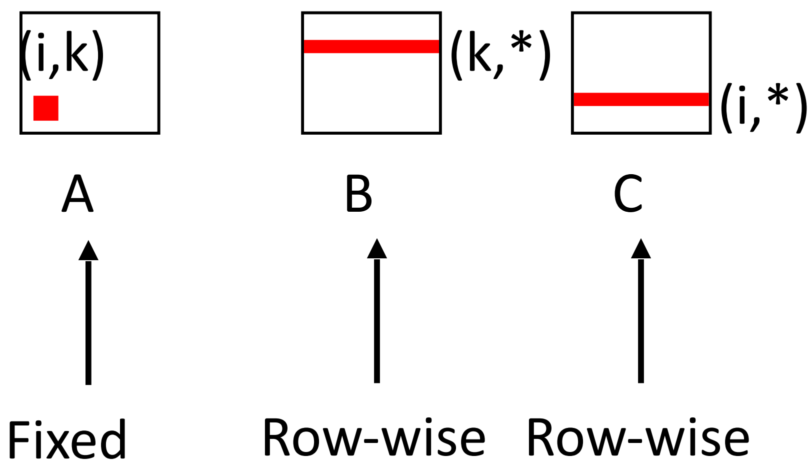
A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

matmult/mm.c

Inner loop:



Misses per inner loop iteration:

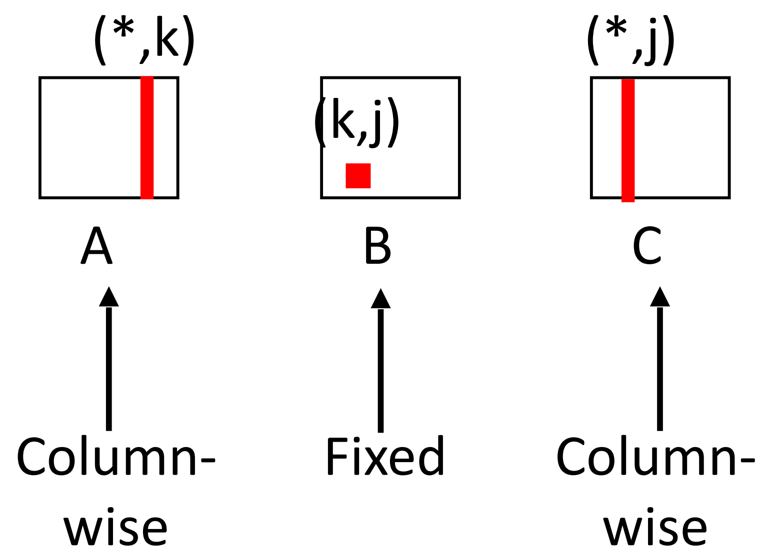
A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

matmult/mm.c

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
                    matmult/mm.c
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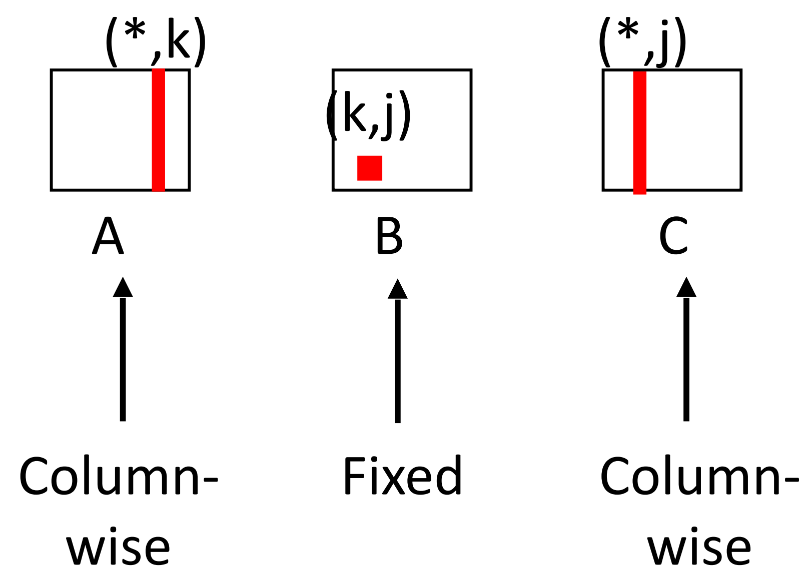
Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):

