CSC 252: Computer Organization Spring 2021: Lecture 3

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University of Rochester

Announcement

- Programming Assignment 1 is out
  - Details: https://www.cs.rochester.edu/courses/252/spring2021/labs/assignment1.html
  - Due on Feb. 17, 11:59 PM
  - You have 3 slip days

7	8	9	10	11	12	13
		Today				
14	15	16	17	18	19	20
			Due			

Announcement

- Programming assignment 1 is in C language. Seek help from TAs.
- TAs are best positioned to answer your questions about programming assignments!!!
- Programming assignments do NOT repeat the lecture materials. They ask you to synthesize what you have learned from the lectures and work out something new.

Encoding Negative Numbers

- Two’s Complement

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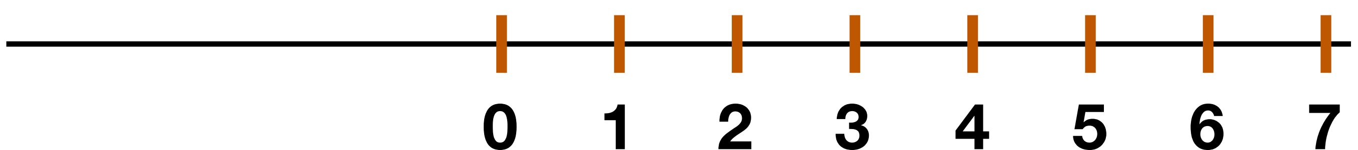

Unsigned	Binary
0	000
1	001
2	010
3	011
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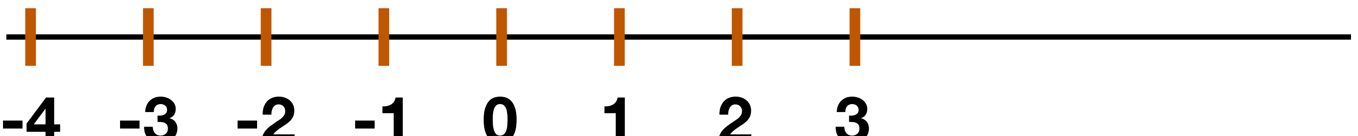

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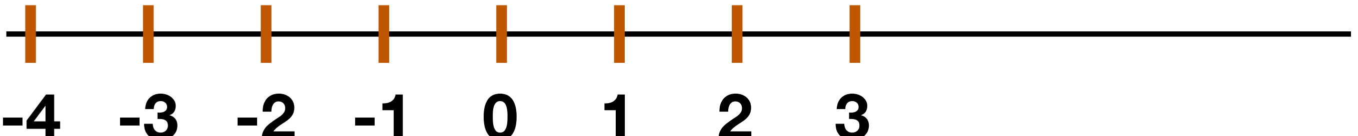

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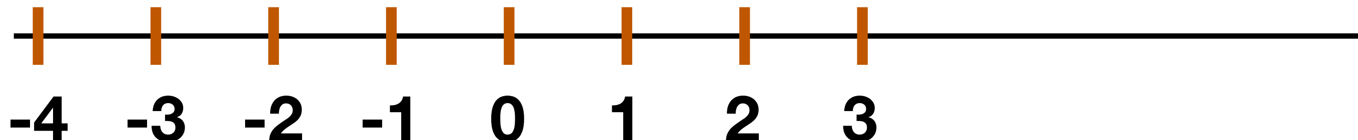

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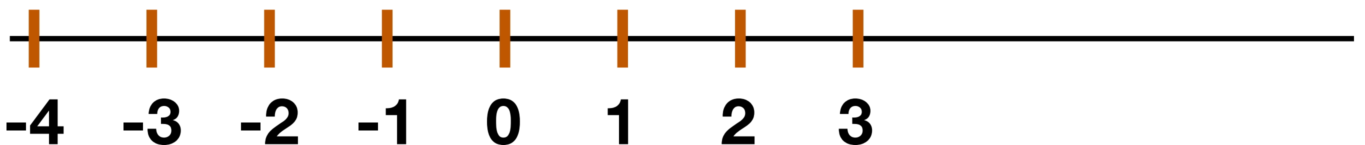

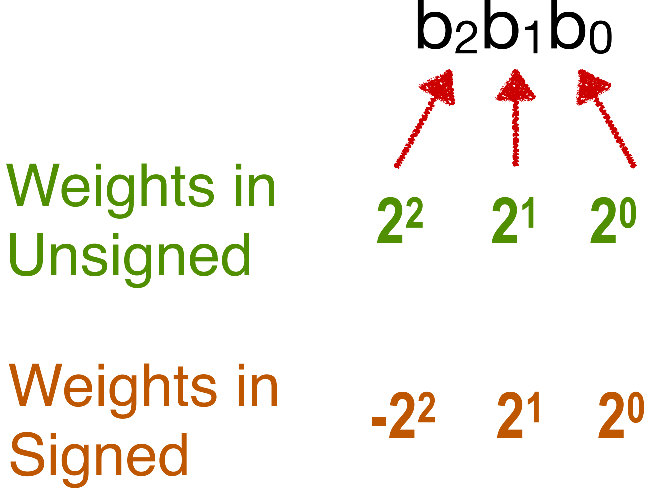

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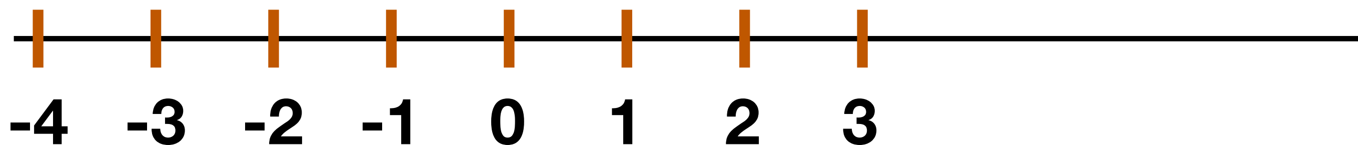

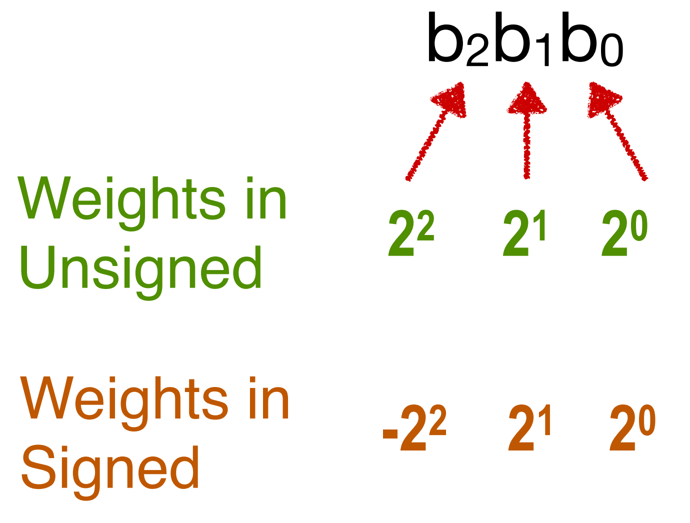

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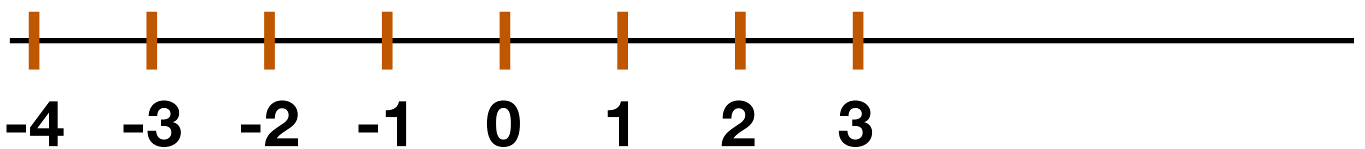

