CSC 252: Computer Organization Spring 2019: Lecture 2

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Department of Computer Science
University of Rochester

Action Items:
- **Programming Assignment 1 is out**
- **Trivia 1 is due on Friday, midnight**

Announcement

- Programming Assignment 1 is out
  - Details: http://cs.rochester.edu/courses/252/spring2019/labs/assignment1.html
  - Due on Feb 1, 11:59 PM
  - Trivia due Friday, 1/25, 11:59 PM
  - You have 3 slip days (not for trivia)

20	21	22	23	24	25 **Trivia**	26
27	28	29	30	31	Feb 1 **Due**	2

Announcement

- TA office hours are all posted. Start from this week.
- TA review sessions schedule to be posted soon…
- Programming assignment 1 is in C language. Seek help from TAs.
- Programming assignments do NOT repeat the lecture materials. They ask you to synthesize what you have learned from the lectures and work out something new.

Previously in 252…

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

Microarchitecture

Circuit

Previously in 252…

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ISA is the contract between software and hardware.

Previously in 252…

Problem

Algorithm

	Renting	Computing
Service provider	Landlord	Hardware
Service receiver	YOU	Software
Contract	Lease	Assembly Program
Contract's language	Natural language (e.g., English)	ISA

ct
e and

Circuit

Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

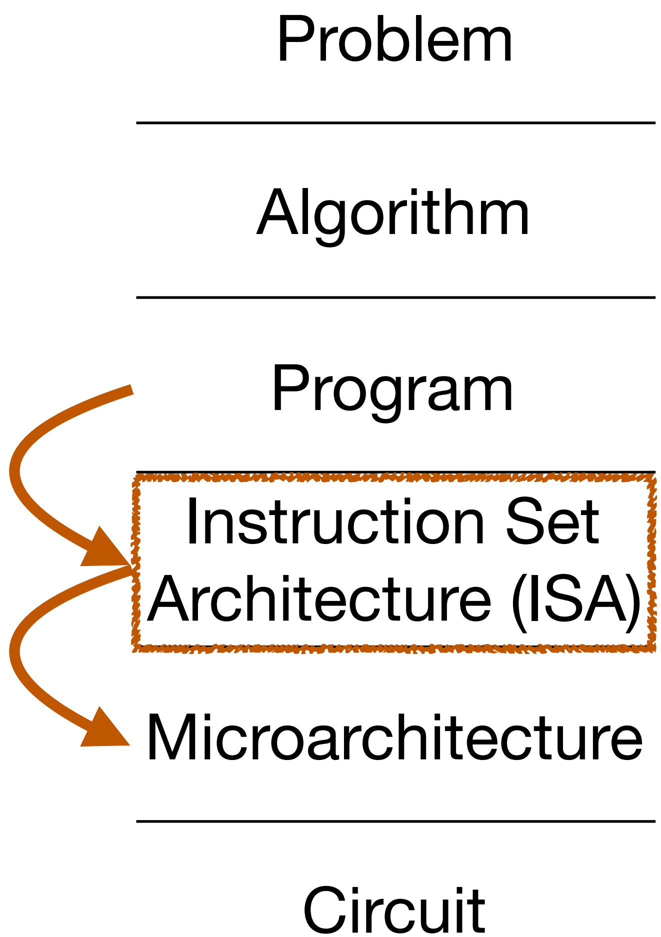
Microarchitecture

Circuit

ISA is the contract between software and hardware.

Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?
- How does a modern computer execute that program?



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Previously in 252…

C Program

```
void add() {
  int a = 1;
  int b = 2;
  int c = a + b;
}
```

Assembly program

```
movl    $1, -4(%rbp)
movl    $2, -8(%rbp)
movl    -4(%rbp), %eax
addl    -8(%rbp), %eax
```

Previously in 252…

Assembly program		*Executable Binary*
movl $1, -4(%rbp)		00011001 …
movl $2, -8(%rbp)	→	01101010 …
movl -4(%rbp), %eax		11010101 …
addl -8(%rbp), %eax		01110001 …