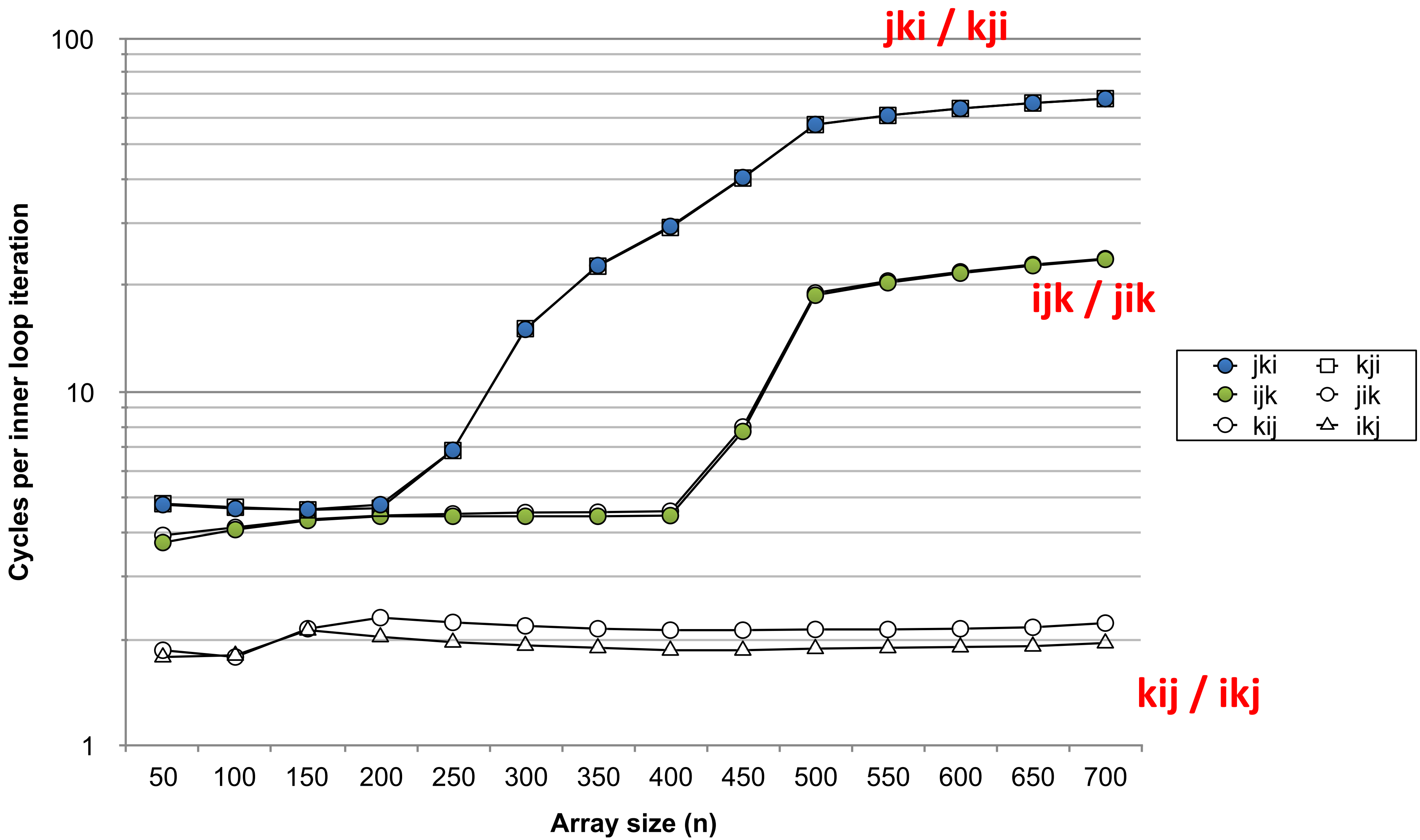
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):

- 2 loads, 1 store
- misses/iter = **2.0**

Core i7 Matrix Multiply Performance



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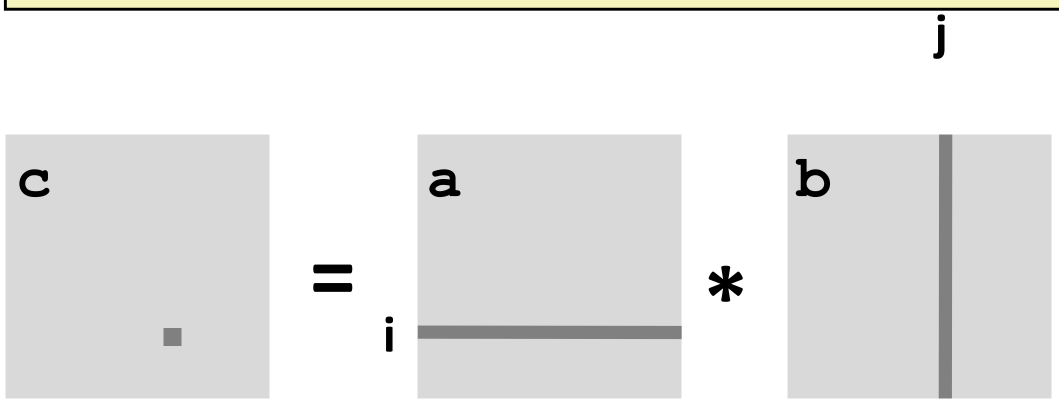
Optimizing Irregular and dynamic applications [32]