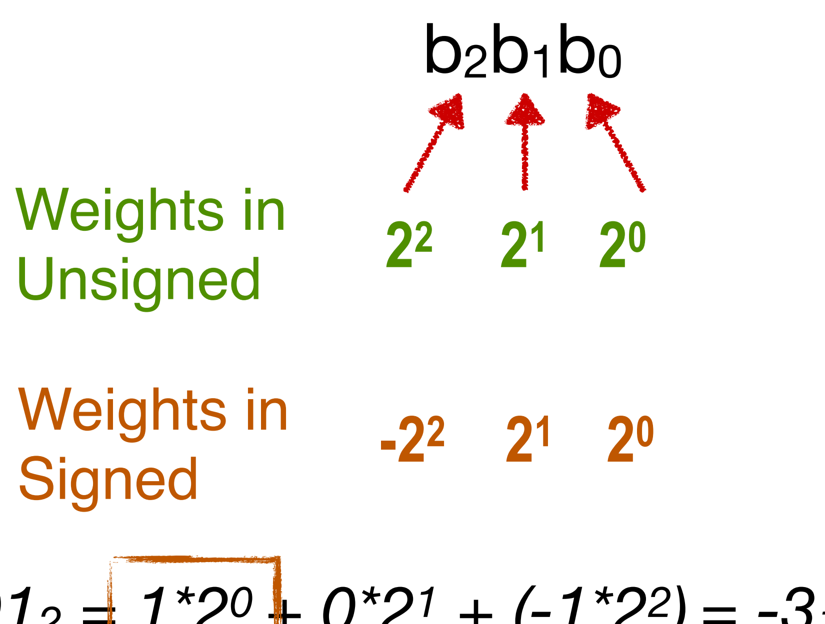

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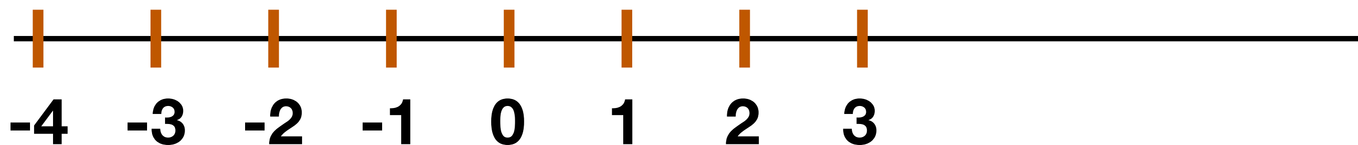

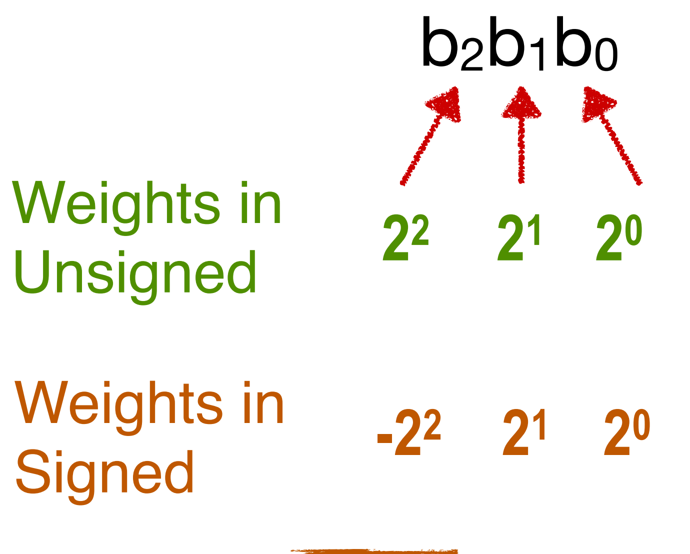

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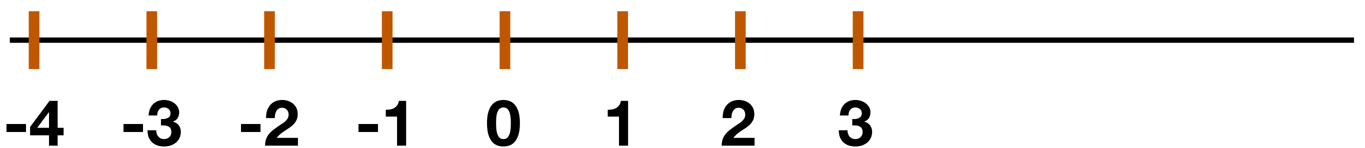

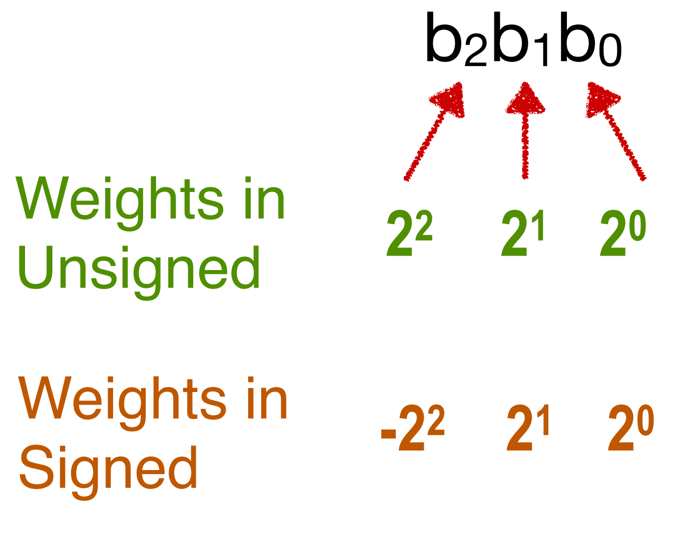

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- Only 1 zero
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- 3 + 1 becomes -4 (called overflow. More on it later.)