Previously in 252…

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Executable Binary

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00011001   …
01101010   …
11010101   …
01110001   …
```

- What's the difference between an assembly program and an executable binary?
  - They refer to the same thing — a list of instructions that the software asks the hardware to perform
  - They are just different representations
- Instruction = Operator + Operand(s)

Previously in 252…

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Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

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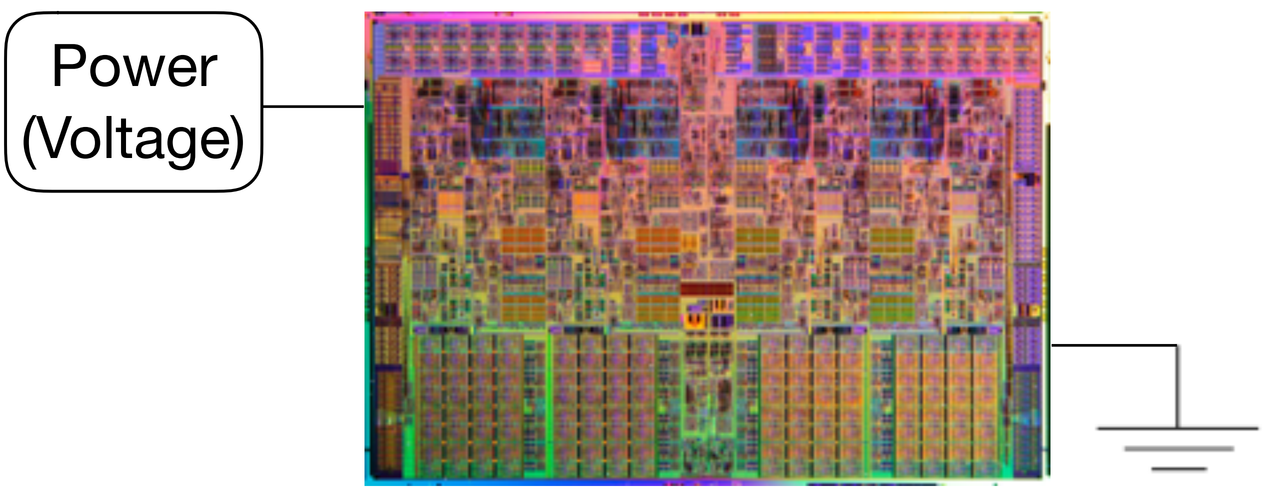

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    - Use high voltage to represent 1
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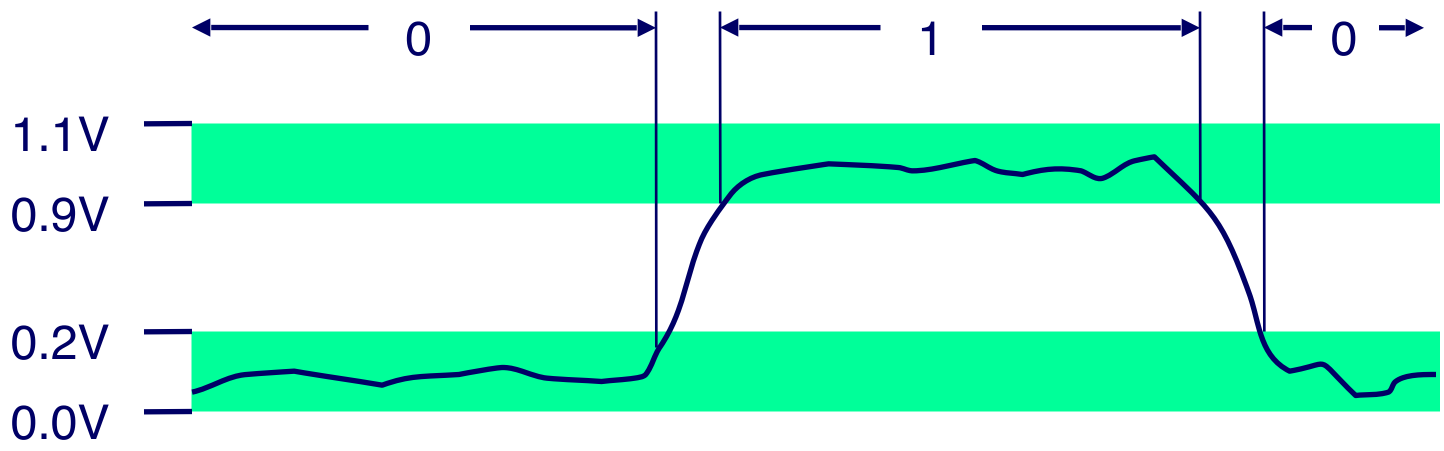

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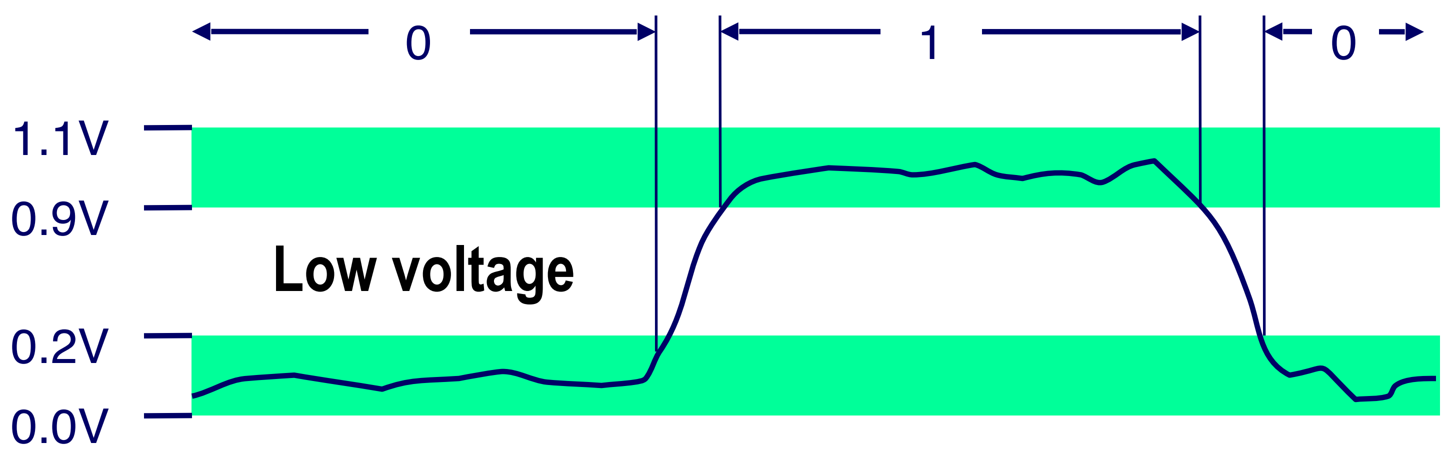

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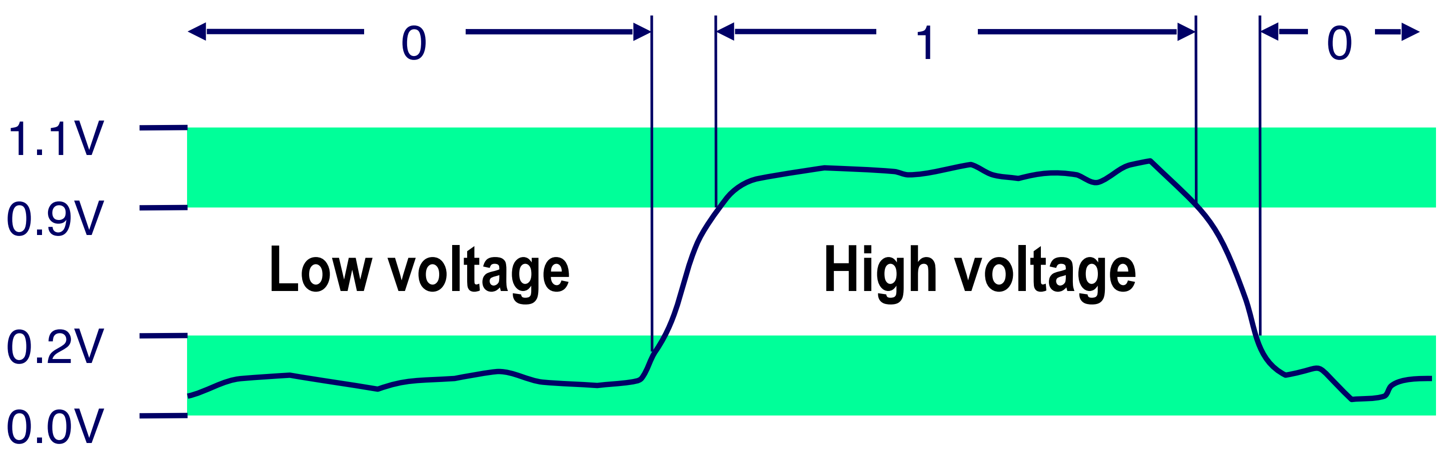

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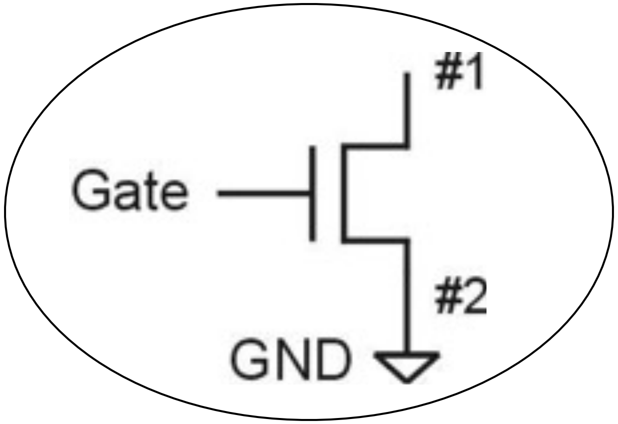

Terminal #2 must be connected to GND (0V).

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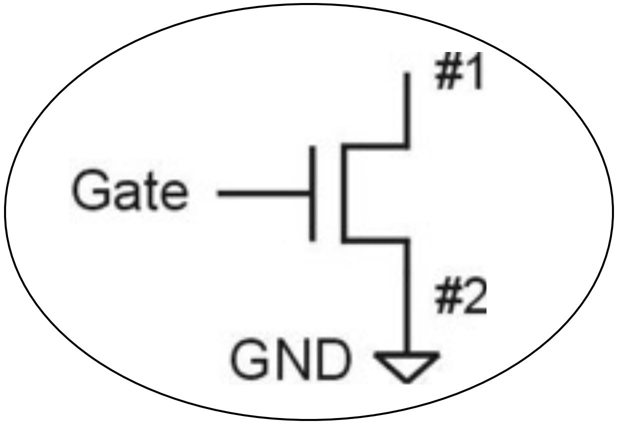

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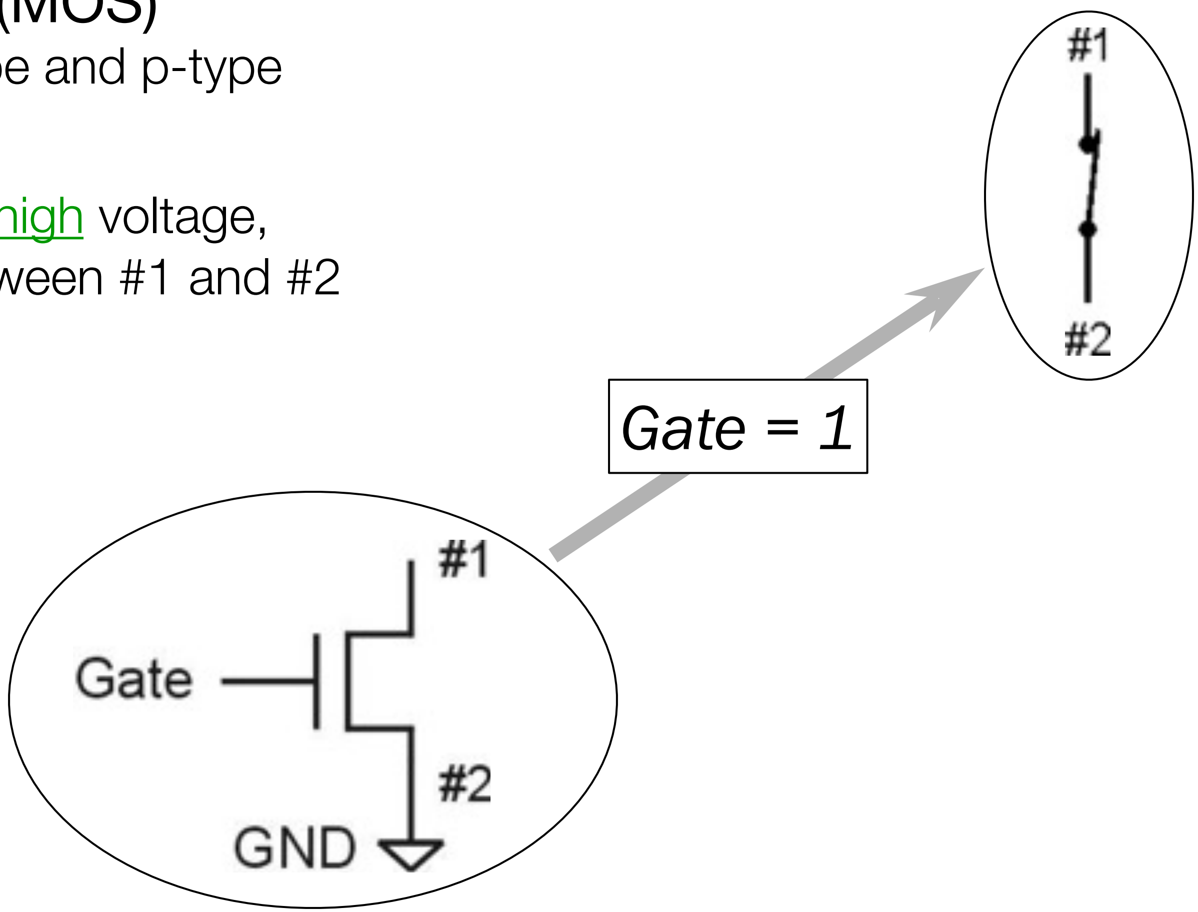

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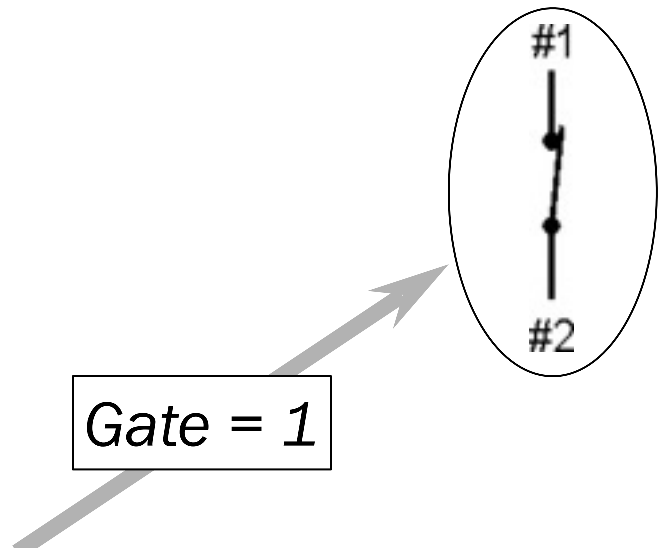

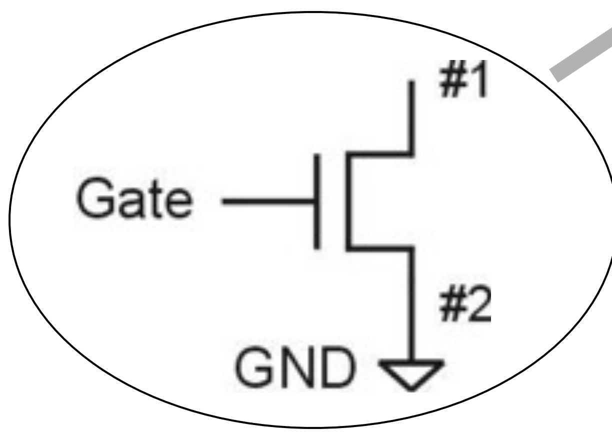

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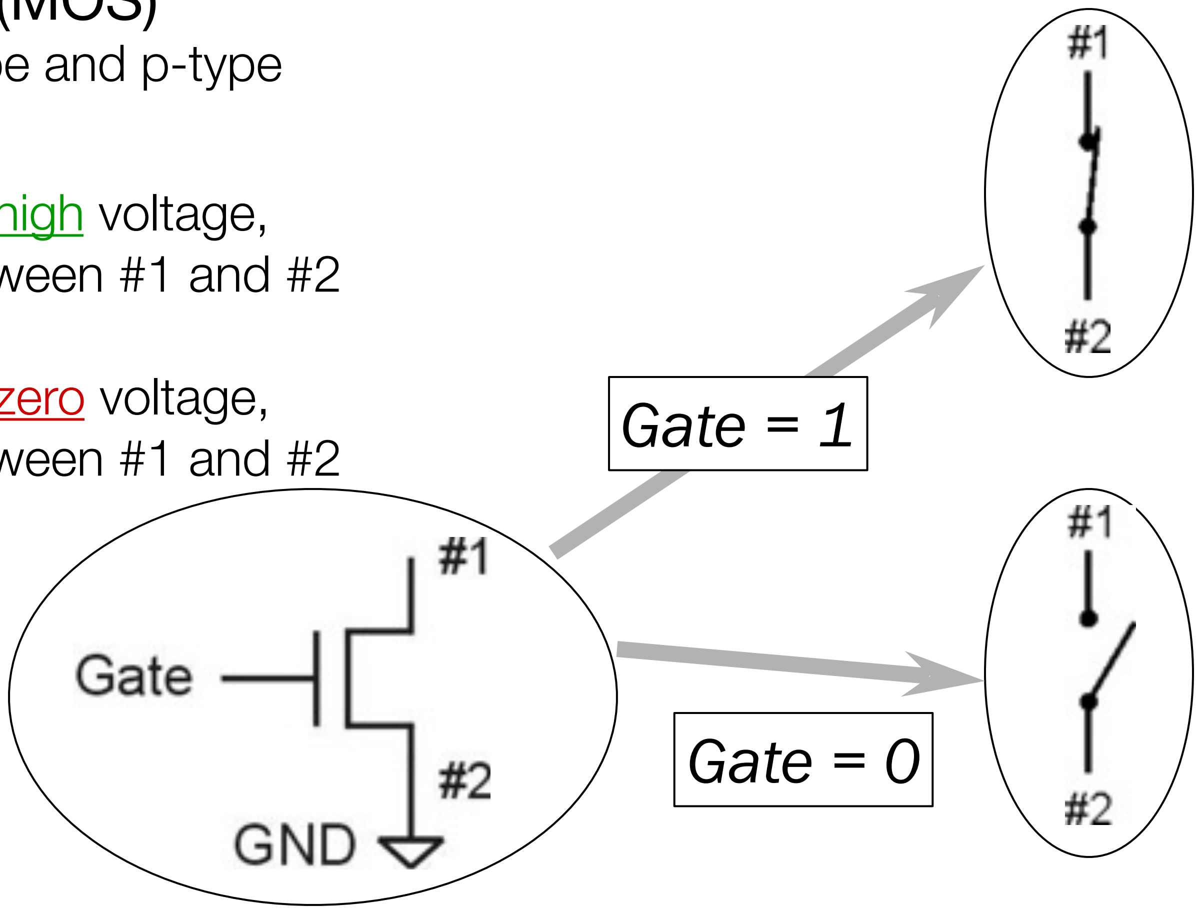