Example: Matrix Multiplication

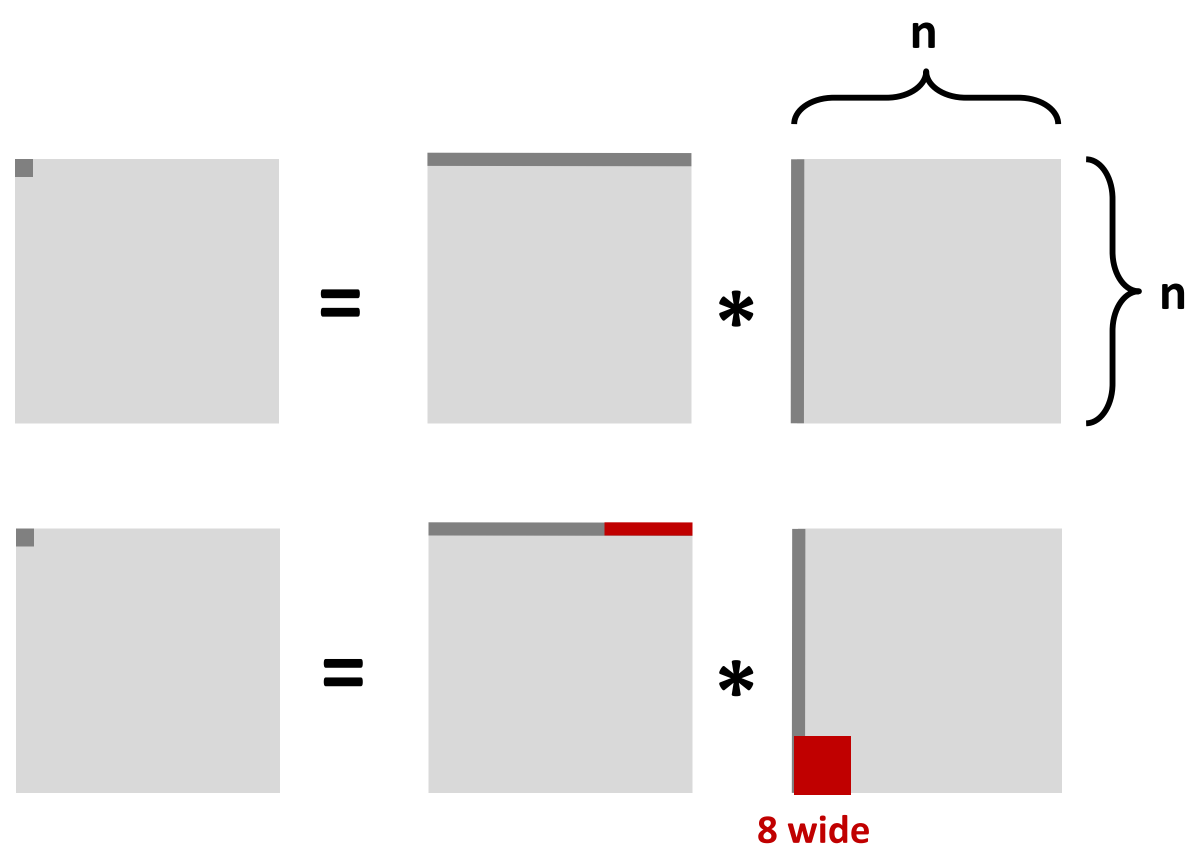
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)

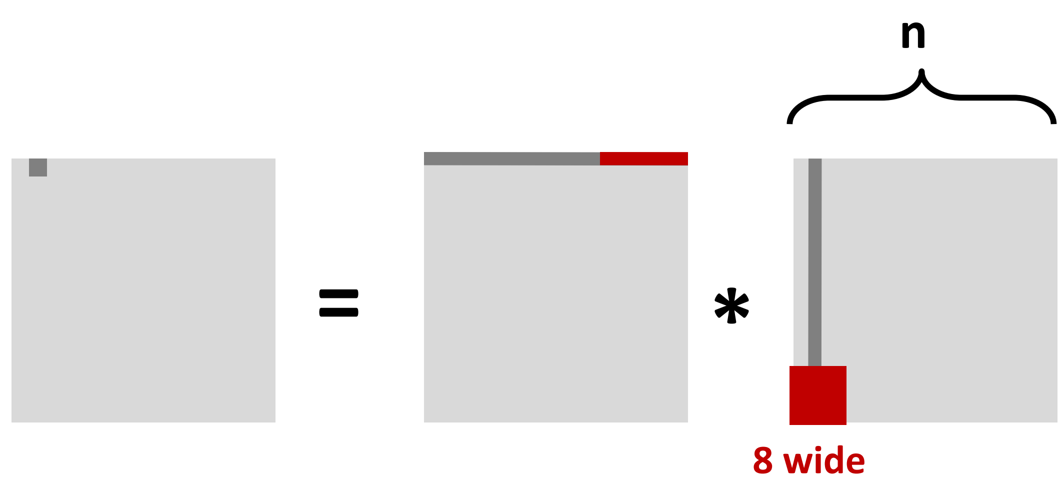
- First iteration:
  - n/8 + n = 9n/8 misses



Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)

- Second iteration:
  - Again:
    $n/8 + n = 9n/8$ misses

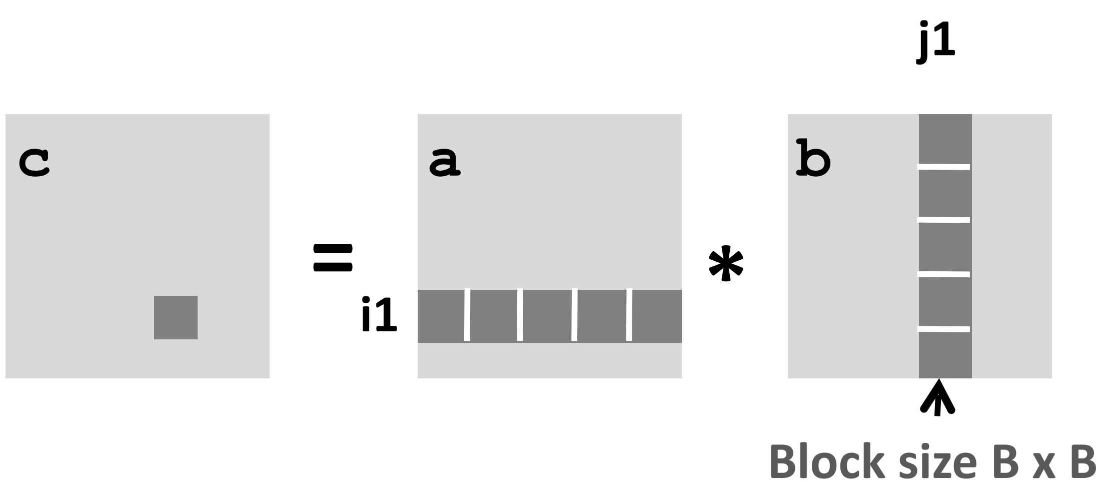


- Total misses:
  - $9n/8 * n^2 = (9/8) * n^3$

Blocked Matrix Multiplication

```c
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

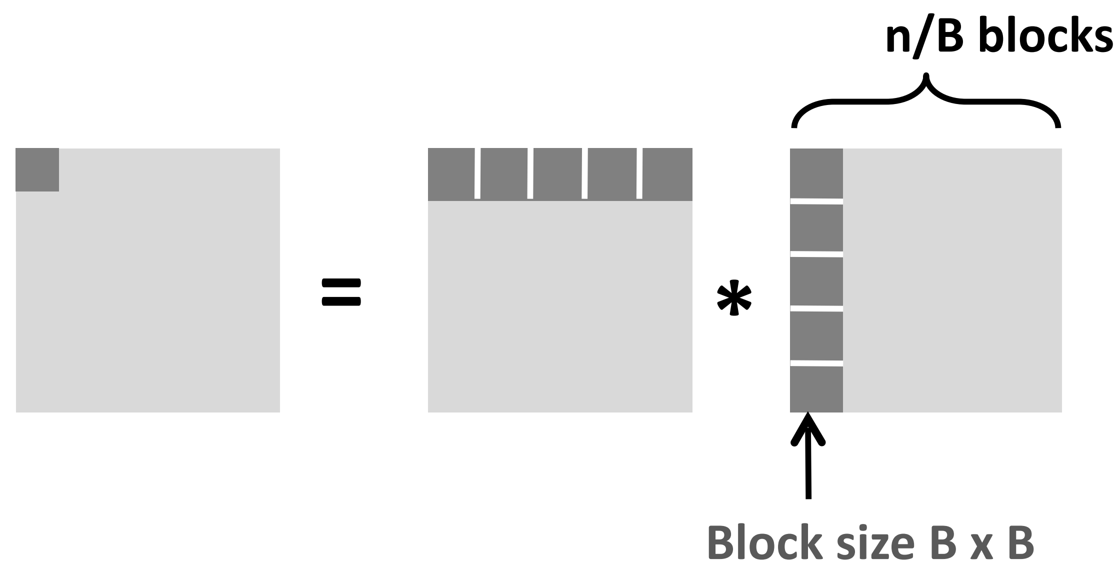
matmult/bmm.c



Cache Miss Analysis

- Assume:
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$

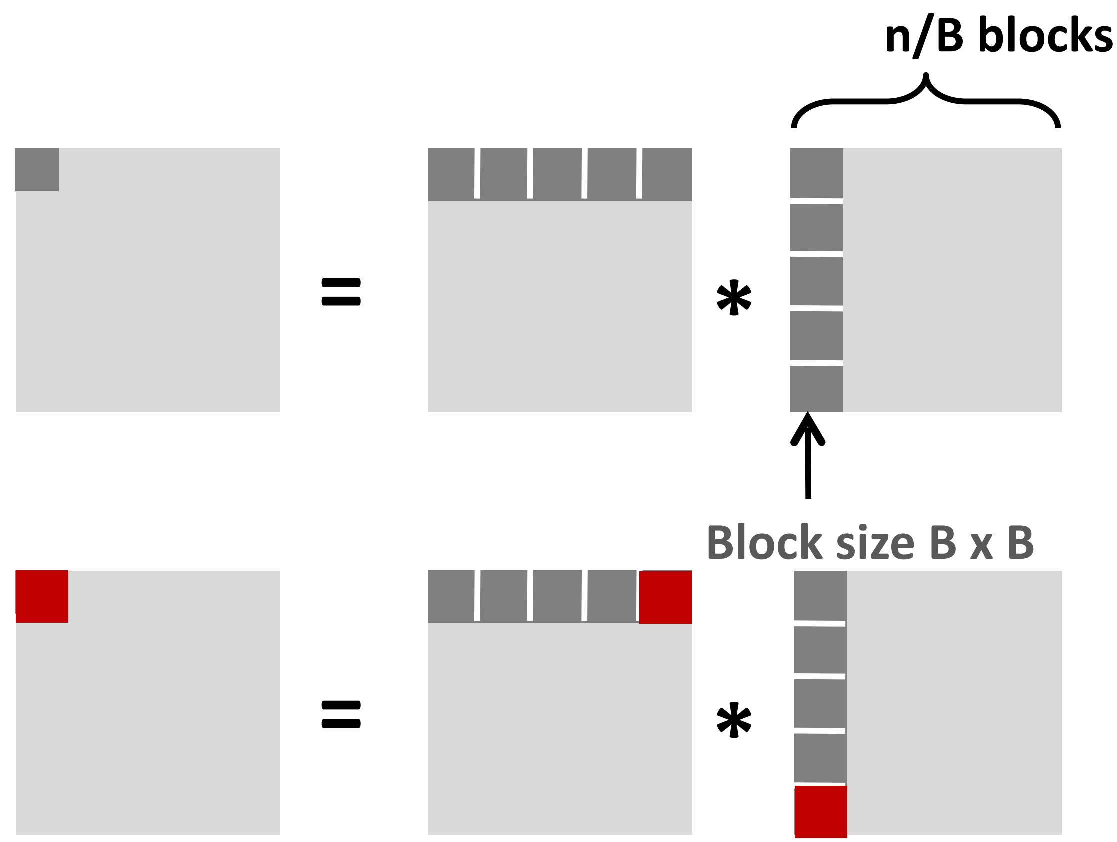
- First (block) iteration:



Cache Miss Analysis

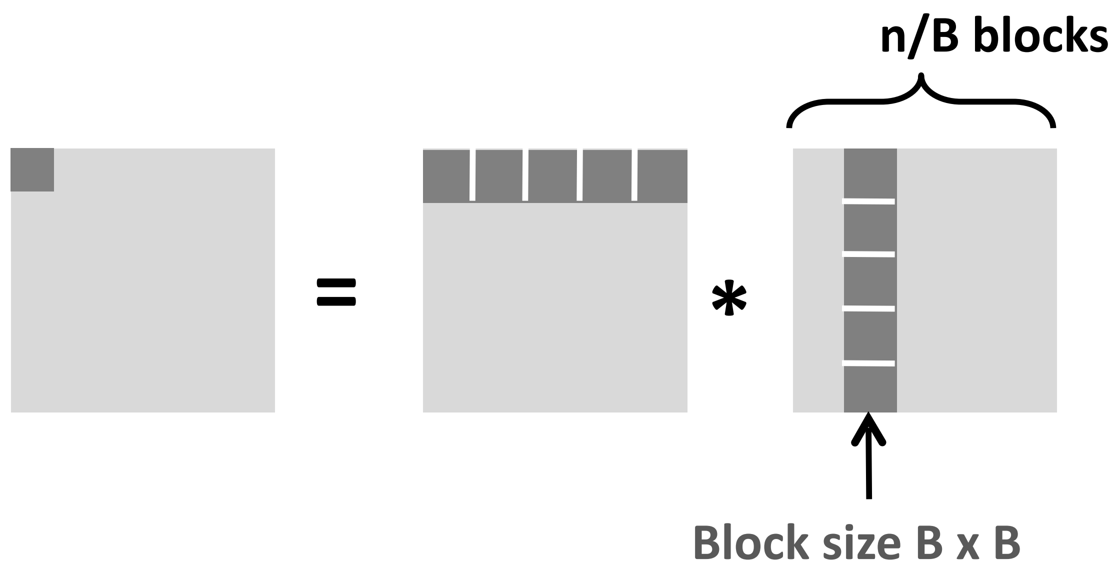
- Assume:
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$

- First (block) iteration:
  - $B^2/8$ misses for each block
  - $2n/B * B^2/8 = nB/4$



Cache Miss Analysis

- Assume:
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$
- Second (block) iteration:
  - Same as first iteration
  - $2n/B * B^2/8 = nB/4$



- Total misses:
  - $nB/4 * (n/B)^2 = n^3/(4B)$

Blocking Summary

- No blocking: $(9/8) * n^3$
- Blocking: $1/(4B) * n^3$

- Suggest largest possible block size B, but limit $3B^2 < C$!

- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

Cache Summary

- Cache memories can have significant performance impact

- You can write your programs to exploit this!
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it’s read from memory.

Dynamic Optimizations

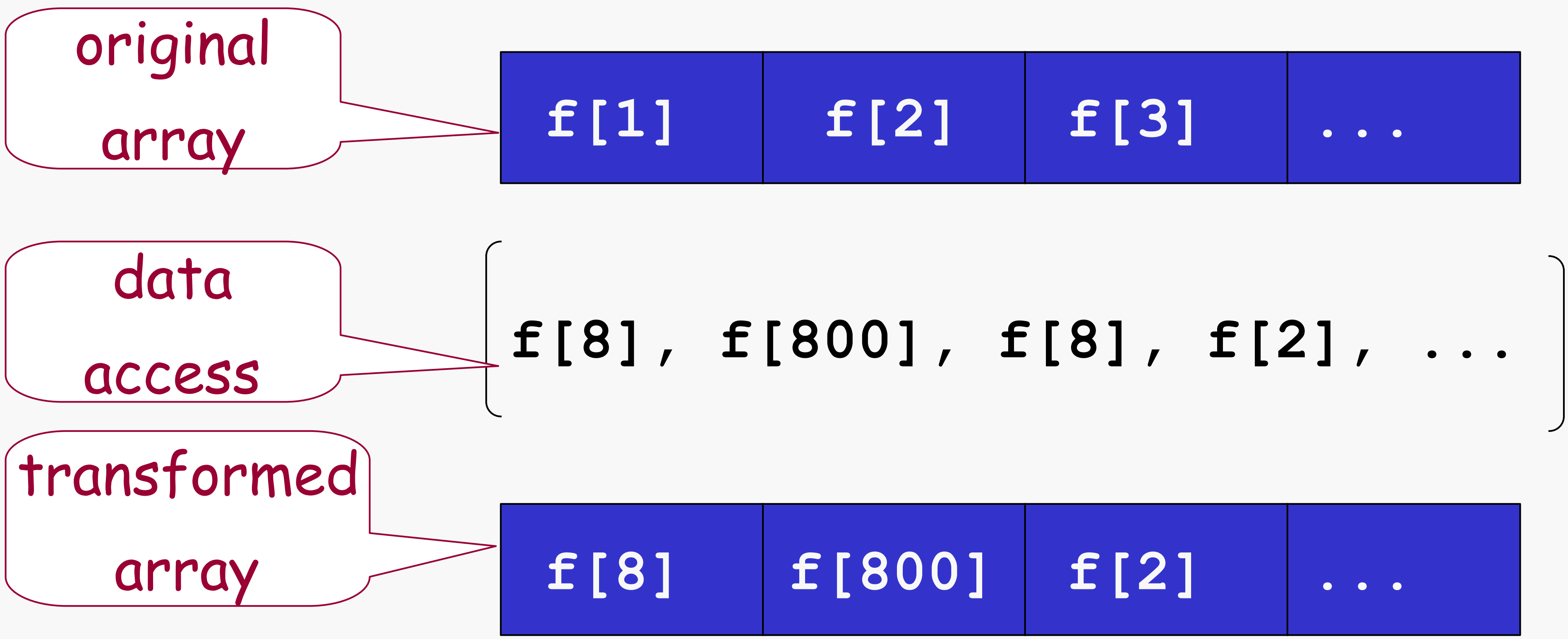
[PLDI'99, with Ken Kennedy]

Unknown Access

"Every problem can be solved by adding one more level of indirection."

- Irregular and dynamic applications
  - Irregular data structures are unknown until run time
  - Data and their uses may change during the computation
- For example
  - Molecular dynamics
  - Sparse matrix
- Problems
  - How to optimize at run time?
  - How to automate?

Example packing