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  - Integer
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- Define a data type that captures all these attributes: `unsigned char` in C
  - Internally, an **`unsigned char`** variable is represented as a 8-bit, non-negative, binary number

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```
int x = -5, y = 4;
int z = x + y;
fprintf(stdout, "%d\n", z);
fprintf(stdout, "%u\n", z);
```

Data Types (in C)

C Data Type	32-bit	64-bit
`(unsigned) char`	1	1
`(unsigned) short`	2	2
`(unsigned) int`	4	4
`(unsigned) long`	4	8

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	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
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- C Language
  - `#include <limits.h>`
  - Declares constants, e.g.,
    - `ULONG_MAX`
    - `LONG_MAX`
    - `LONG_MIN`
  - Values platform specific

Can We Represent Fractions in Binary?

- What does 10.01_2 mean?
- C.f., Decimal
  - $12.45 = 1*10^1 + 2*10^0 + 4*10^{-1} + 5*10^{-2}$
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Decimal	Binary
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0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
1.25	01.01
1.5	01.10
1.75	01.11
2	10.00
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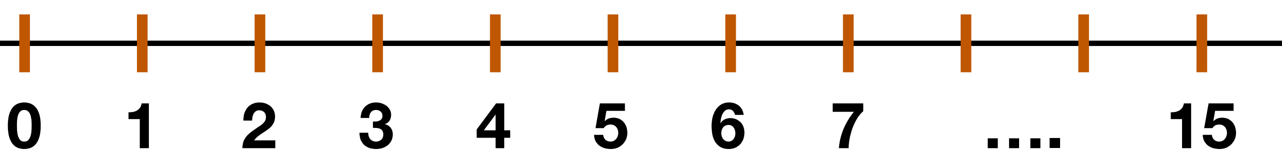

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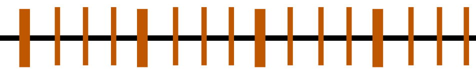

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0.5	00.10
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  10.11
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  1.50
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0 1 2 3

Integer Arithmetic Still Works!

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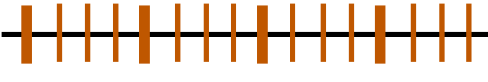
Fixed-Point Representation

- Fixed interval between two representable numbers as long as the binary point stays fixed
    - Each bit represents 0.25_{10}
- Fixed-point representation of numbers
    - Integer is one special case of fixed-point

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Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

One Bit Sequence, Two Interpretations

- A sequence of bits can be interpreted as either a signed integer or an unsigned integer

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Signed vs. Unsigned Conversion in C

- What happens when we convert between signed and unsigned numbers?
- Casting (In C terminology)
  - Explicit casting between signed & unsigned

```
int tx, ty = -4;
unsigned ux = 7, uy;
tx = (int) ux; // U2T
uy = (unsigned) ty; // T2U
```

  - Implicit casting
    - e.g., assignments, function calls