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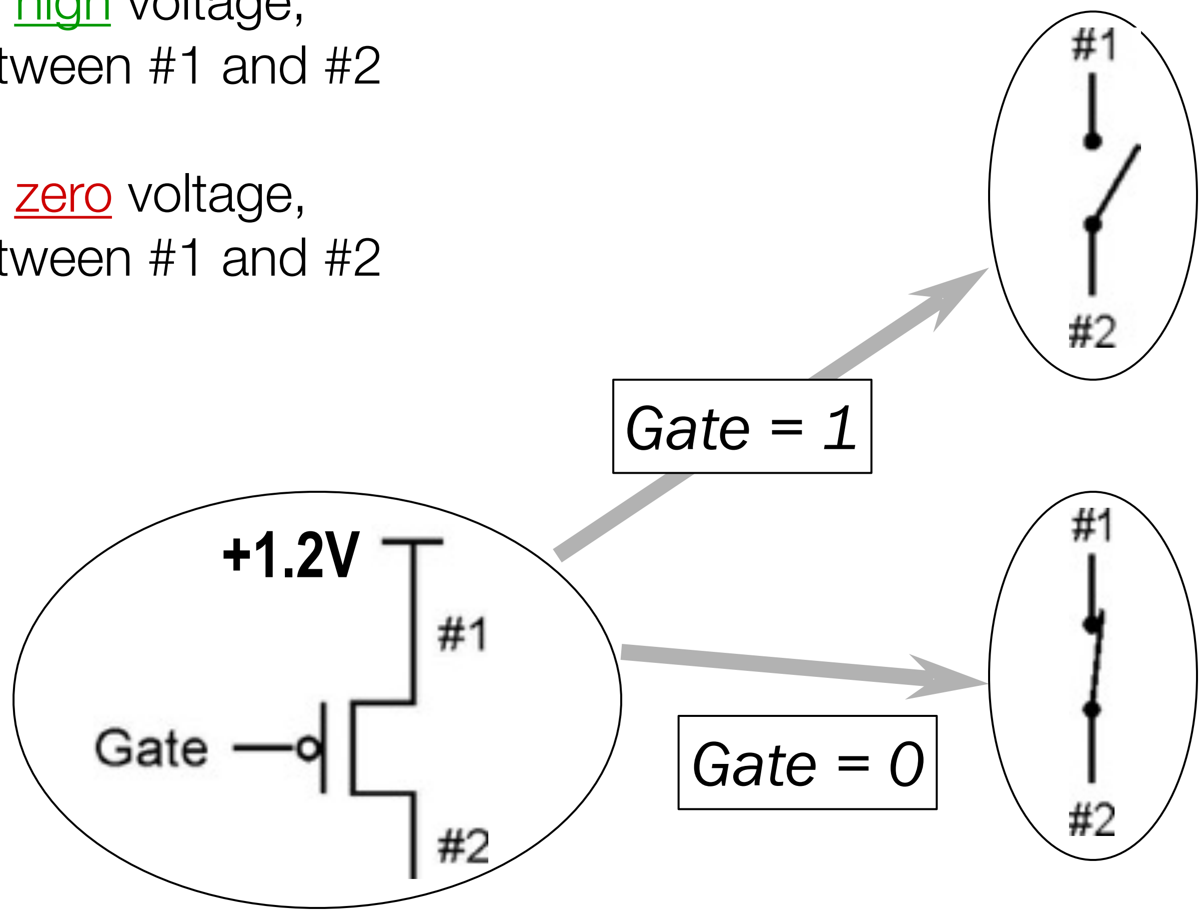
Why Bits?

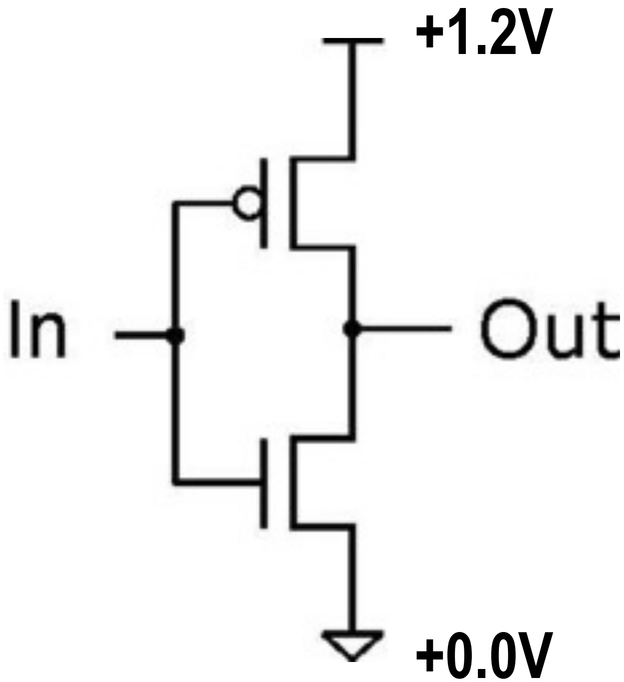
p-type is *complementary* to n-type (PMOS)

- when Gate has high voltage, open circuit between #1 and #2 (switch open)
- when Gate has zero voltage, short circuit between #1 and #2 (switch closed)



Terminal #1 must be connected to +1.2V

Inverter



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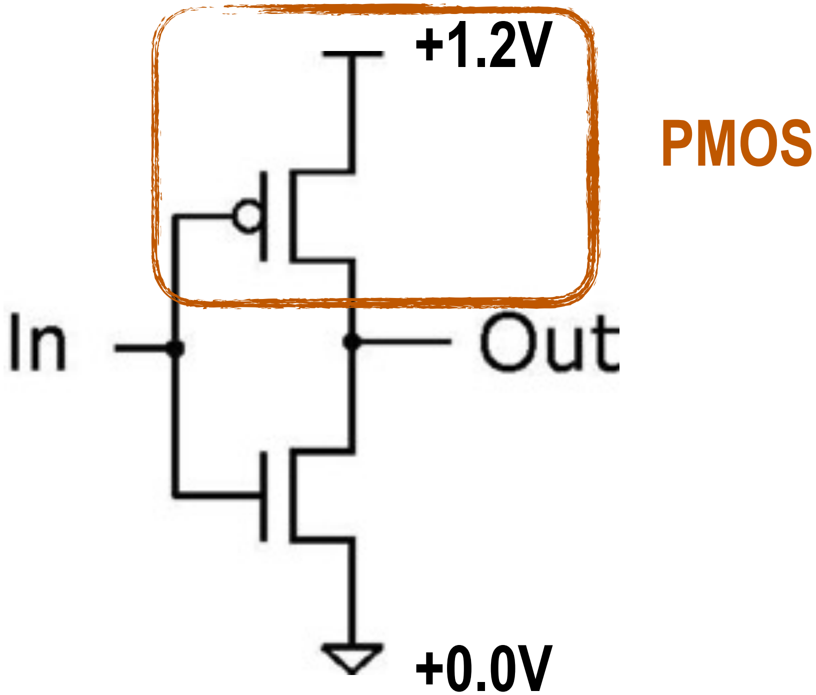

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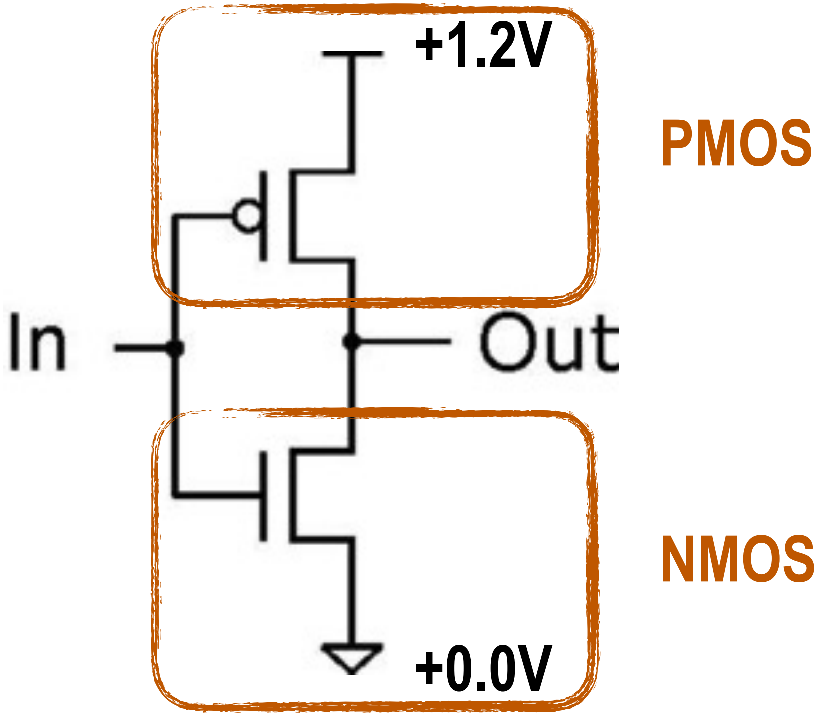

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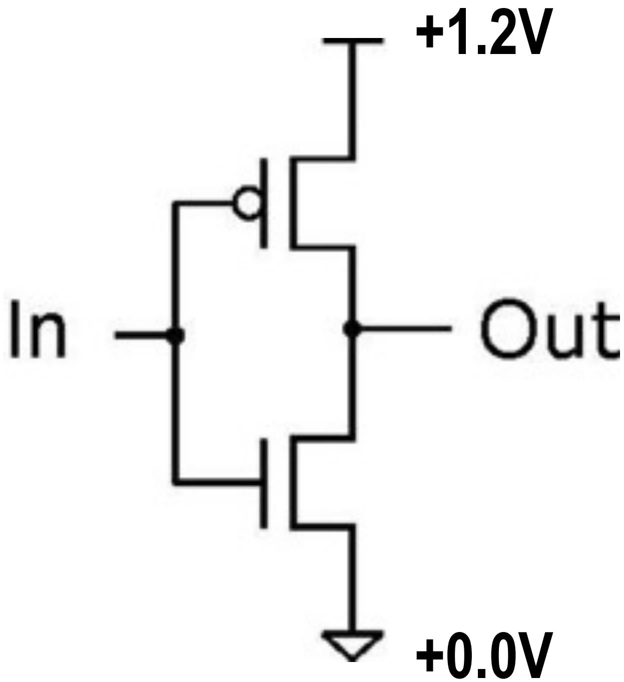

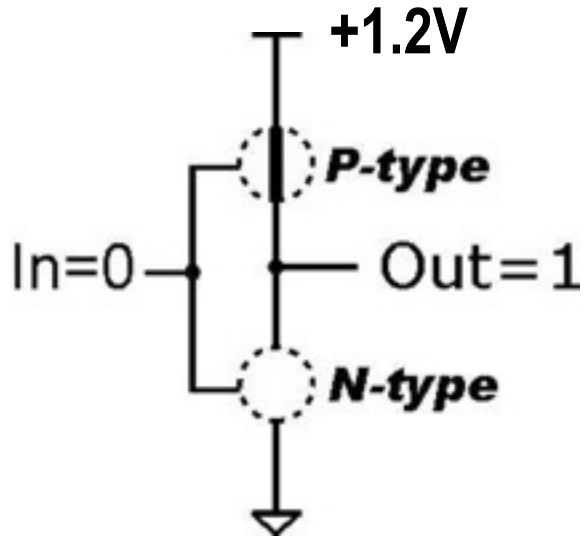

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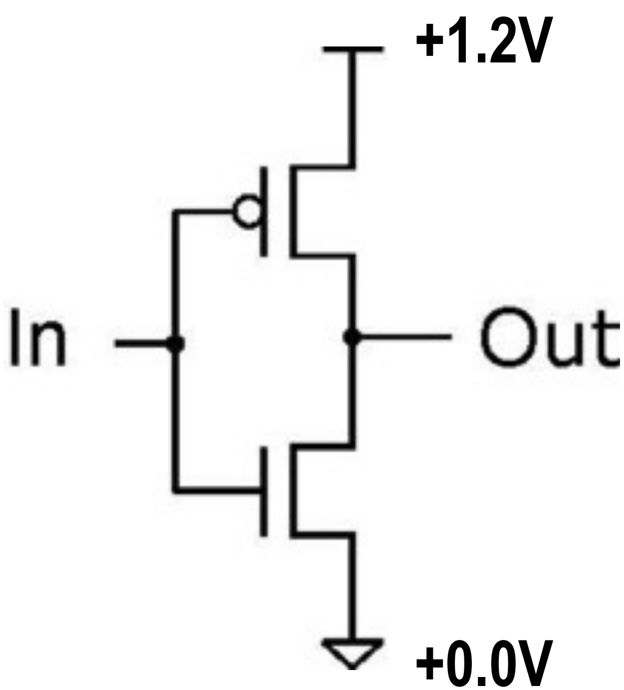
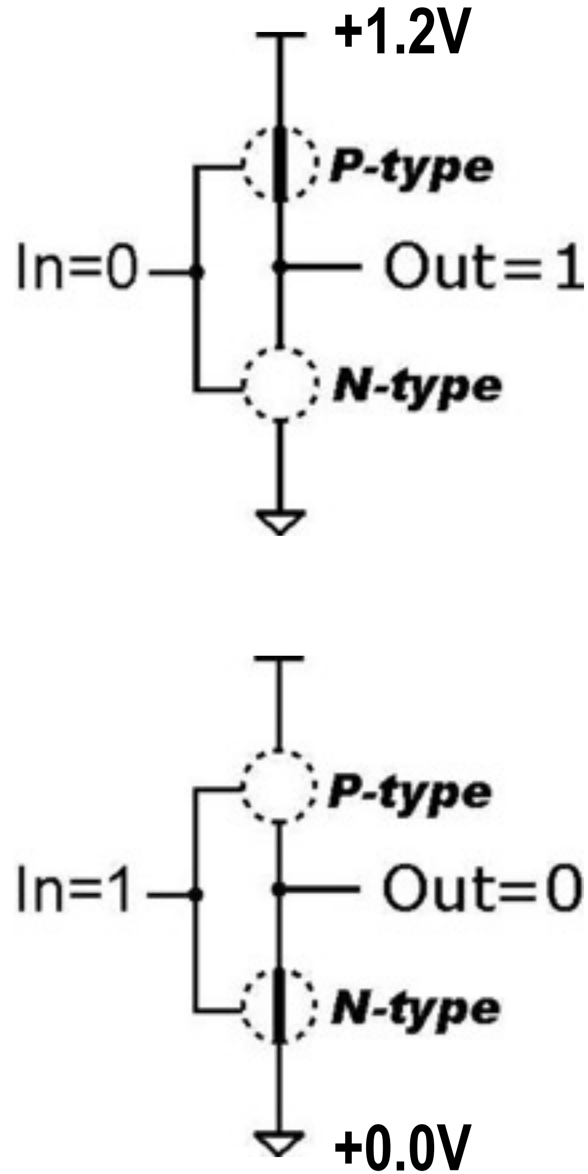
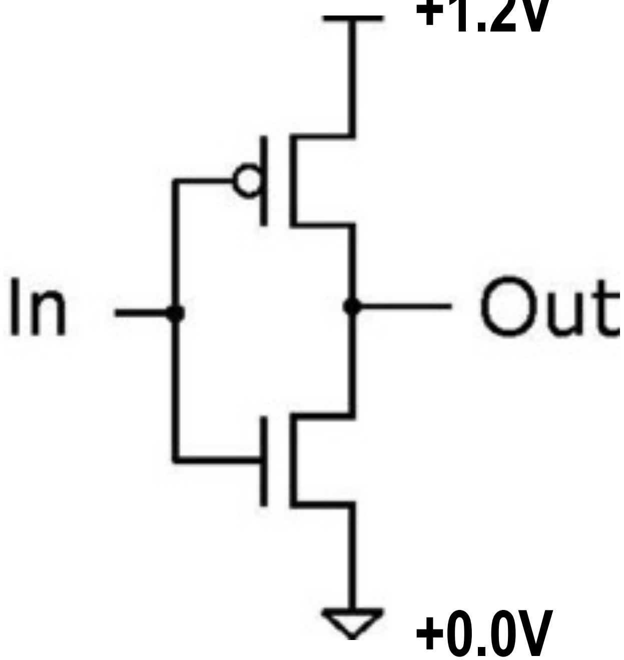


+1.2V
In
Out
+0.0V

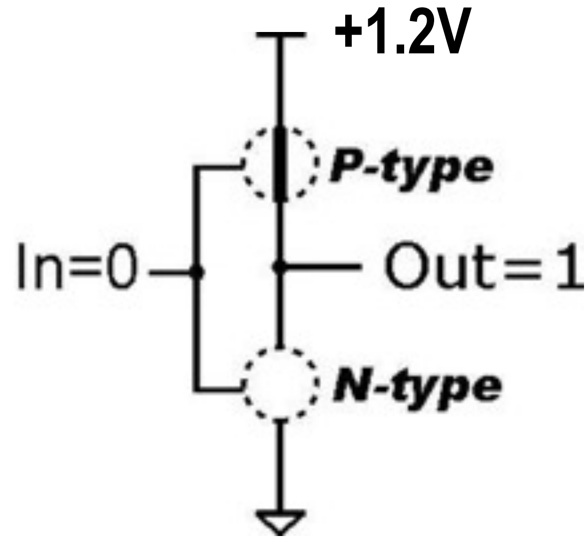


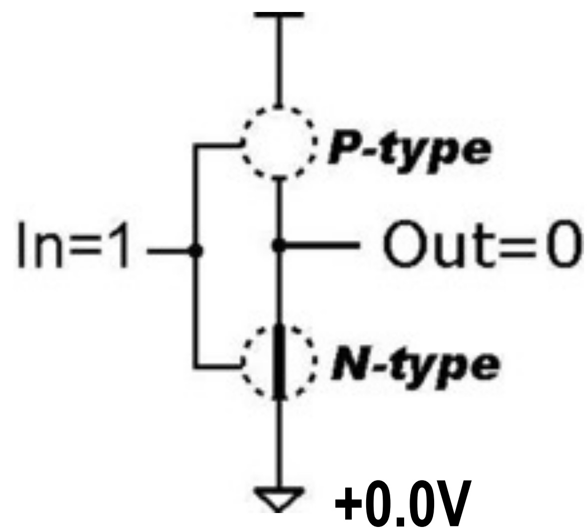
+1.2V
P-type
In=0
Out=1
N-type
P-type
In=1
Out=0
N-type
+0.0V

Inverter



In	Out
0	1
1	0





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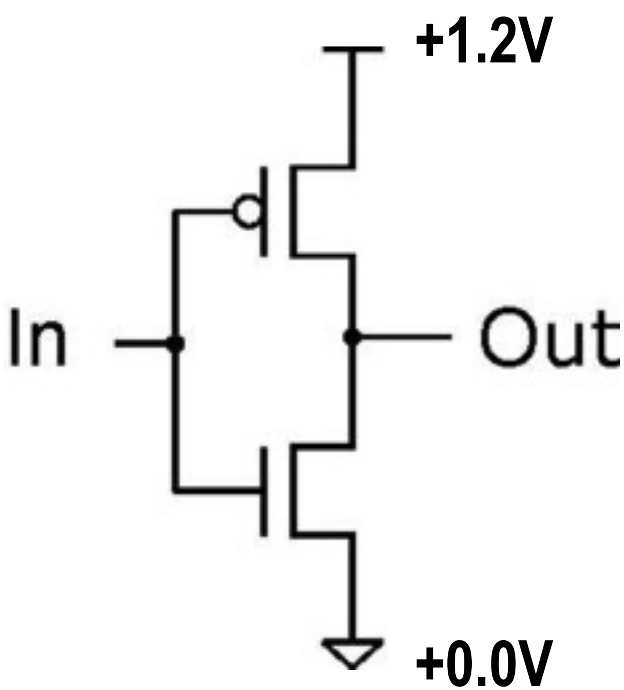

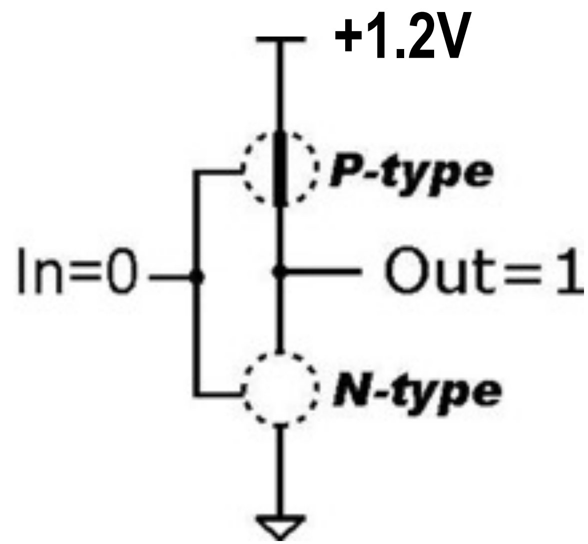

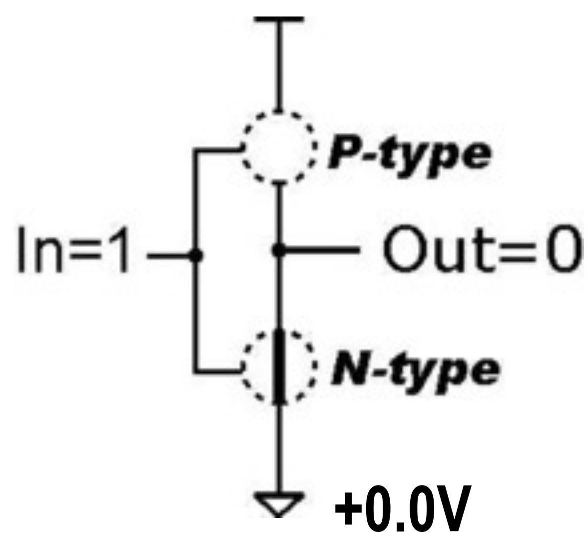

In	Out
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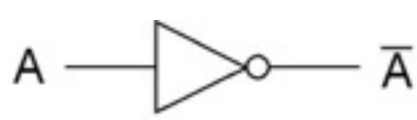
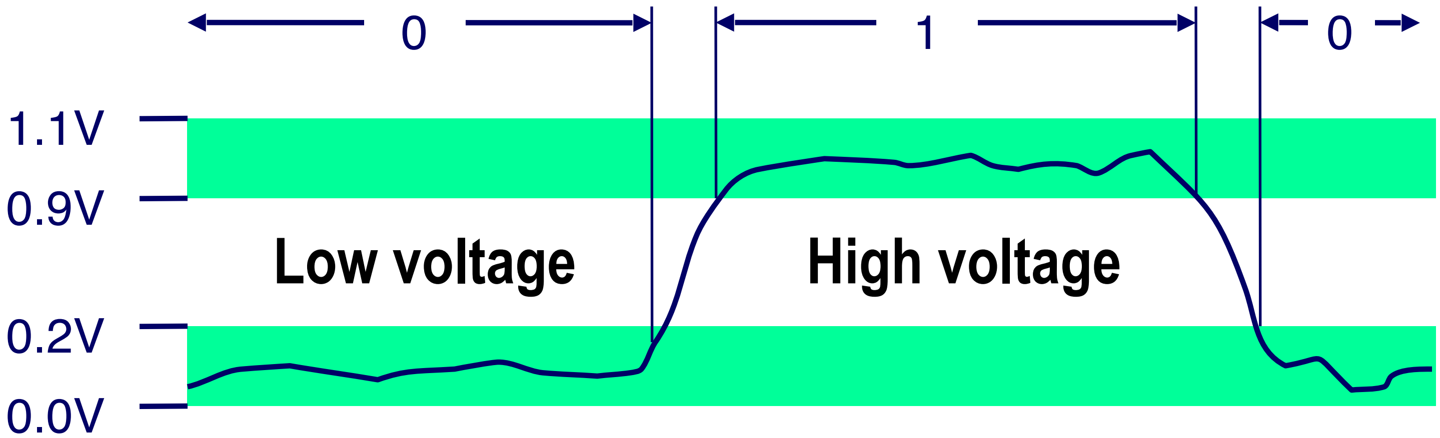
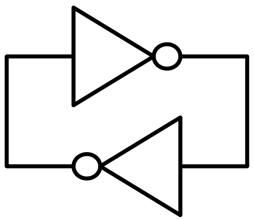


Store/Access Data





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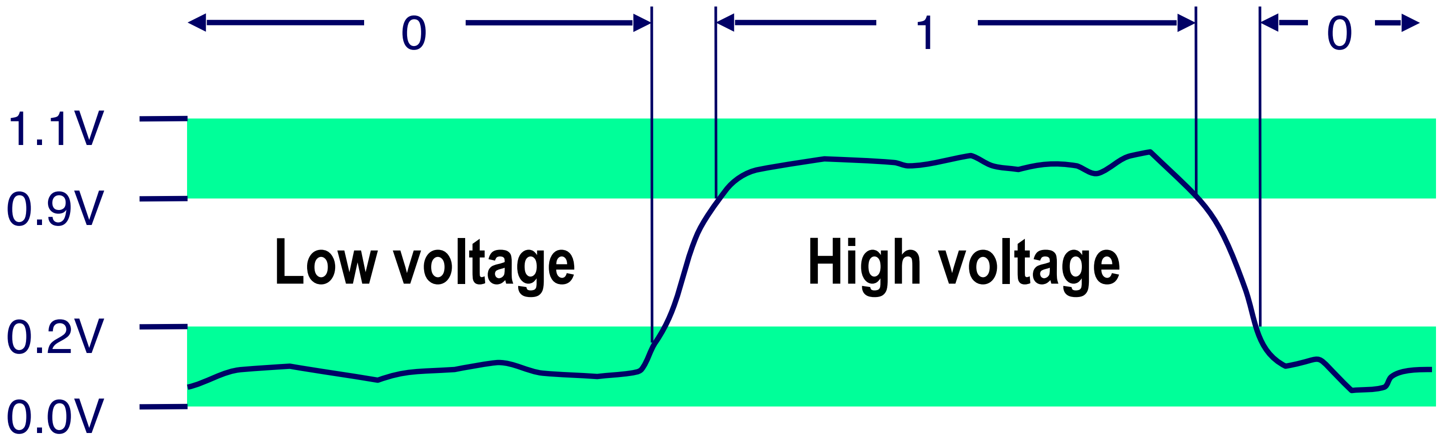