Software remapping:

f[t[i]] → f[remap[t[i]]] → f[t'[i]]

f[i] → f[remap[i]] → f[i]

Dynamic Optimizations

- Locality grouping & Dynamic packing
  - run-time versions of computation fusion & data grouping
  - linear time and space cost
- Compiler support
  - analyze data indirections
  - find all optimization candidates
  - use run-time maps to guarantee correctness
  - remove unnecessary remappings
    - pointer update
    - array alignment
- The first set of compiler-generated run-time transformations

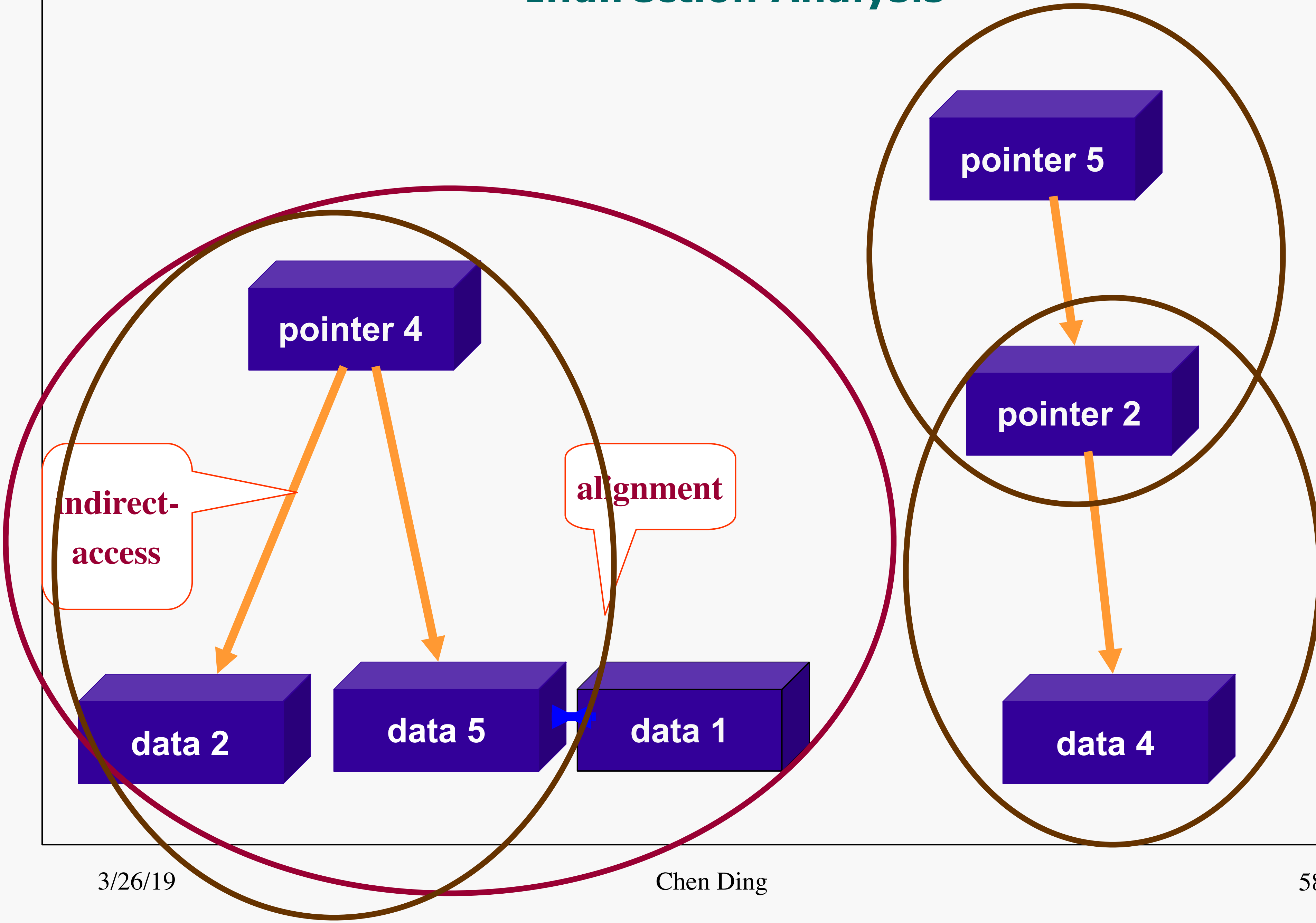
```
packing Directive: apply packing using interactions