```
tx = ux;
uy = ty;
```

Mapping Between Signed & Unsigned

- Mappings between unsigned and two's complement numbers: Keep bit representations and reinterpret

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Mapping Signed ↔ Unsigned

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0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
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1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
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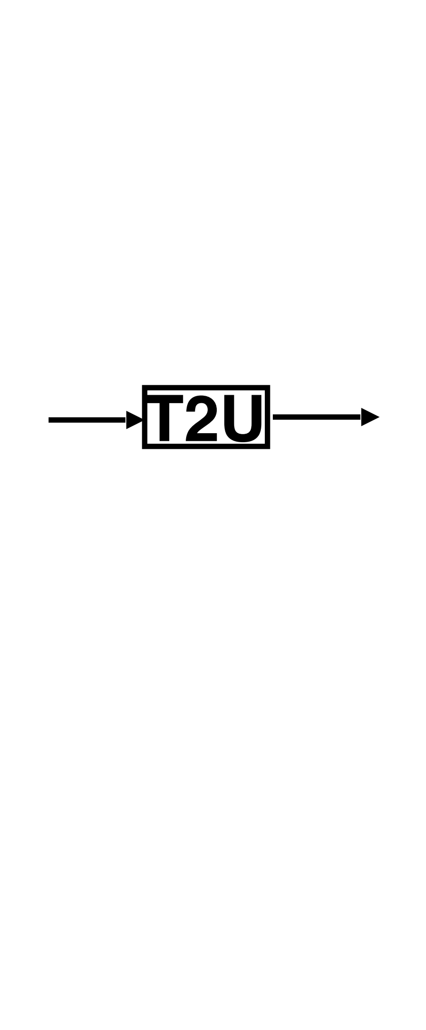

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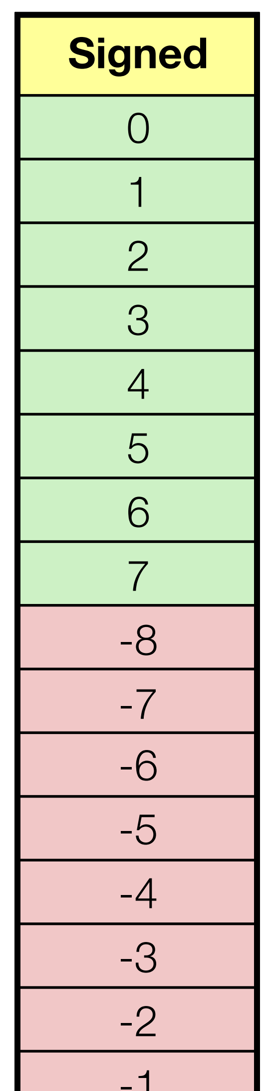

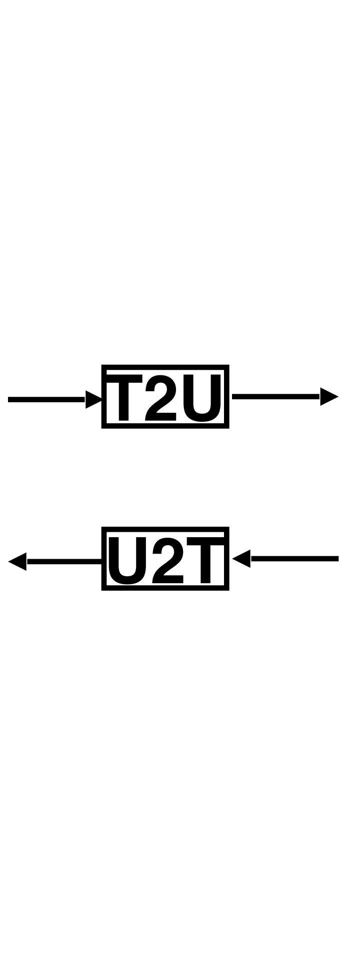

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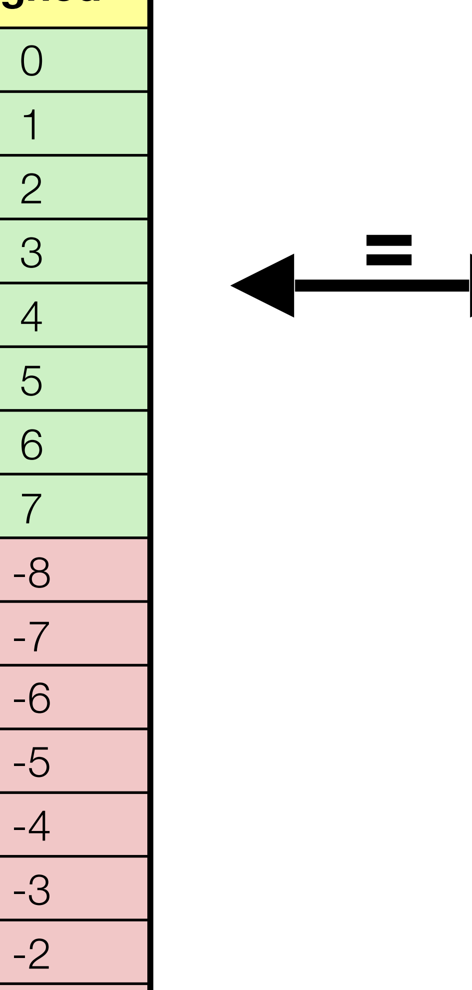