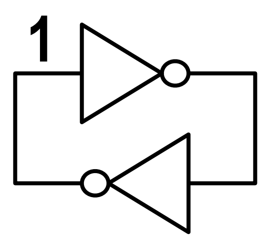

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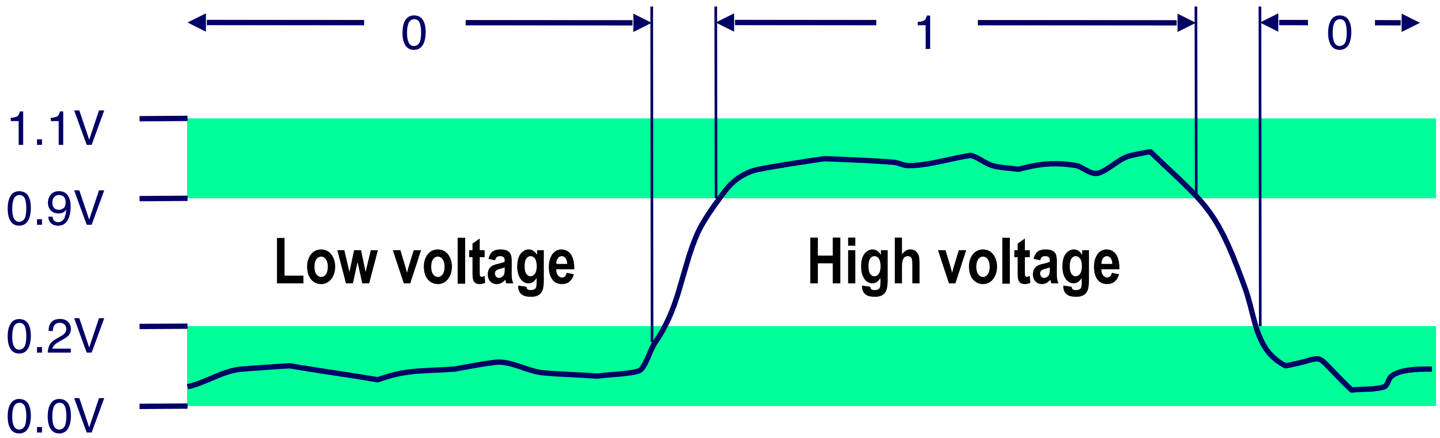

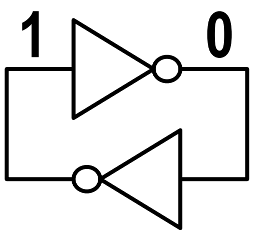

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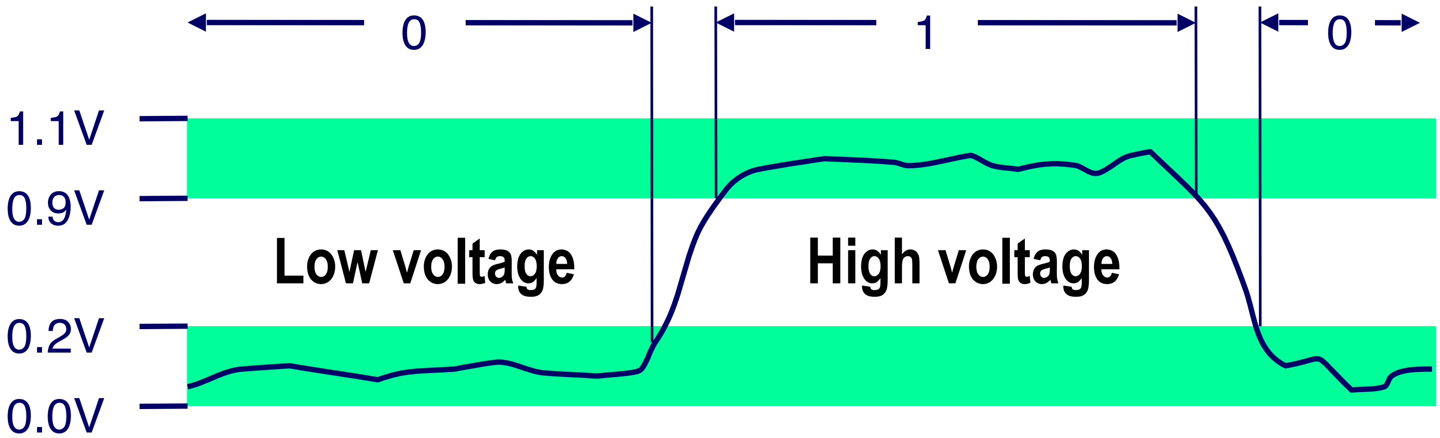

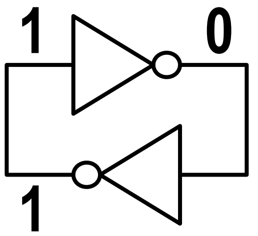

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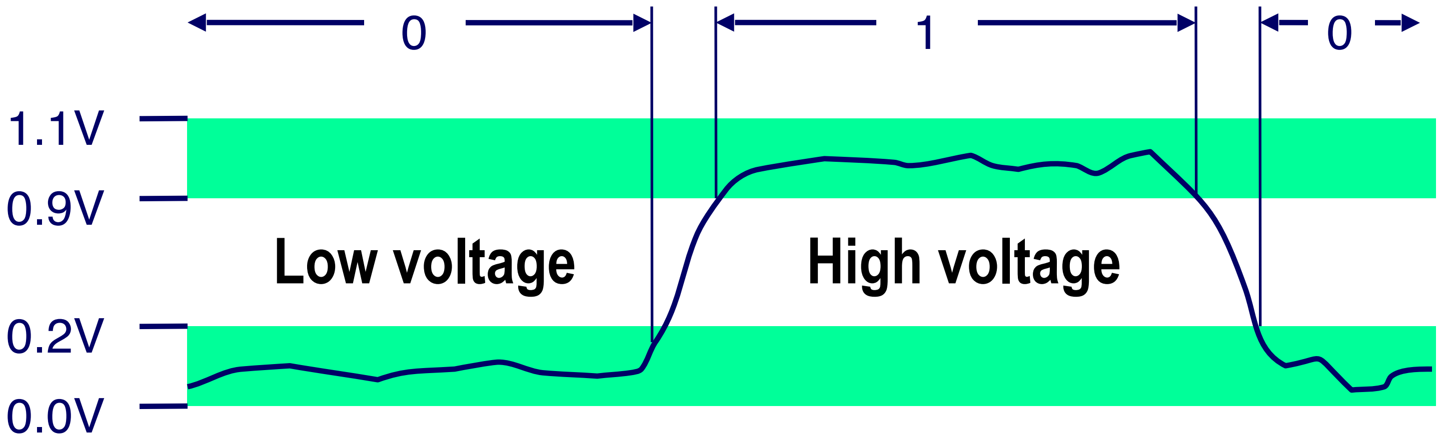
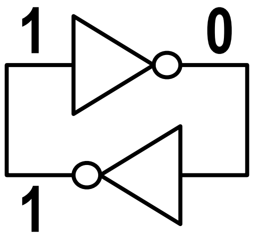


- Two cross coupled inverters store a single bit
  - Feedback path persists the value in the “cell”



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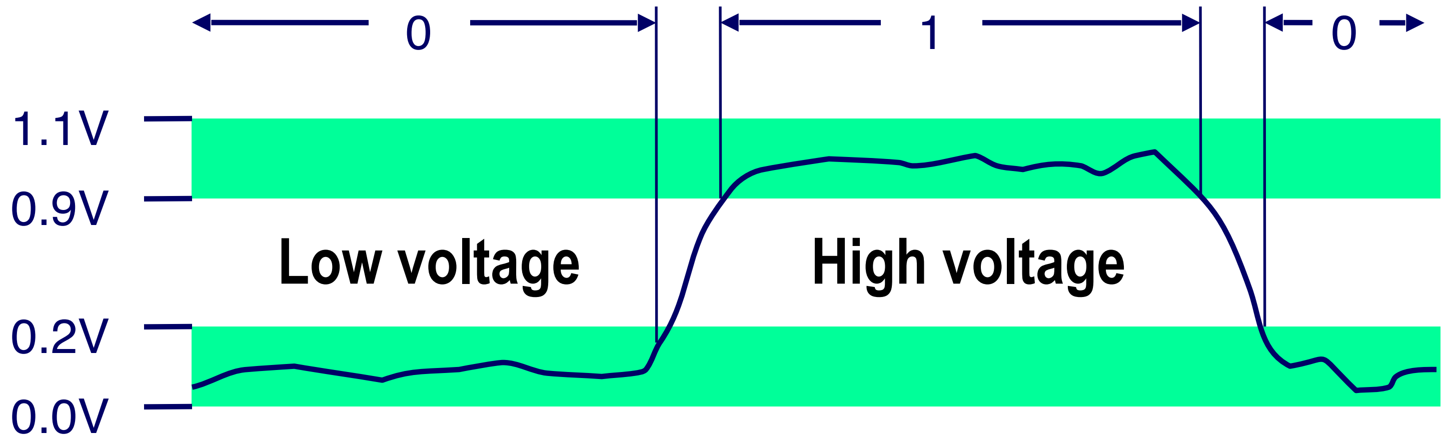

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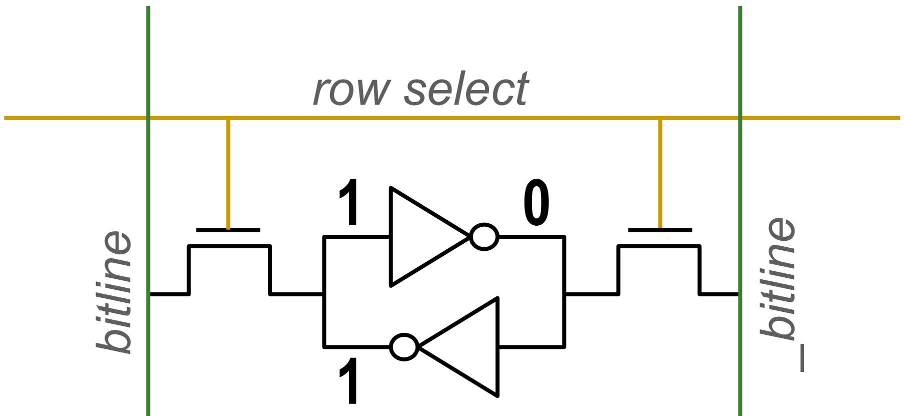

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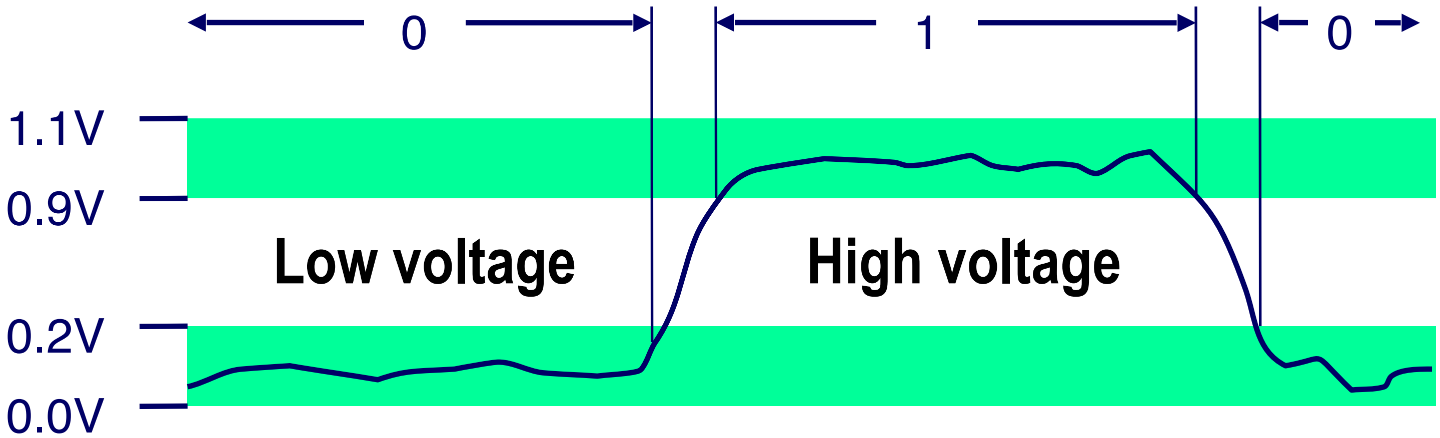

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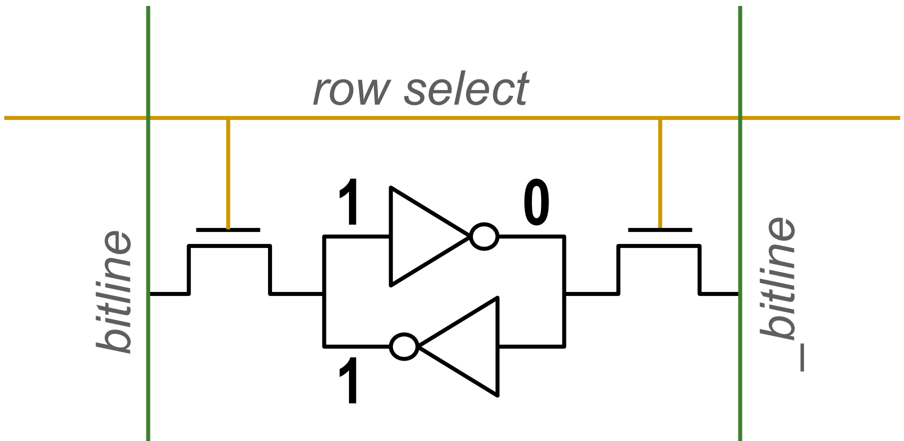

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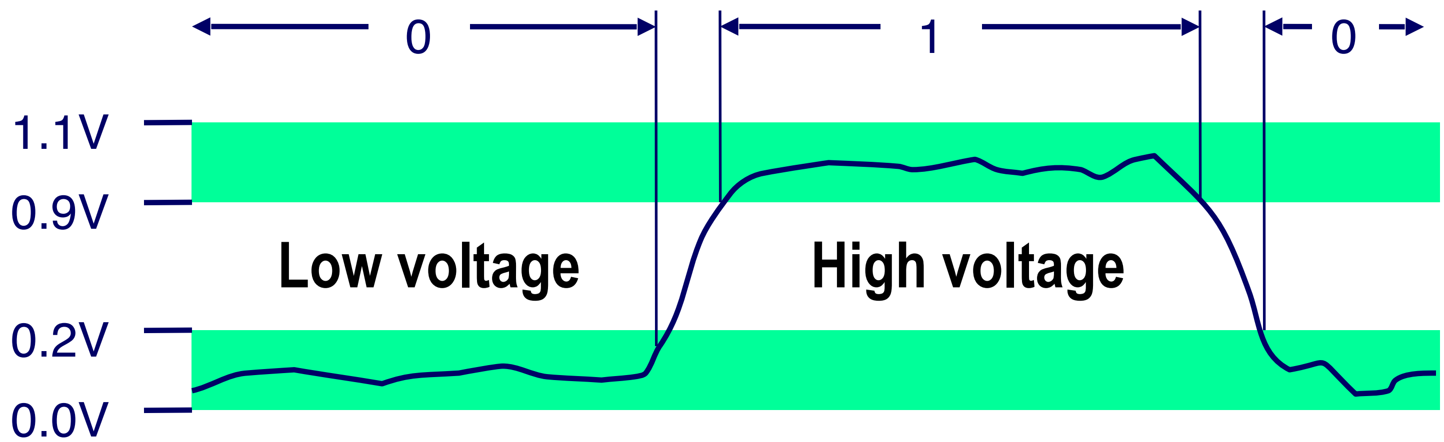

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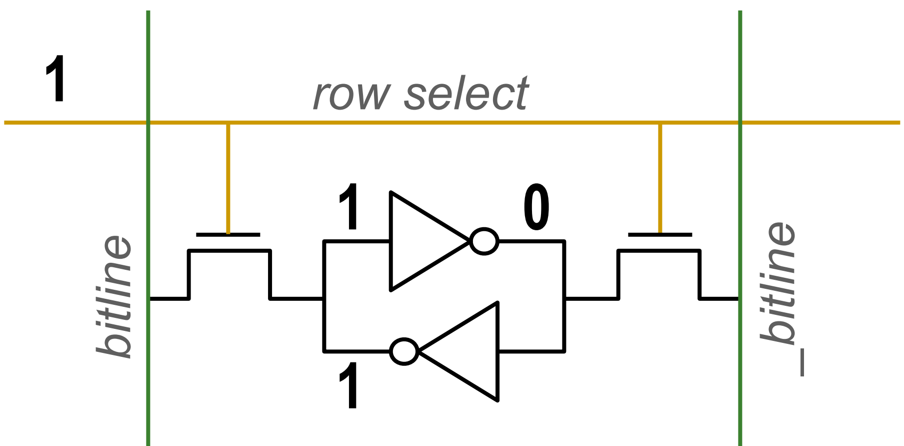

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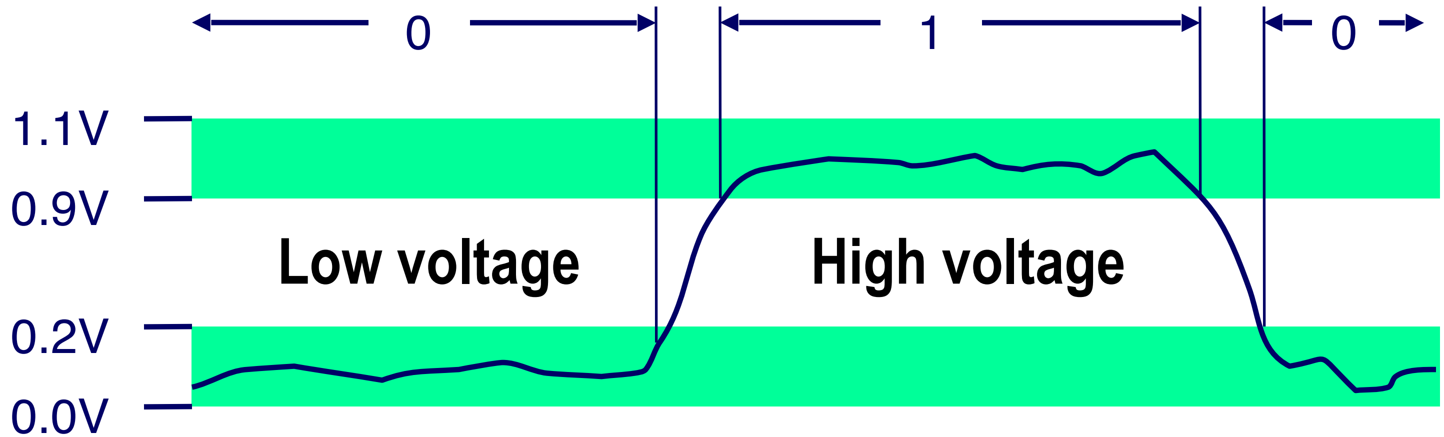

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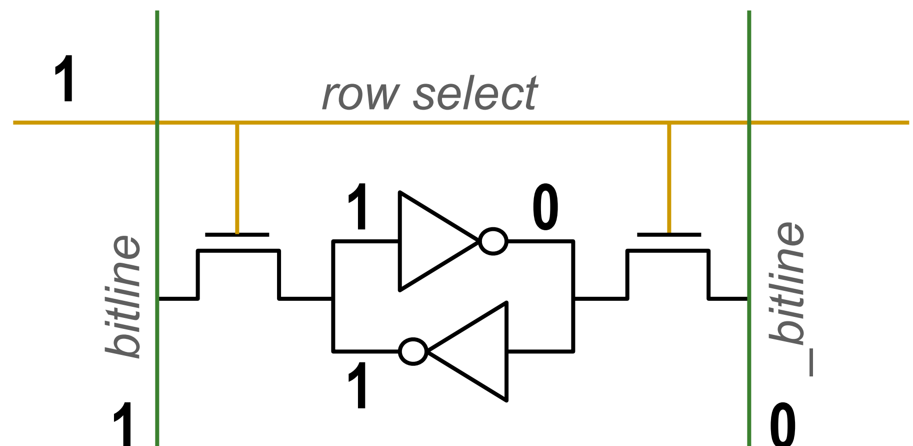

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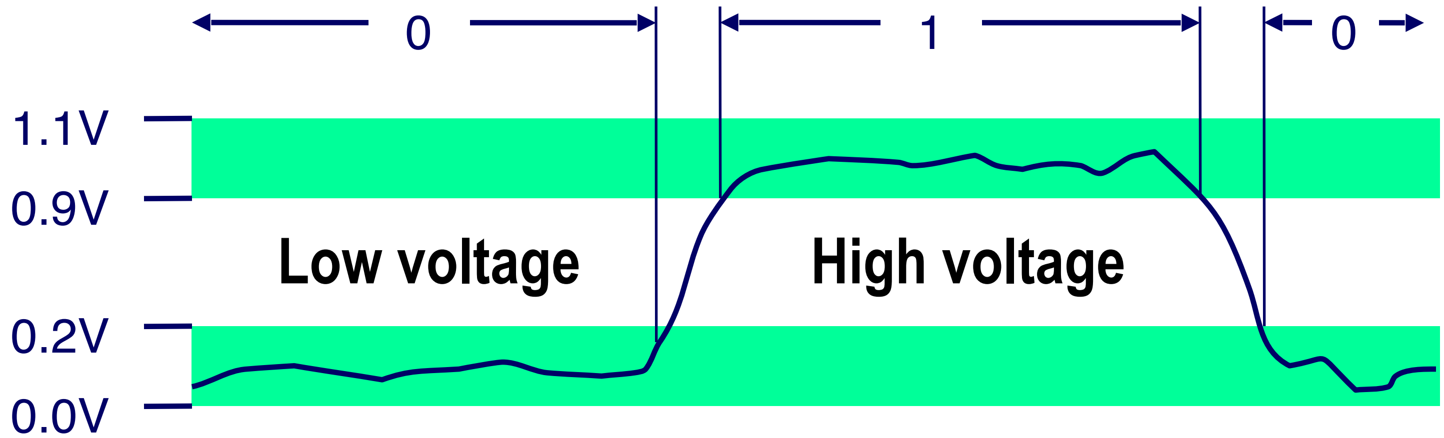

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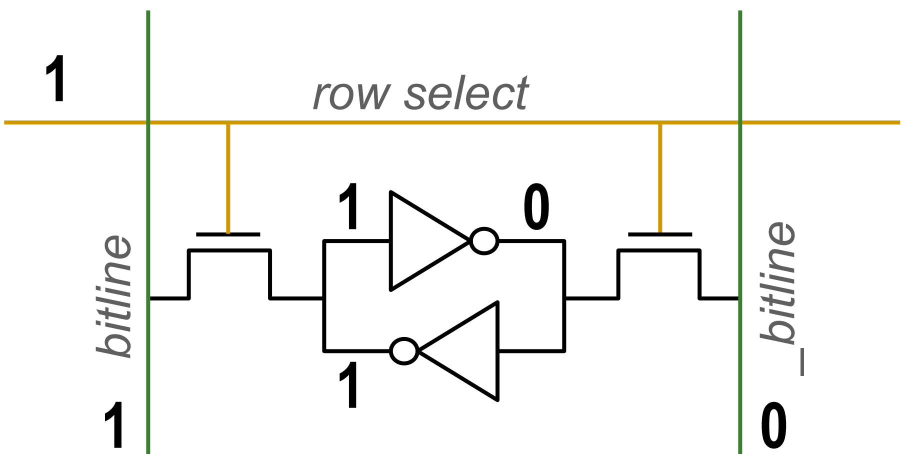
  - 2 transistors for access