for each pair (i,j) in interactions
   compute_force( force[i], force[j] )
end for

for each object i
   update_location( location[i], force[i] )
end for
```

```
apply_packing(interactions[*],force[*],inter_map[*])
for each pair (i,j) in interactions
   compute_force( force[inter_map[i]],
                  force[inter_map[j]] )
end for

for each object i
   update_location(location[i],force[inter_map[i]])
end for
```

indirections

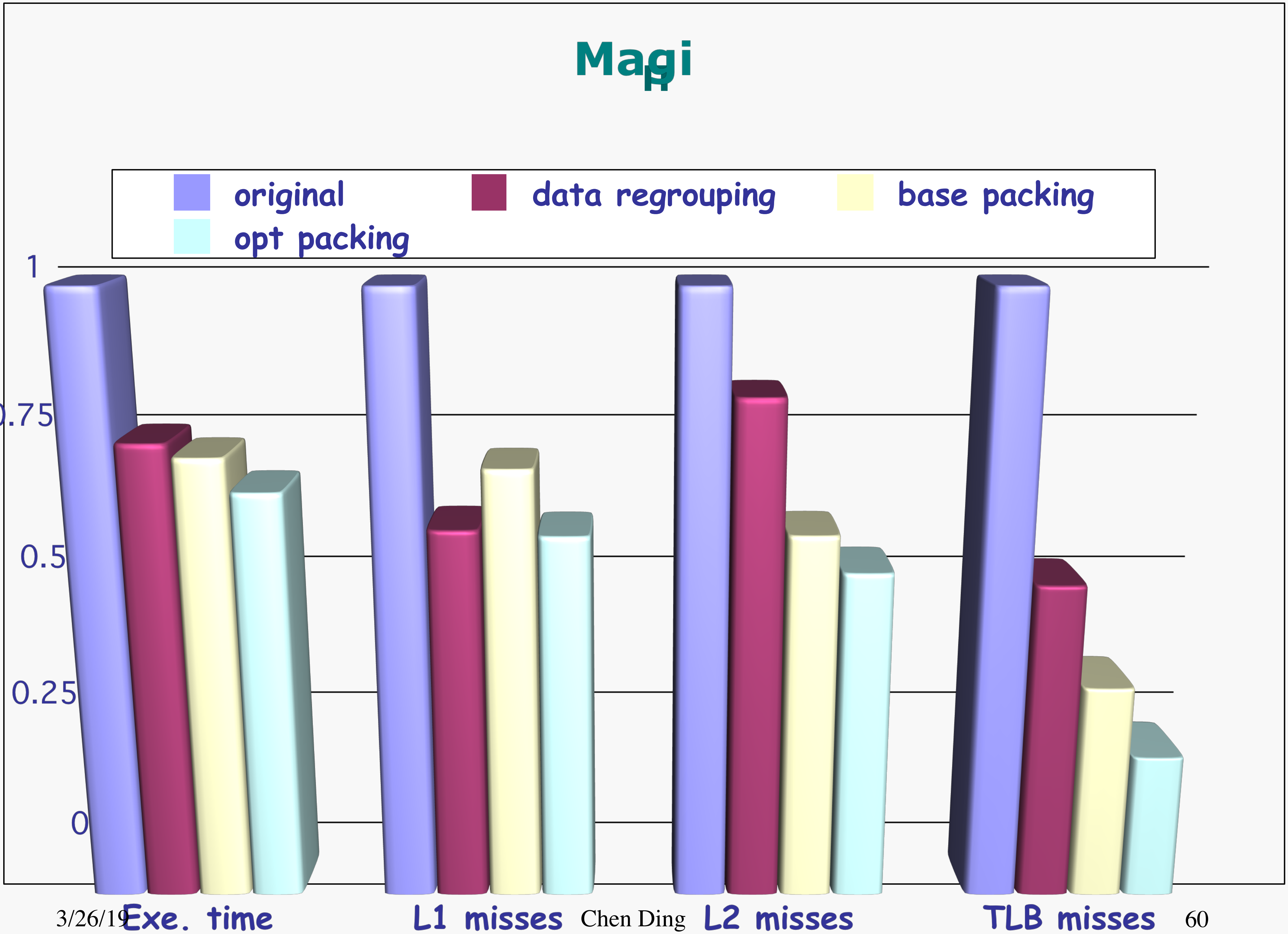
```
apply_packing(interactions[*],force[*],
              inter_map[*], update_map[*])
update_indirection_array(interactions[*],
                         update_map[*])
transform_data_array(location[*],update_map[*])

for each pair (i,j) in interactions
   compute_force( force[i], force[j] )
end for

for each object i
   update_location( location[i], force[i] )
end for
```

Indirection Analysis



DoD/Magi

- A real application from DoD Philips Lab
  - particle hydrodynamics
  - almost 10,000 lines of code
  - user supplied input of 28K particles
  - 22 arrays in major phases, split into 26
- Optimizations
  - grouped into 6 arrays
  - inserted 1114 indirections to guarantee correctness
  - optimization reorganized 19 more arrays
  - removed 379 indirections in loops
  - reorganized 45 arrays 4 times during execution


Magi
original
data regrouping
base packing
opt packing
1
0.75
0.5
0.25
0
Exe. time
L1 misses
L2 misses
TLB misses