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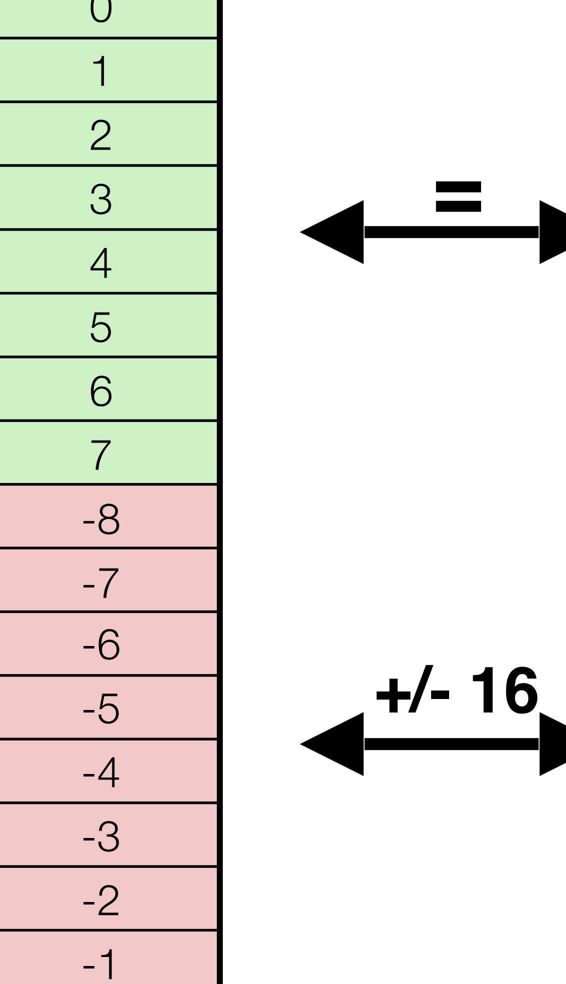

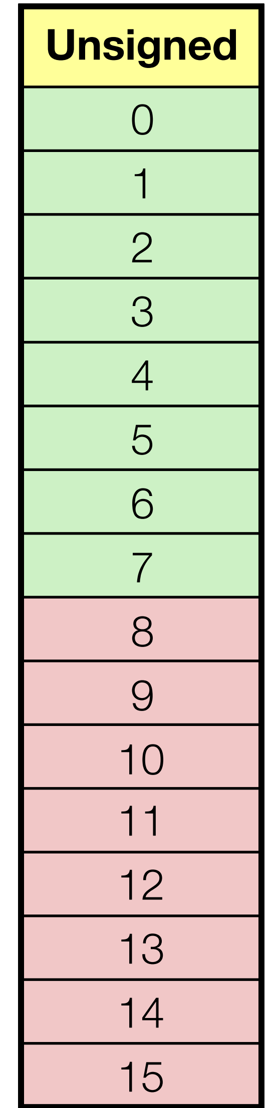

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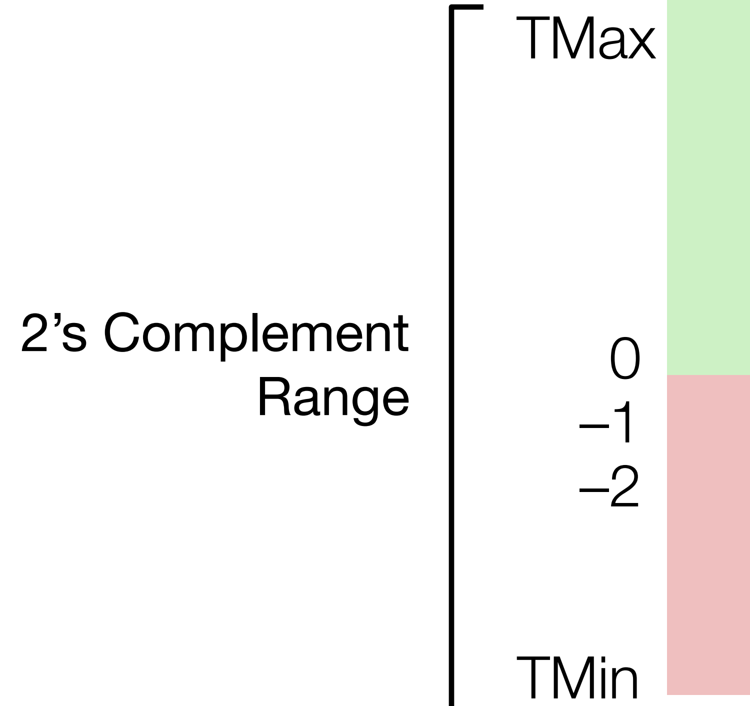
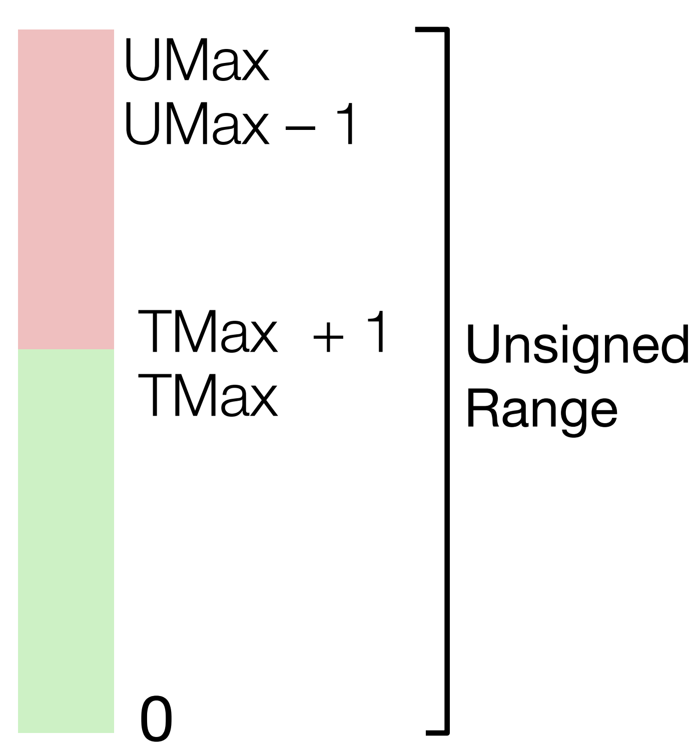
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

Conversion Visualized

- Signed → Unsigned
  - Ordering Inversion
  - Negative → Big Positive





Conversion Visualized

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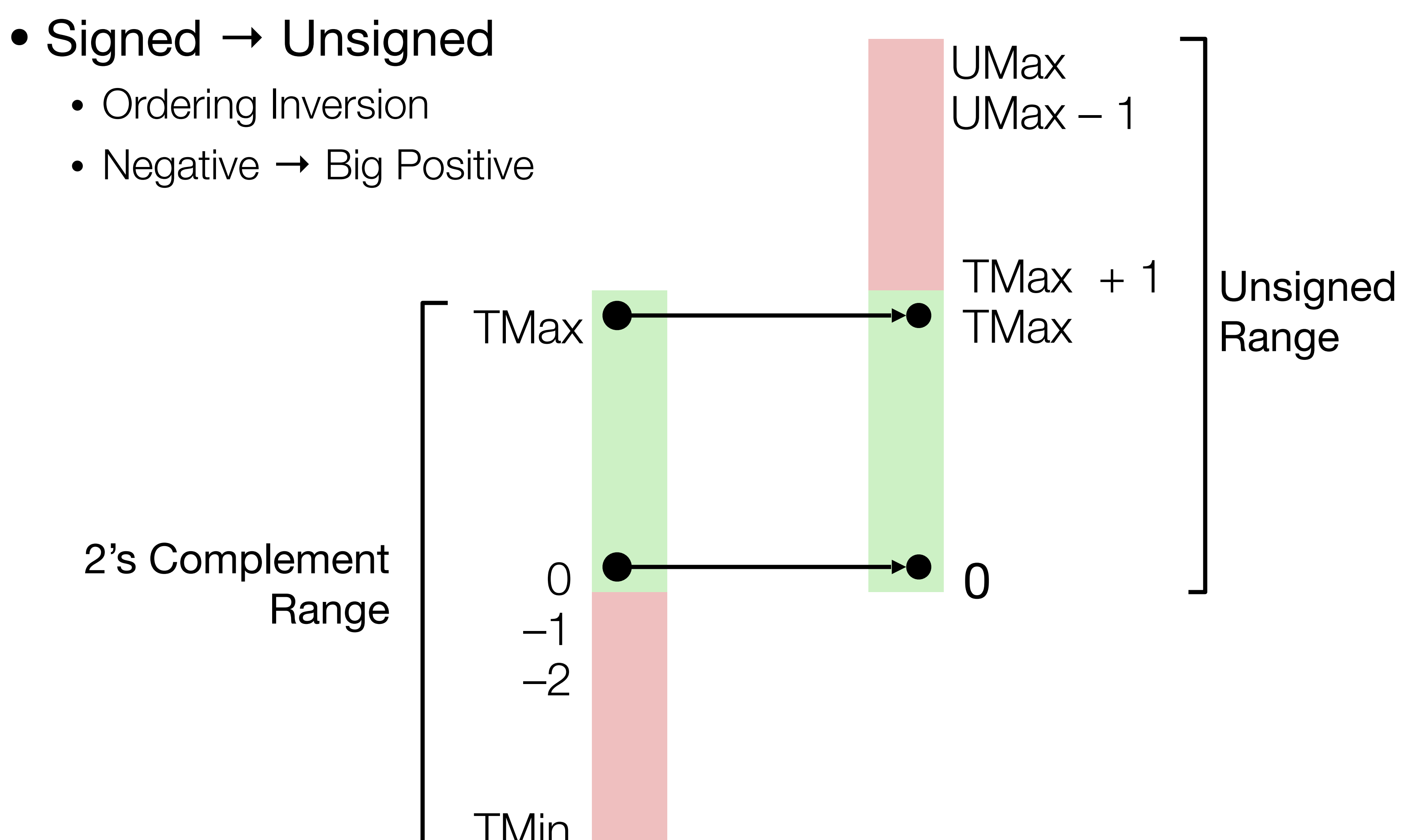

Conversion Visualized

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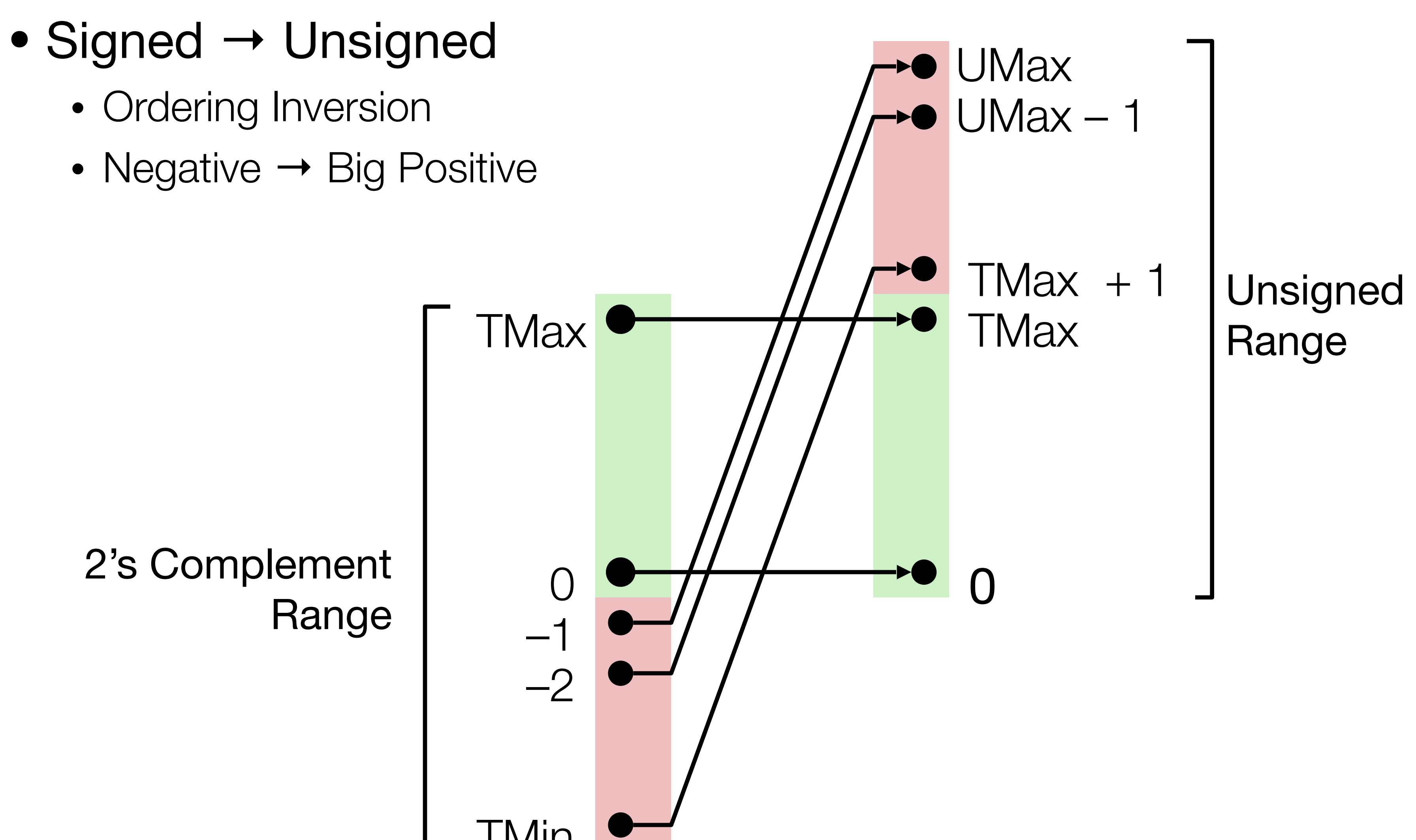

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

The Problem