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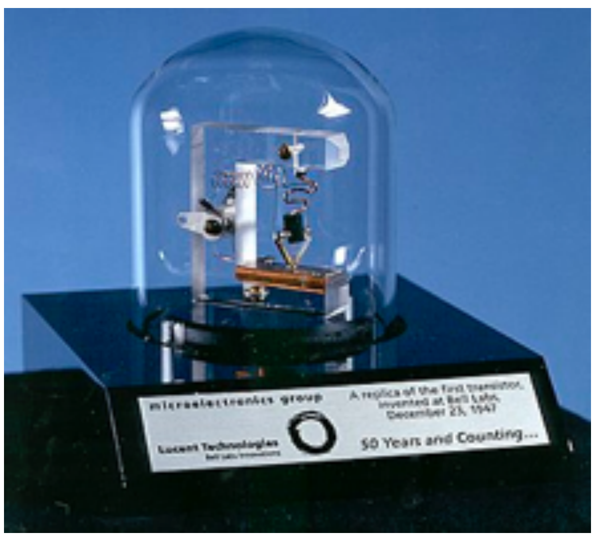
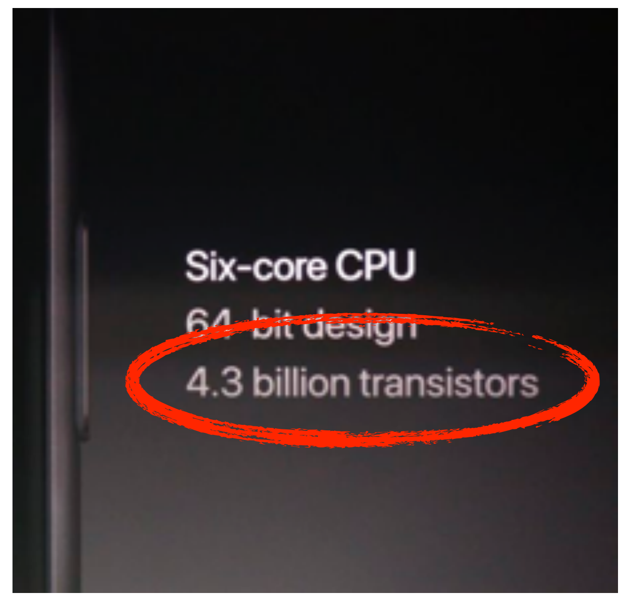
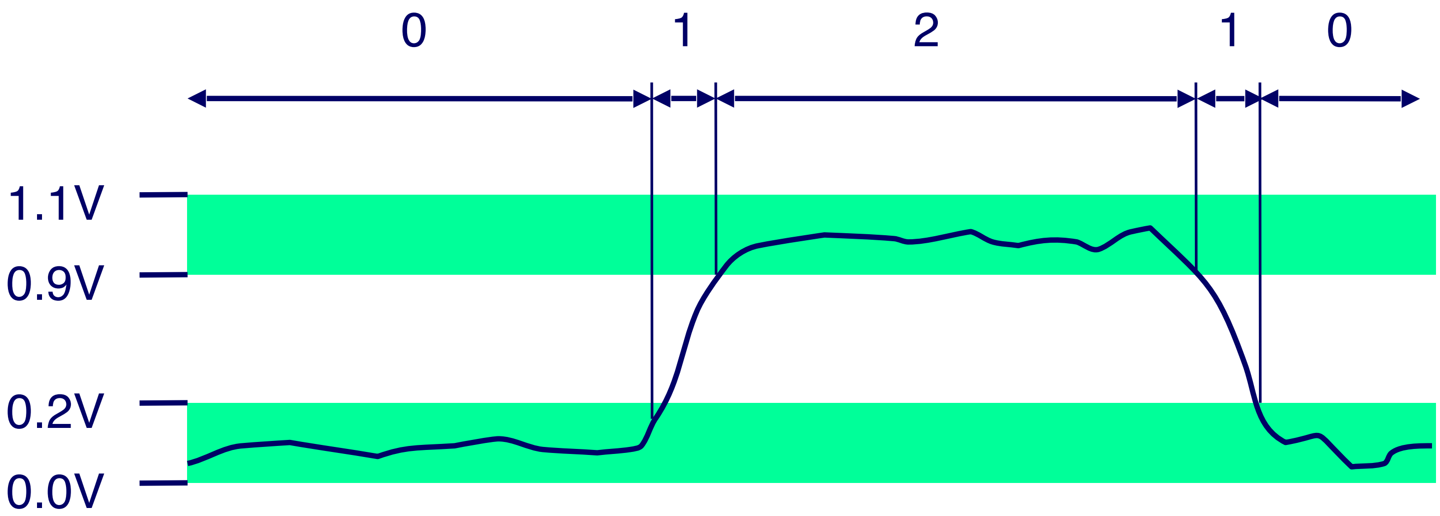




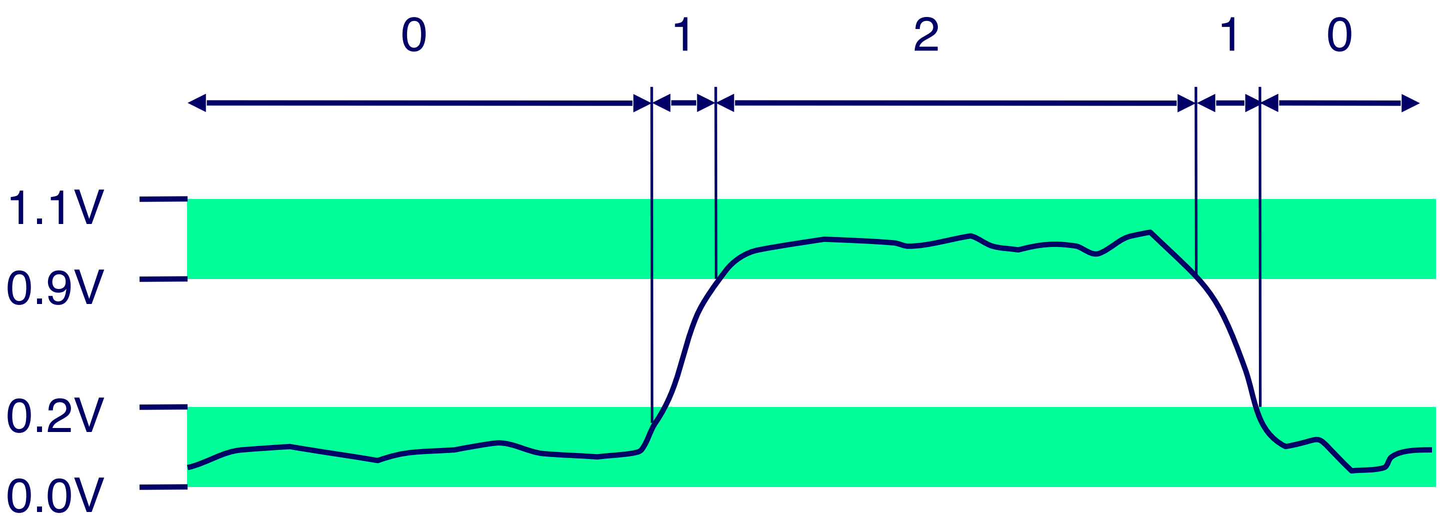
Aside: Why Limit Ourselves Only to Binary?

- Voltage is continuous. Why interpret it only as 0s and 1s?



Aside: Why Limit Ourselves Only to Binary?

- Voltage is continuous. Why interpret it only as 0s and 1s?
- Answer: Noise



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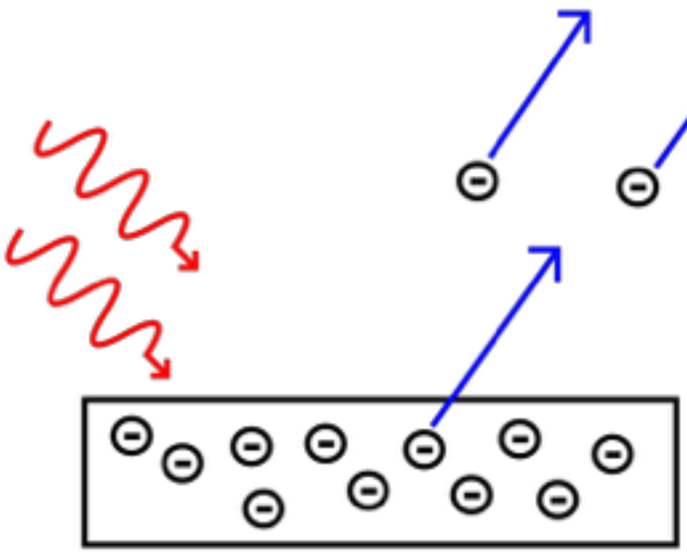
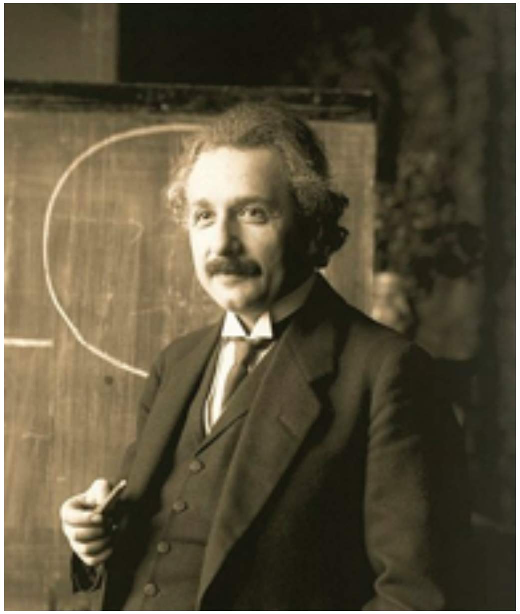
- Classic Example: Camera Sensor
    - Photoelectric Effect





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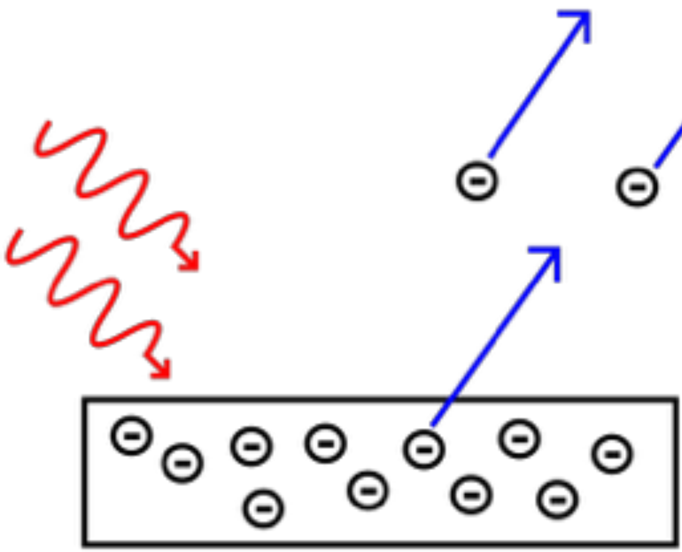

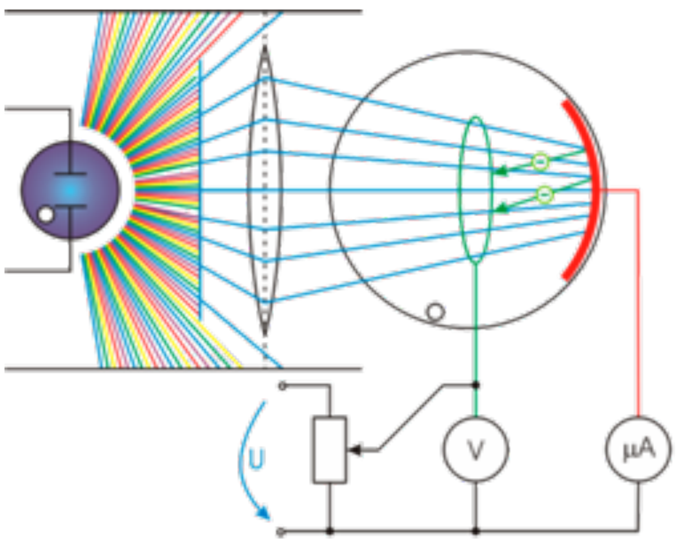

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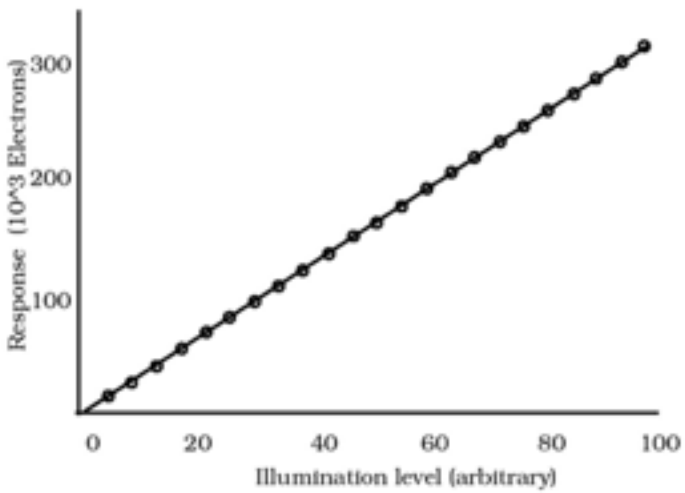
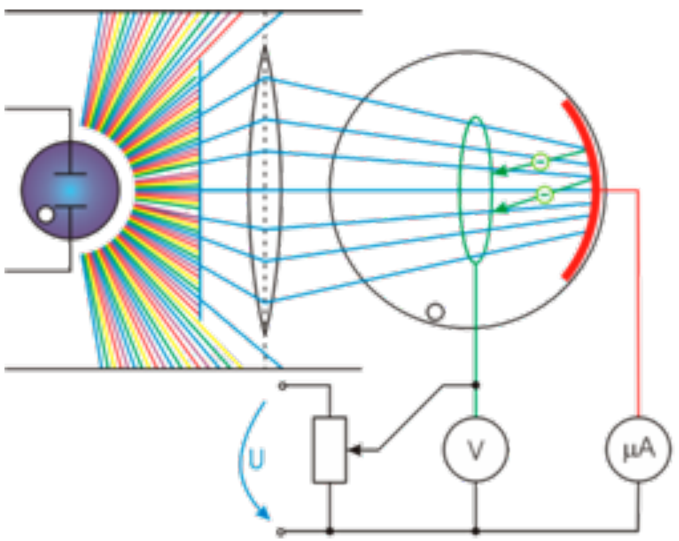


(Epperson, P.M. et al. Electro-optical characterization of the Tektronix TK5 ...., Opt Eng., 25, 1987)



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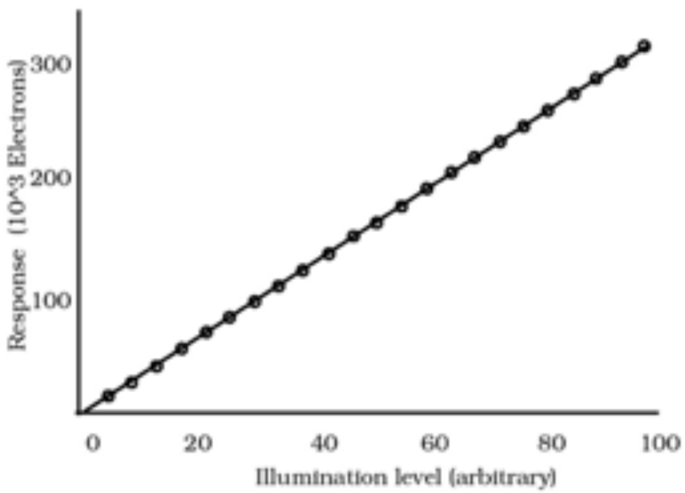

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Binary Notation

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- Base 2 Number Representation (Binary)

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- C.f., Base 10 number representation (Decimal)

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Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

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```
  0110
+ 0101
------
  1011
```

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0	0000
1	0001
2	0010
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4	0100
5	0101
6	0110
7	0111
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```
  0110
+ 0101
------
  1011
```

```
  6
+ 5
----
 11
```

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Hexdecimal (Hex) Notation

- Base 16 Number Representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Four bits per Hex digit
  - $11111110_2 = FE_{16}$
- Write $FA1D37B_{16}$ in C as
  - 0xFA1D37B
  - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit, Byte, Word

- Byte = 8 bits
  - Binary 00000000_2 to 11111111_{2}; Decimal: 0_{10} to 255_{10}; Hex: 00_{16} to FF_{16}
  - Least Significant Bit (LSb) vs. Most Significant Bit (MSb)

10110111

MSb **LSb**

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MSb **LSb**

- Word = 4 Bytes (32-bit machine) / 8 Bytes (64-bit machine)
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  - Binary 00000000_2 to 11111111_{2}; Decimal: 0_{10} to 255_{10}; Hex: 00_{16} to FF_{16}
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$\boxed{1}011011\boxed{1}$

MSb **LSb**

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Questions?

Today: Representing Information in Binary

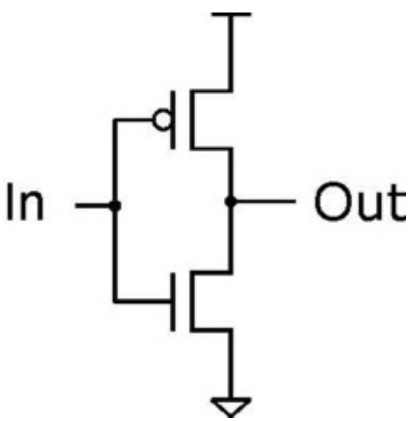
- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Bit-level manipulations

Not

- **~A = 1 when A=0**

~	
0	1
1	0



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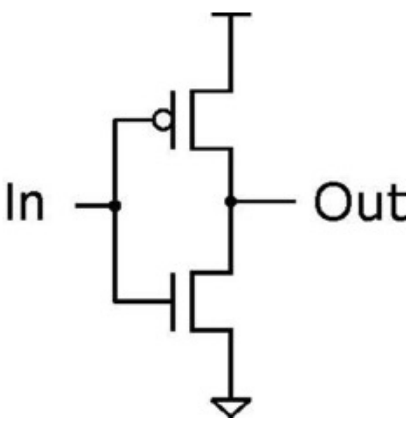

Or

- A|B = 1 when either A=1 or B=1

$\vert$	0	1
0	0	1
1	1	1

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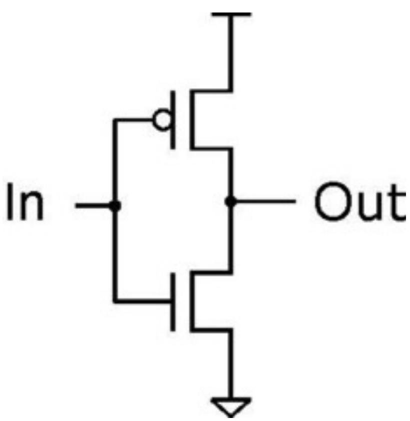

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And

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&	0	1
0	0	0
1	0	1

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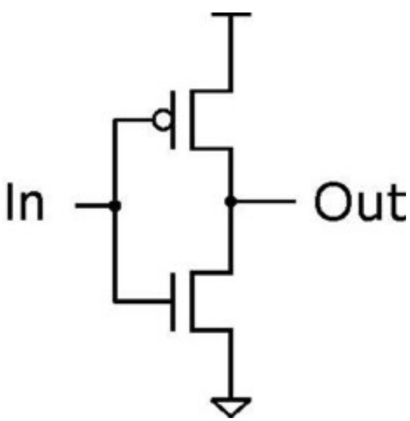

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- A&B = 1 when both A=1 and B=1

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0	0	0
1	0	1

Exclusive-Or (Xor)

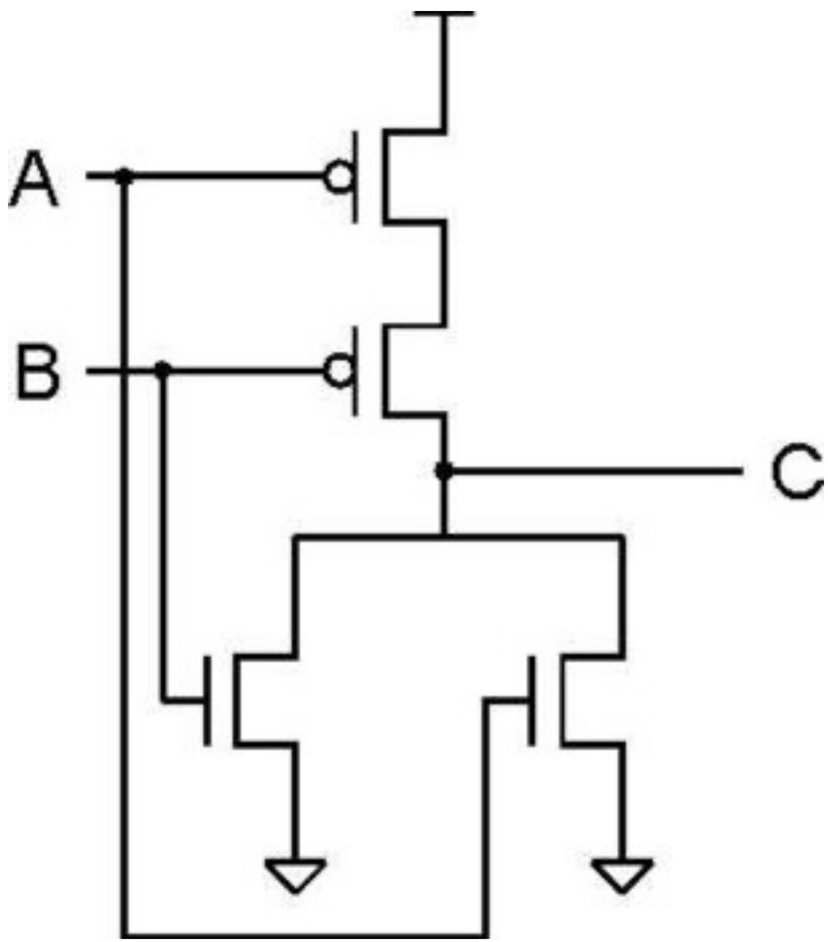
- A^B = 1 when either A=1 or B=1, but not both

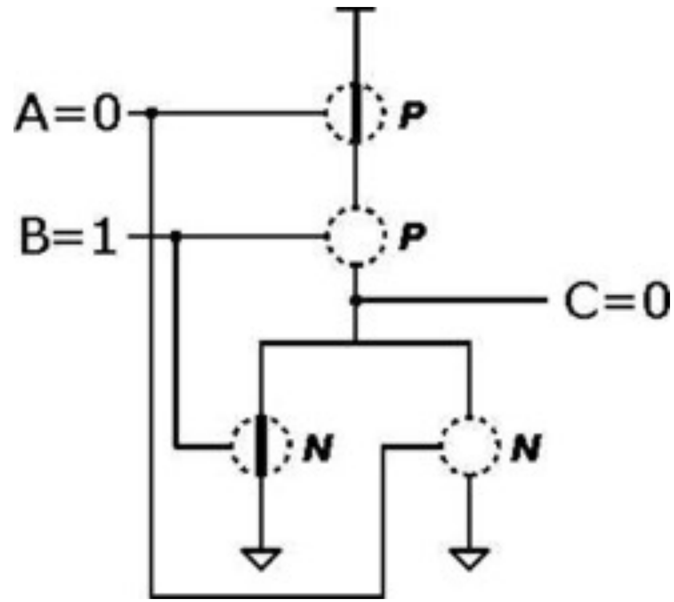
^	0	1
0	0	1
1	1	0

NOR (OR + NOT)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

NOR (OR + NOT)





A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Bit Vector Operations

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
__________    __________    __________    __________
```