```
short int x =  15213;
int       ix = (int) x;
short int y = -15213;
int       iy = (int) y;
```

C Data Type	64-bit
`char`	1
`short`	2
`int`	4
`long`	8

The Problem

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- Converting from smaller to larger integer data type
- Should we preserve the value?
- Can we preserve the value?
- How?

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short int x =  15213;
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`long`	8

- Converting from smaller to larger integer data type
- Should we preserve the value?
- Can we preserve the value?
- How?

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

Signed Extension

- Task:
  - Given w-bit signed integer x
  - Convert it to $(w+k)$-bit integer with same value

Signed Extension

- Task:
  - Given *w*-bit signed integer *x*
  - Convert it to (*w*+*k*)-bit integer with same value
- Rule:
  - Make *k* copies of sign bit:
  - $X' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \ldots, x_0$

Signed Extension

- Task:
  - Given w-bit signed integer x
  - Convert it to $(w+k)$-bit integer with same value
- Rule:
  - Make k copies of sign bit:
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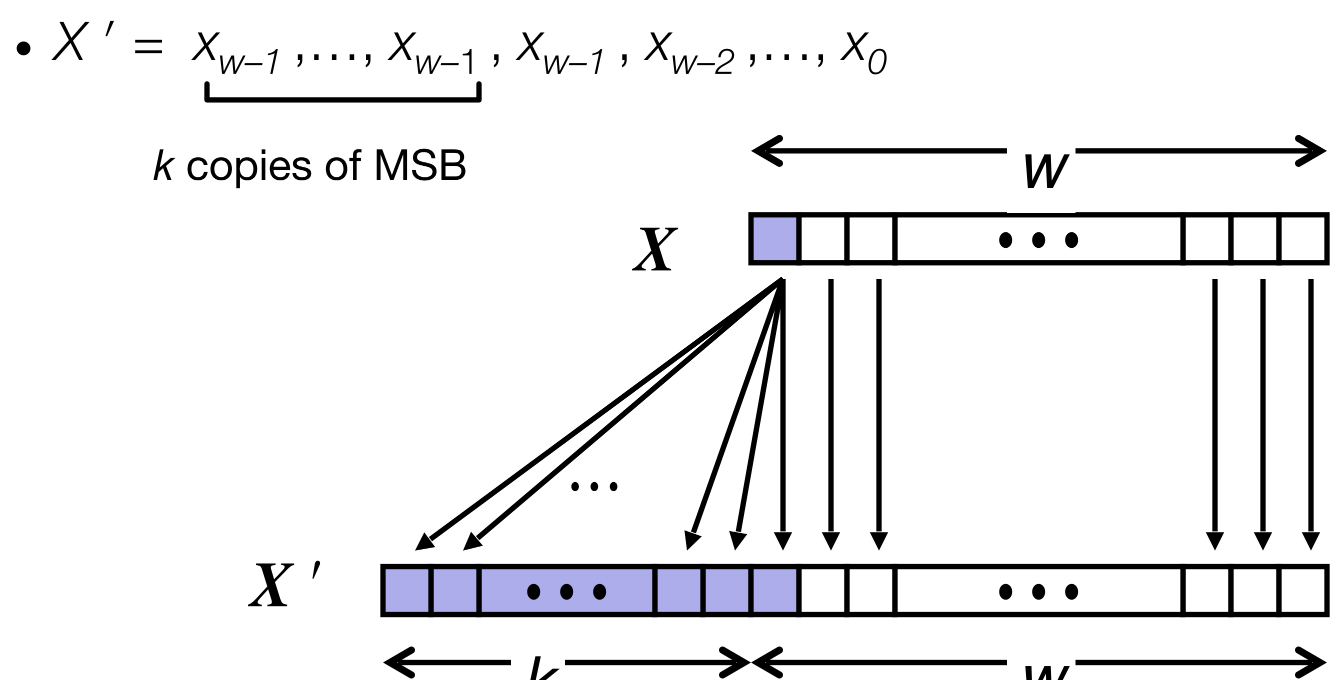


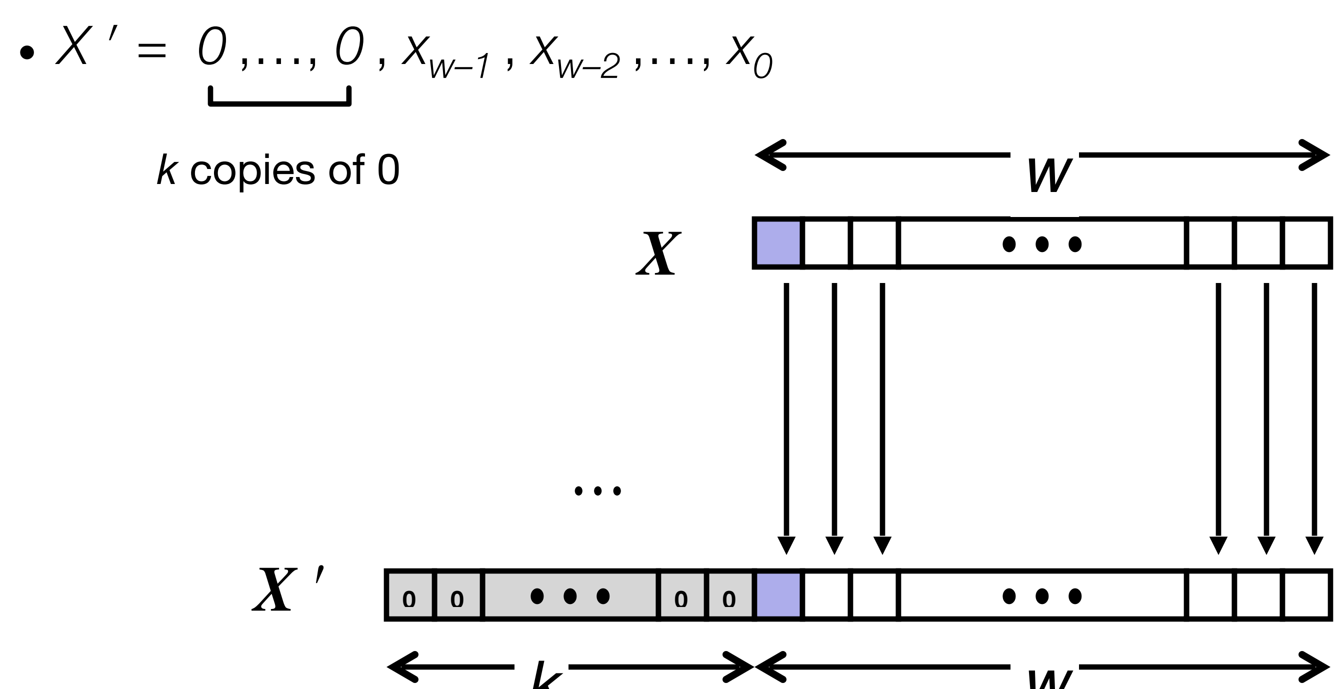
Another Problem