Bit Vector Operations

- Operate on Bit Vectors
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```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  --------      --------      --------      --------
  01000001
```

Bit Vector Operations

- Operate on Bit Vectors
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```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------     --------
  01000001     01111101
```

Bit Vector Operations

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```
  01101001     01101001     01101001
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  --------     --------     --------     --------
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```

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- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
----------    ----------    ----------    ----------
  01000001      01111101      00111100      10101010
```

Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - `~0x41` → `0xBE`
    - `~01000001`$_2$ → `10111110`$_2$
  - `~0x00` → `0xFF`
    - `~00000000`$_2$ → `11111111`$_2$
  - `0x69 & 0x55` → `0x41`
    - `01101001`$_2$ `&` `01010101`$_2$ → `01000001`$_2$
  - `0x69 | 0x55` → `0x7D`
    - `01101001`$_2$ `|` `01010101`$_2$ → `01111101`$_2$

Contrast: Logic Operations in C

- Contrast to Logical Operators
  - `&&, ||, !`
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination (e.g., 0 && 1 && 1)
- Examples (char data type)
  - `!0x41` → `0x00`
  - `!0x00` → `0x01`
  - `!!0x41` → `0x01`

  - `0x69 && 0x55` → `0x01`
  - `0x69 || 0x55` → `0x01`
  - `p && *p` (avoids null pointer access)

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
      - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
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  - Logical shift
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  - Arithmetic shift
    - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount < 0 or ≥ word size

Argument x	`01100010`
<< 3	
Log. >> 2	
Arith. >> 2	

Argument x	`10100010`
<< 3	
Log. >> 2	
Arith. >> 2	

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Arith. >> 2	*`00`*`011000`

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Arith. >> 2	

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Arith. >> 2	

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<< 3	00010*000*
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Log. >> 2	*00*101000
Arith. >> 2	

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Log. >> 2	*00*011000
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Log. >> 2	*00*101000
Arith. >> 2	101000

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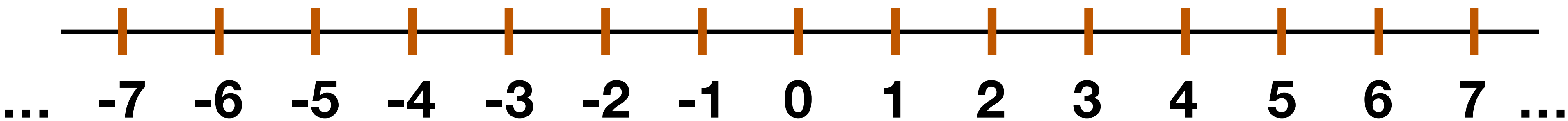
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

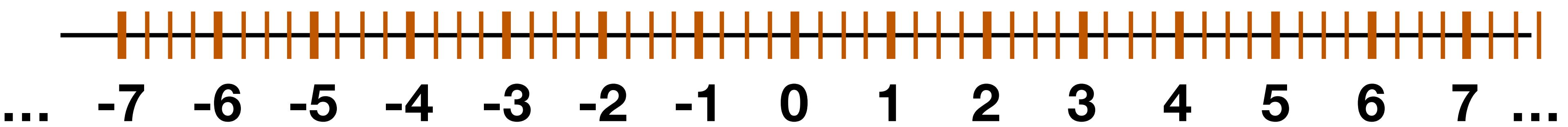
Representing Numbers in Binary

- What numbers do we need to represent in bits?
  - Integer (Negative and Non-negative)
  - Fractions
  - Irrationals



Representing Numbers in Binary

- What numbers do we need to represent in bits?
  - Integer (Negative and Non-negative)
  - Fractions
  - Irrationals



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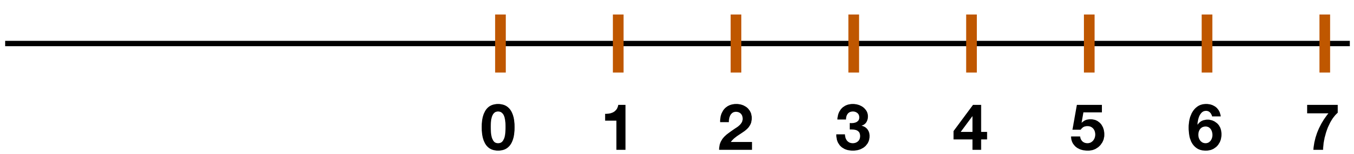

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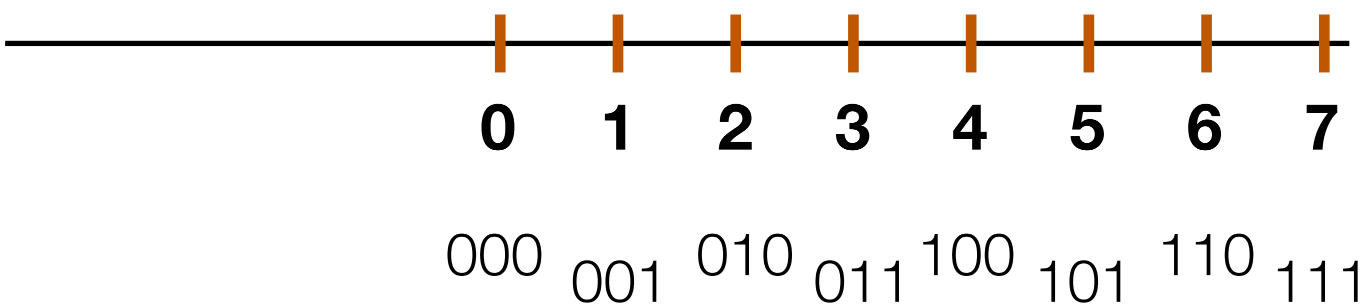

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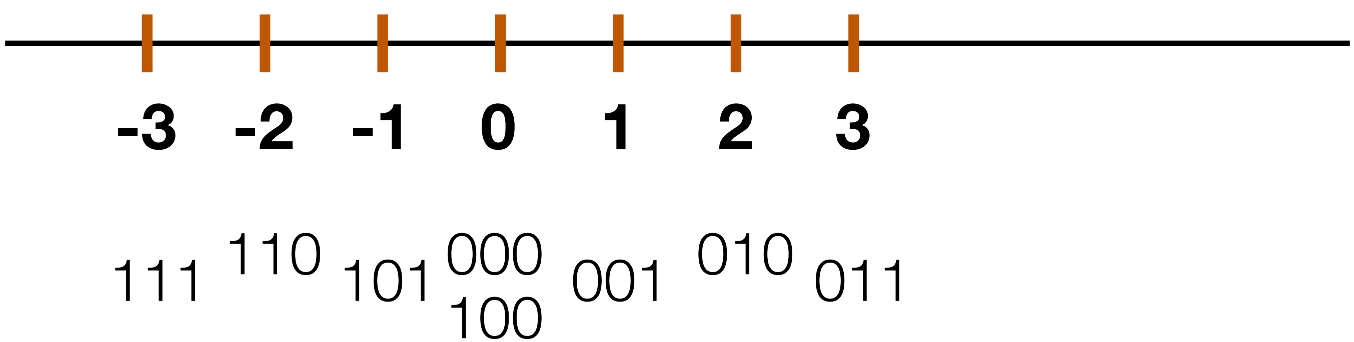

Sign-Magnitude Implications

- Bits have different semantics
  - Two zeros…
  - Normal arithmetic doesn't work
  - Make hardware design harder

Signed Value	Binary
0	000
1	001
2	010
3	011
-0	100
-1	101
-2	110
-3	111

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```
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-------
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~~2~~
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Encoding Negative Numbers

- Solution 2: Two’s Complement

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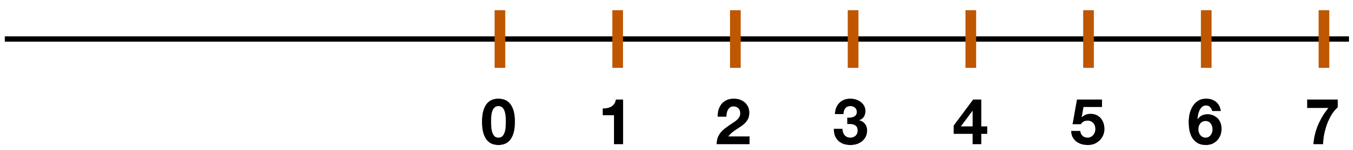

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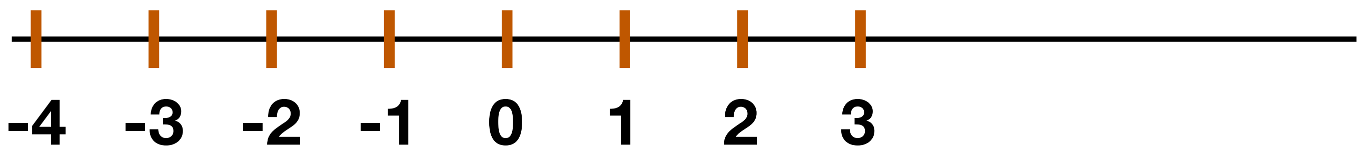

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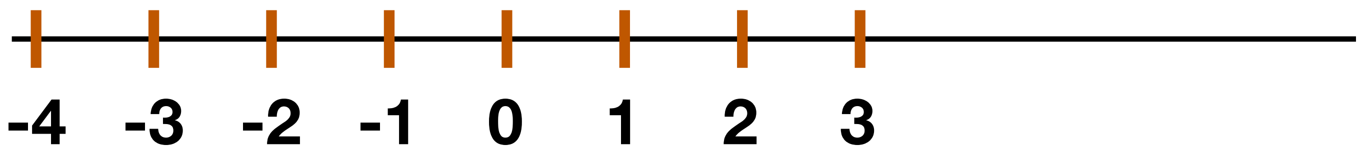

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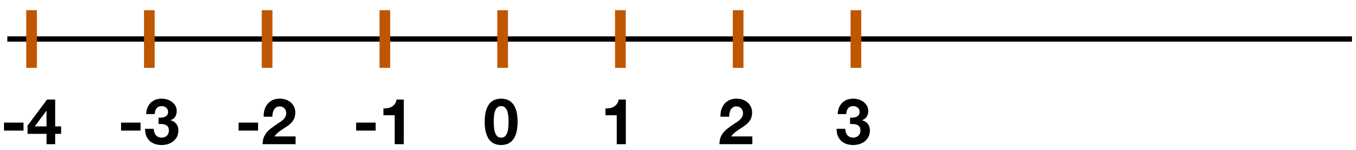

Signed Weight	Unsigned Weight	Bit Position
2^0	2^0	0
2^1	2^1	1
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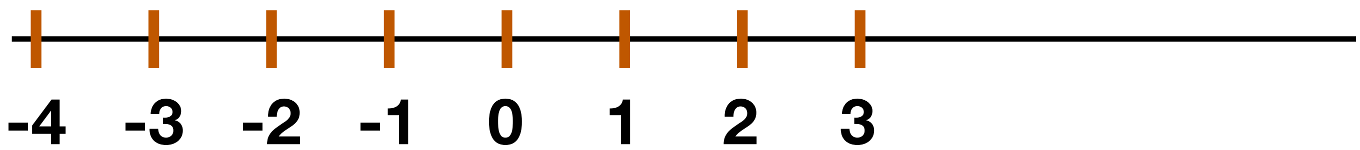

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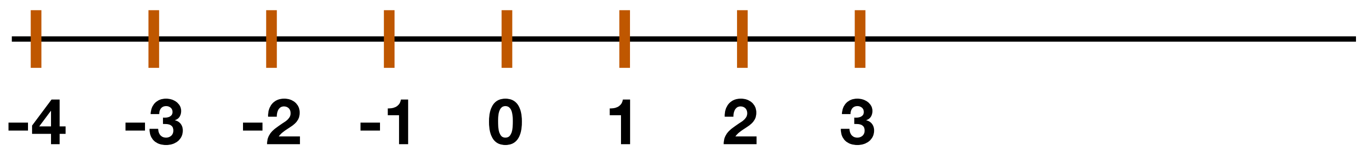

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$$101_2 = 1*2^0 + 0*2^1 - 1*2^2 = -3_{10}$$

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2	2	010
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-4	4	100
-3	5	101
-2	6	110
-1	7	111

Two-Complement Encoding Example

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Two-Complement Implications

- Only 1 zero
- Usual arithmetic still works
- There is a bit that represents sign!
- Most widely used in today's machines

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    111
```

```
     2
+)  -3
-------
    -1
```

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- Unsigned Values
  - *UMin* = 0

    000…0
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Values for *W* = 16

	Decimal	Hex	Binary
UMax	**65535**	**FF FF**	**11111111 11111111**
TMax	**32767**	**7F FF**	**01111111 11111111**
TMin	**-32768**	**80 00**	**10000000 00000000**
-1	**-1**	**FF FF**	**11111111 11111111**
0	**0**	**00 00**	**00000000 00000000**

Numeric Ranges

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  - *UMin* = 0

    000…0
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- Two's Complement Values
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    100…0
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    011…1
- Other Values
  - Minus 1

    111…1

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-1	**-1**	**FF FF**	11111111 11111111
0	**0**	00 00	00000000 00000000

Data Representations in C (in Bytes)

- By default variables are signed
- Unless explicitly declared as unsigned (e.g., `unsigned int`)
- Signed variables use two-complement encoding

C Data Type	32-bit	64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8

Data Representations in C (in Bytes)

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
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- C Language
  - `#include <limits.h>`
  - Declares constants, e.g.,
    - `ULONG_MAX`
    - `LONG_MAX`
    - `LONG_MIN`
  - Values platform specific

Can We Represent Fractions in Binary?

- What does 10.01_2 mean?
- C.f., Decimal
    - $12.45 = 1*10^1 + 2*10^0 + 4*10^{-1} + 5*10^{-2}$
- $10.01_2 = 1*2^1 + 0*2^0 + 0*2^{-1} + 1*2^{-2} = 2.25_{10}$

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Decimal	Binary
0	00.00
0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
1.25	01.01
1.5	01.10
1.75	01.11
2	10.00
2.25	10.01
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2.75	10.11
3	11.00
3.25	11.01
3.5	11.10
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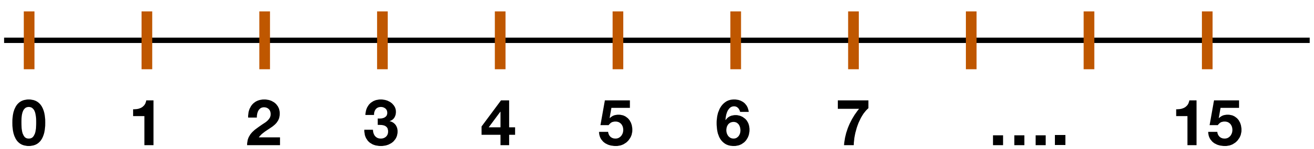

Decimal	Binary
0	00.00
0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
1.25	01.01
1.5	01.10
1.75	01.11
2	10.00
2.25	10.01
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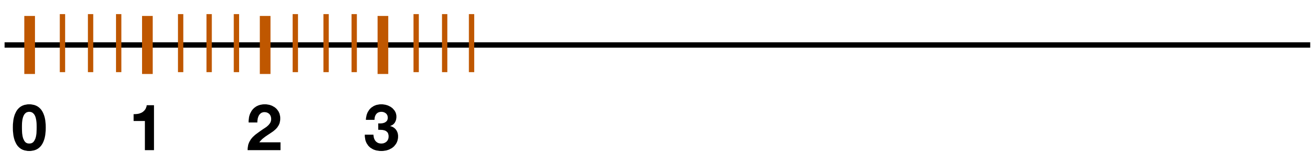

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0.75	00.11
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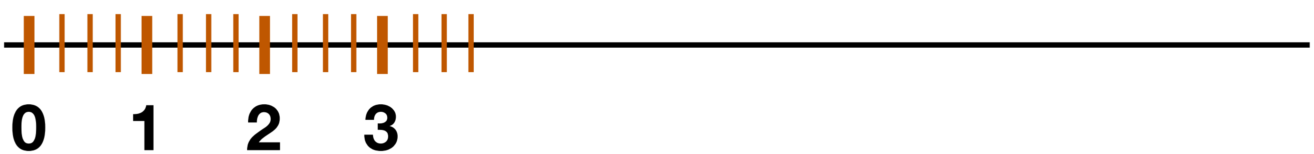


```
   01.10
+  01.01
--------
   10.11
```

```
   1.50
+  1.25
-------
   2.75
```