```
unsigned short x = 47981;
unsigned int  ux = x;
```

	Decimal	Hex	Binary
x	47981	BB 6D	10111011 01101101
ux	47981	00 00 BB 6D	00000000 00000000 10111011 01101101

Unsigned (Zero) Extension

- Task:
  - Given w-bit unsigned integer x
  - Convert it to $(w+k)$-bit integer with same value
- Rule:
  - Simply pad zeros:
  - $X' = \underbrace{0, \ldots, 0}_{k \text{ copies of } 0}, x_{w-1}, x_{w-2}, \ldots, x_0$



Yet Another Problem

```
int    x =  53191;
short sx = (short) x;
```

	Decimal	Hex	Binary
x	53191	00 00 CF C7	00000000 00000000 11001111 11000111
sx	-12345	CF C7	11001111 11000111

Yet Another Problem

```
int    x =  53191;
short sx = (short) x;
```

	Decimal	Hex	Binary
x	53191	00 00 CF C7	00000000 00000000 11001111 11000111
sx	-12345	CF C7	11001111 11000111

- Truncating (e.g., int to short)
  - C’s implementation: leading bits are truncated, results reinterpreted
  - So can’t always preserve the numerical value

Today: Representing Information in Binary

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- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Unsigned Addition

Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Unsigned Addition

- Similar to Decimal Addition

Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Unsigned Addition

- Similar to Decimal Addition
- Suppose we have a new data type that is 3-bit wide (c.f., **`short`** has 16 bits)

Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Normal Case

```
    010          2
+)  101     +)   5
--------    --------
    111          7
```

Unsigned Addition

- Similar to Decimal Addition
- Suppose we have a new data type that is 3-bit wide (c.f., **`short`** has 16 bits)
- Might overflow: result can't be represented within the size of the data type

Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Normal Case

```
    010         2
+)  101    +)   5
-------    ------
    111         7
```

Overflow Case

```
    110         6
+)  101    +)   5
-------    ------
   1011        11
```

Unsigned Addition

- Similar to Decimal Addition
- Suppose we have a new data type that is 3-bit wide (c.f., **`short`** has 16 bits)
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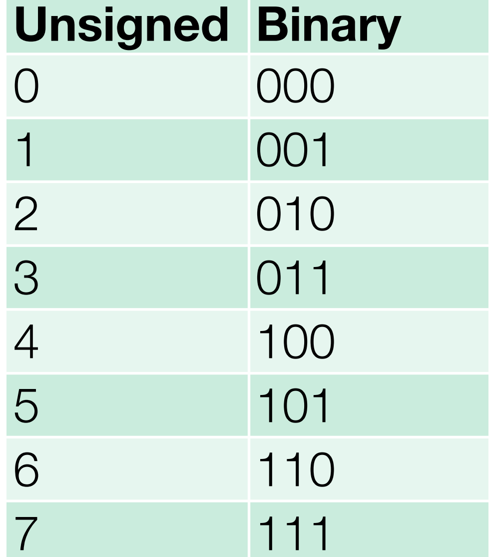


Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Normal Case

```
     010          2
 +)  101     +)   5
 --------    ------
     111          7
```

Overflow Case

```
     110          6
 +)  101     +)   5
 --------    ------
    1011         11
```

True Sum

Unsigned Addition

- Similar to Decimal Addition
- Suppose we have a new data type that is 3-bit wide (c.f., **`short`** has 16 bits)
- Might overflow: result can't be represented within the size of the data type

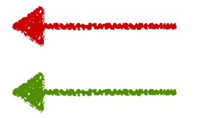
Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Normal Case

```
     010          2
 +)  101      +)  5
 ---------    --------
     111          7
```

Overflow Case

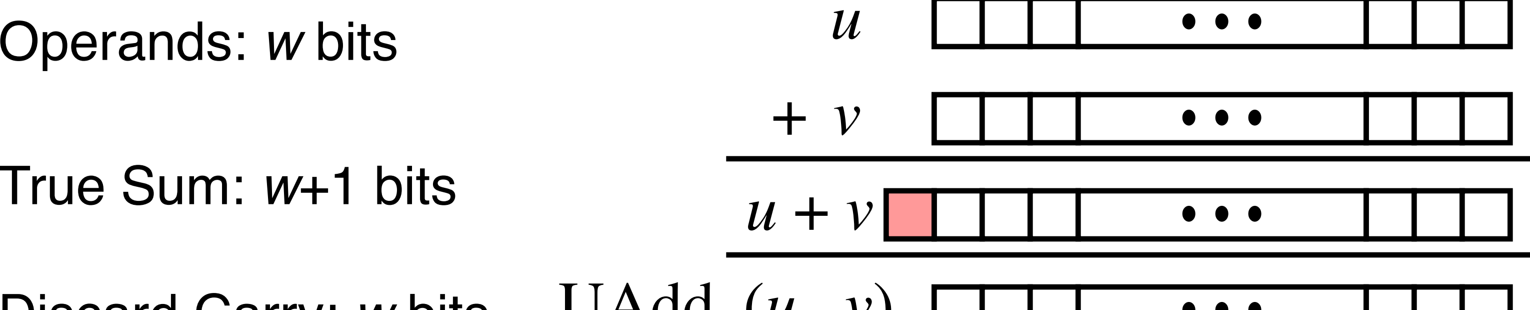
```
     110          6
 +)  101      +)  5
 ---------    --------
    1011         11
     011          3
```



True Sum

Sum with same bits

Unsigned Addition in C



Two's Complement Addition

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Two's Complement Addition

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Two's Complement Addition

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Normal Case

```
    010          2
+)  101     +)  -3
-------     ------
    111         -1
```

Two's Complement Addition

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Normal Case

```
    010          2
+)  101      +) -3
-------      -----
    111         -1
```

Overflow Case

```
    110         -2
+)  101      +) -3
-------      -----
   1011         -5
```

Two's Complement Addition

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Normal Case

```
    010          2
+)  101     +)  -3
-------     ------
    111         -1
```

Overflow Case

```
    110         -2
+)  101     +)  -3
-------     ------
   1011         -5
    011          3
```

Two's Complement Addition

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Min → -4

Normal Case

```
    010          2
+)  101      +) -3
-------     ------
    111         -1
```

Overflow Case

```
    110         -2
+)  101      +) -3
-------     ------
   1011         -5
    011          3
```

Negative Overflow

Two's Complement Addition

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Min → -4 (100)

Normal Case