Decimal	Binary
0	00.00
0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
1.25	01.01
1.5	01.10
1.75	01.11
2	10.00
2.25	10.01
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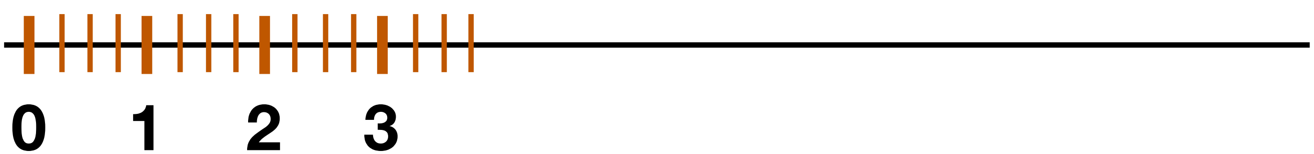

Integer Arithmetic Still Works!

```
  01.10
+ 01.01
-------
  10.11
```

```
  1.50
+ 1.25
------
  2.75
```

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0.25	00.01
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2.25	10.01
2.5	10.10
2.75	10.11
3	11.00
3.25	11.01
3.5	11.10
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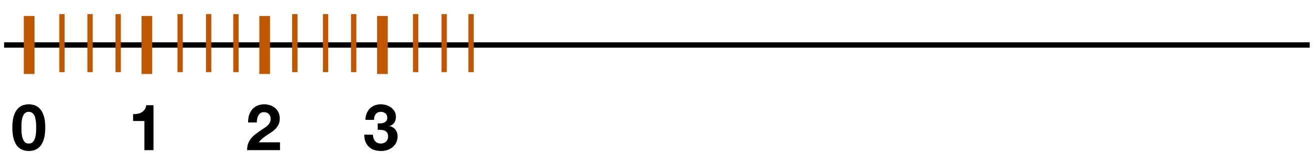
Fixed-Point Representation

- Fixed interval between two representable numbers as long as the binary point stays fixed
  - Each bit represents 0.25_{10}
- Fixed-point representation of numbers
  - Integer is one special case of fixed-point




```
   01.10
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--------
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```

```
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2.25	10.01
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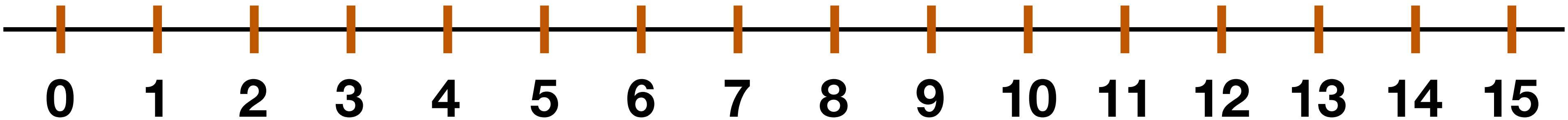
Aside: Quantization



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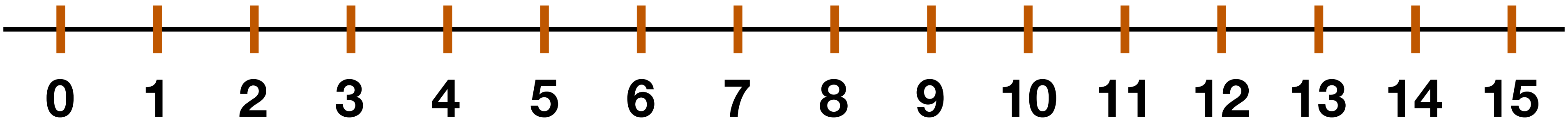

- Representing all integers precisely requires 4 bits

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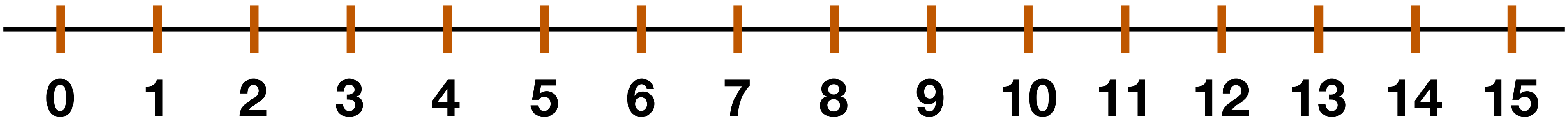

- Representing all integers precisely requires 4 bits
- What if we can tolerate some imprecisions
  - 1, 2, 3 are approximated by 0
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  - We would only need 2 bits

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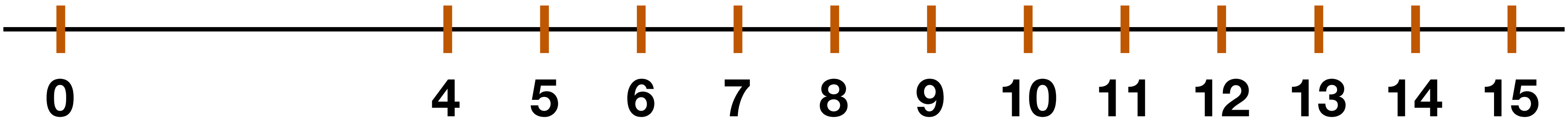

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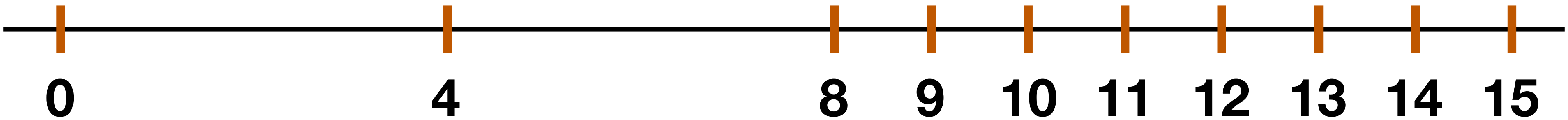

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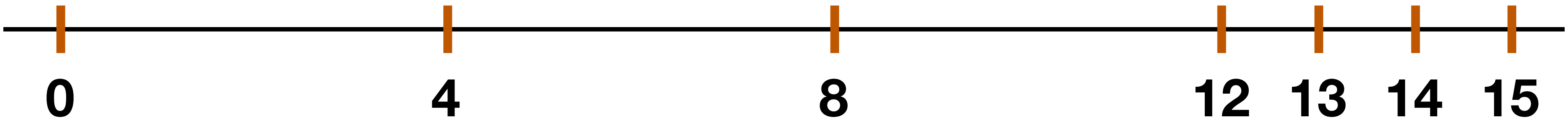

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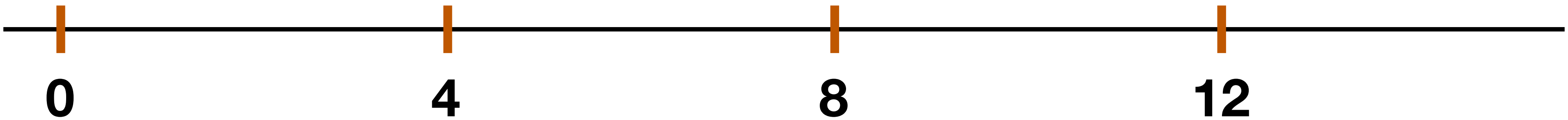

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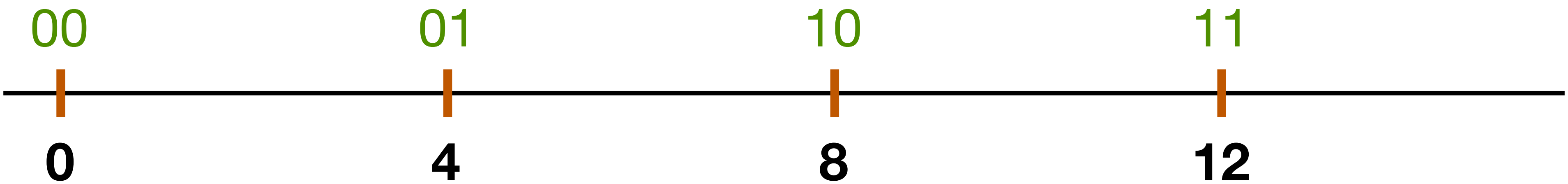

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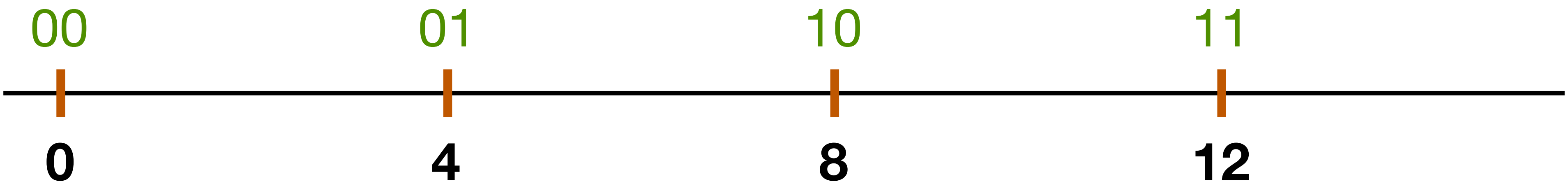

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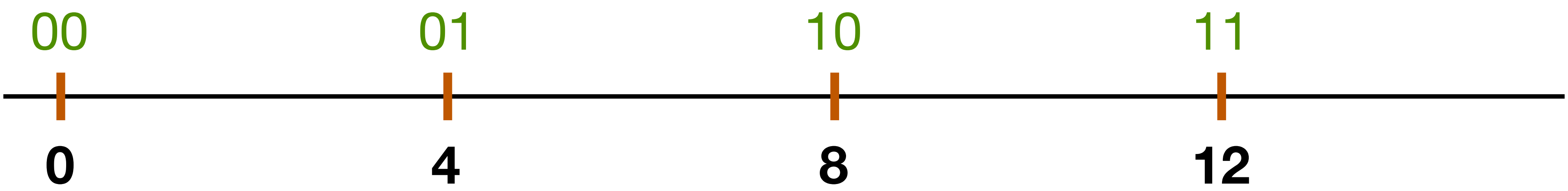
- What if we can tolerate some imprecisions
  - 1, 2, 3 are approximated by 0
  - 5, 6, 7 are approximated by 4…
  - We would only need 2 bits
- That is, 1 bit represents 4_{10}
  - 10_2 becomes $4 * (1 * 2^1) = 8$
  - Every time we increment a bit, the value is incremented by 4
  - 1, 2, 3 are represented approximately by 00_2

Aside: Quantization



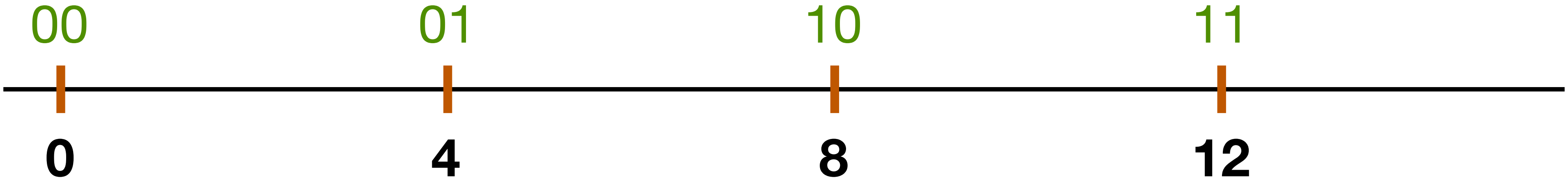
- Representing all integers precisely requires 4 bits
- What i
  - 1, 2
  - 5, 6
  - We

Note that this is different from "base 4"

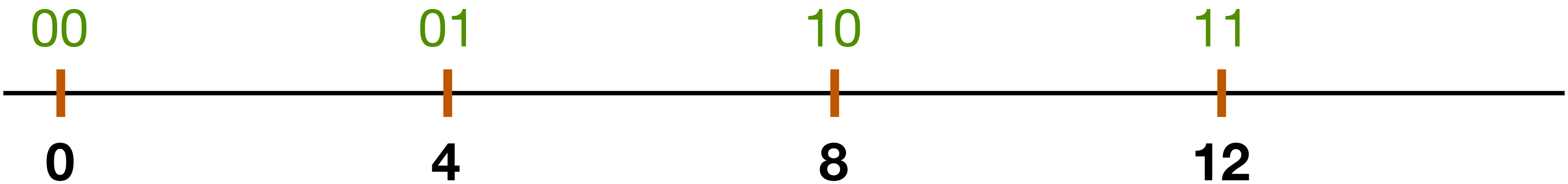
- $10_4 = 1 * 4^1 + 0 * 4^0 = 4$
- Every increment still only increments 1

- That is, 1 bit represents 4_{10}
  - 10_2 becomes $4 * (1 * 2^1) = 8$
  - Every time we increment a bit, the value is incremented by 4
  - 1, 2, 3 are represented approximately by 00_2

Aside: Quantization

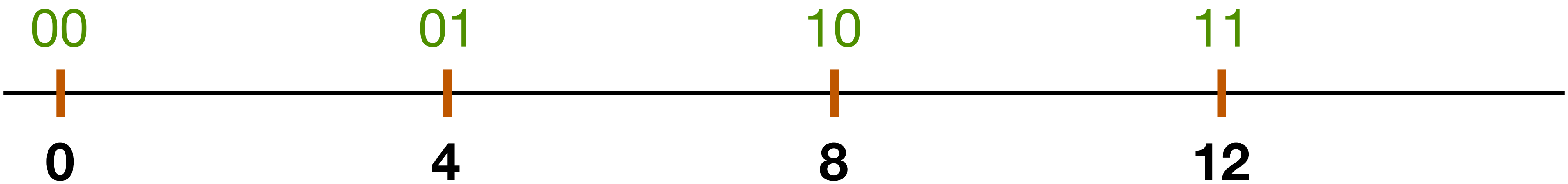


Aside: Quantization



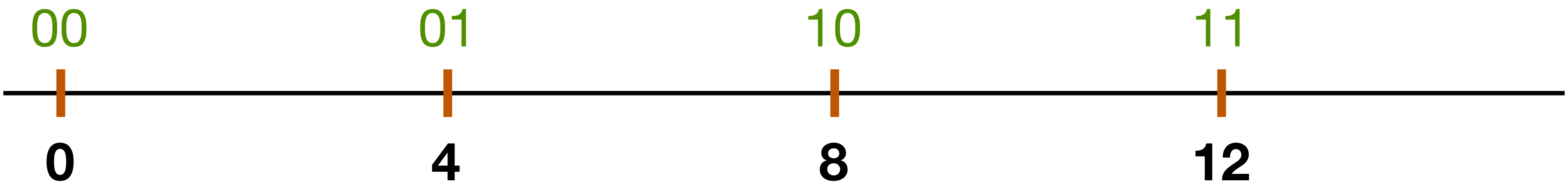
- Saves storage space and improves computation speed
  - 50% space saving
  - 4-bit arithmetic becomes 2-bit arithmetic

Aside: Quantization



- Saves storage space and improves computation speed
  - 50% space saving
  - 4-bit arithmetic becomes 2-bit arithmetic
- Many real-world applications can tolerate imprecisions
  - Image processing
  - Computer vision
  - Real-time graphics
  - Machine learning (Neural networks)

Aside: Quantization



- Saves storage space and improves computation speed
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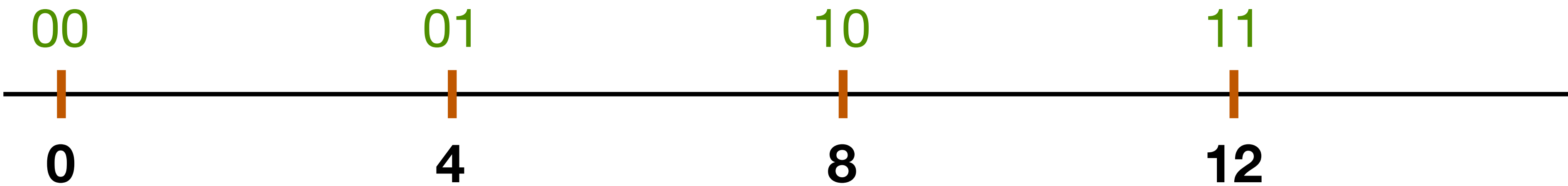
Aside: Quantization

Questions?



- Saves storage space and improves computation speed
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