```
    010          2
+)  101      +) -3
-------      ------
    111         -1
```

Overflow Case

```
    110         -2          011          3
+)  101      +) -3      +)  001      +)  1
-------      ------     -------      -----
   1011         -5         0100          4
    011          3
```

Negative Overflow

Two's Complement Addition

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Min → -4 (100)

Normal Case

```
    010          2
+)  101     +)  -3
-------     ------
    111         -1
```

Overflow Case

```
    110         -2          011          3
+)  101     +)  -3      +)  001     +)   1
-------     ------      -------     ------
   1011         -5         0100          4
    011          3          100         -4
```

Negative Overflow

Two's Complement Addition

- Has identical bit-level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Max → 3 (011)

Min → -4 (100)

Normal Case

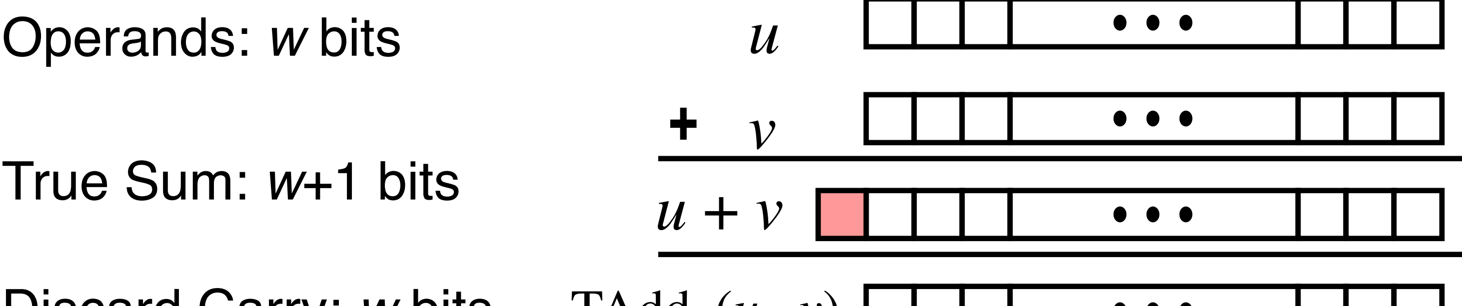
```
    010          2
+)  101     +)  -3
-------     ------
    111         -1
```

Overflow Case

```
    110         -2         011          3
+)  101     +)  -3     +)  001     +)   1
-------     ------     -------     ------
   1011         -5        0100          4
    011          3         100         -4
```

Negative Overflow　　　　Positive Overflow

Two's Complement Addition in C



Is This Signed Addition an Overflow?

```
    111
+)  110
-------
   1101
```

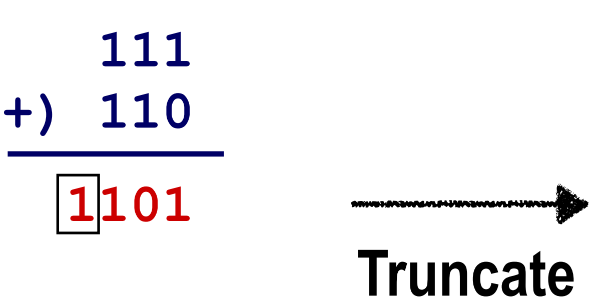
Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Is This Signed Addition an Overflow?

```
    111
+)  110
-------
   1101
```

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

Is This Signed Addition an Overflow?



Signed	Binary
0	000
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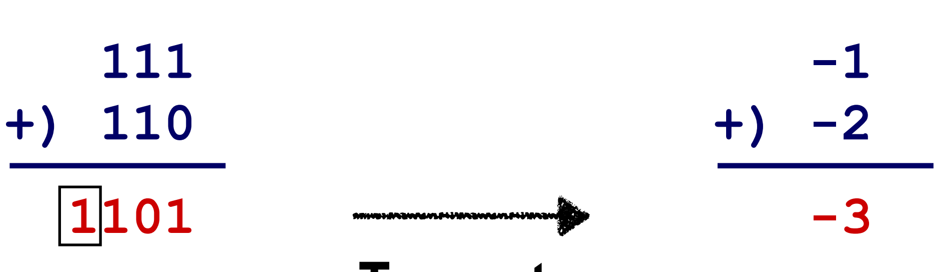
-3	101
-2	110
-1	111

Is This Signed Addition an Overflow?



Signed	Binary
0	000
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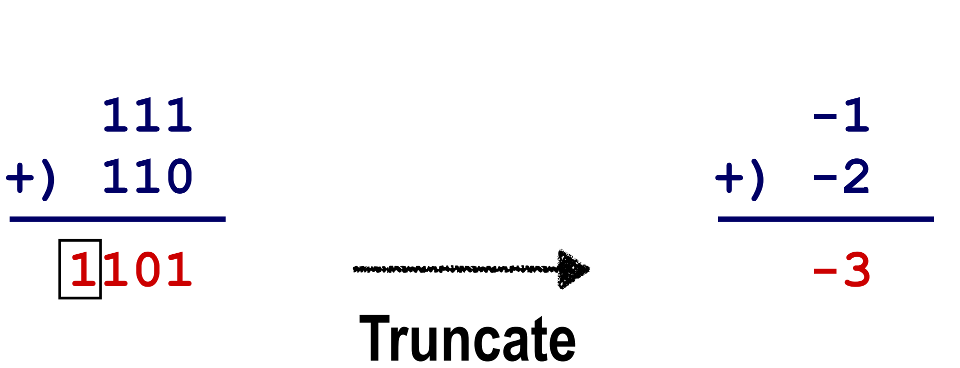
Is This Signed Addition an Overflow?



Signed	Binary
0	000
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2	010
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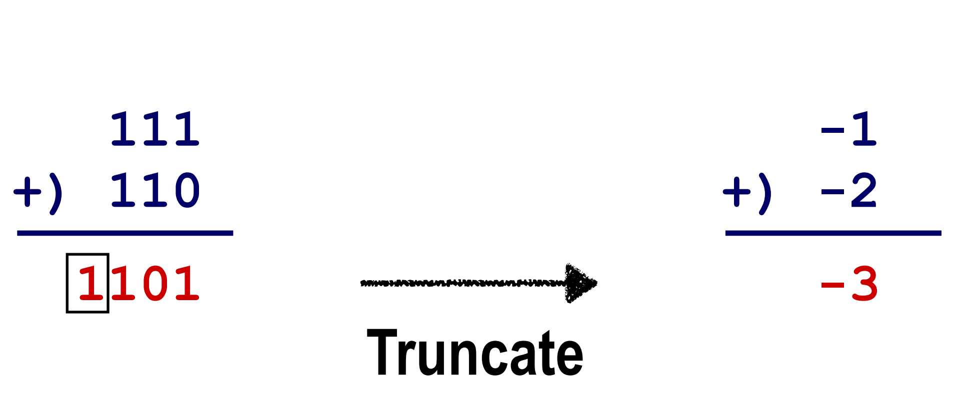
-4	100
-3	101
-2	110
-1	111

- This is not an overflow by definition

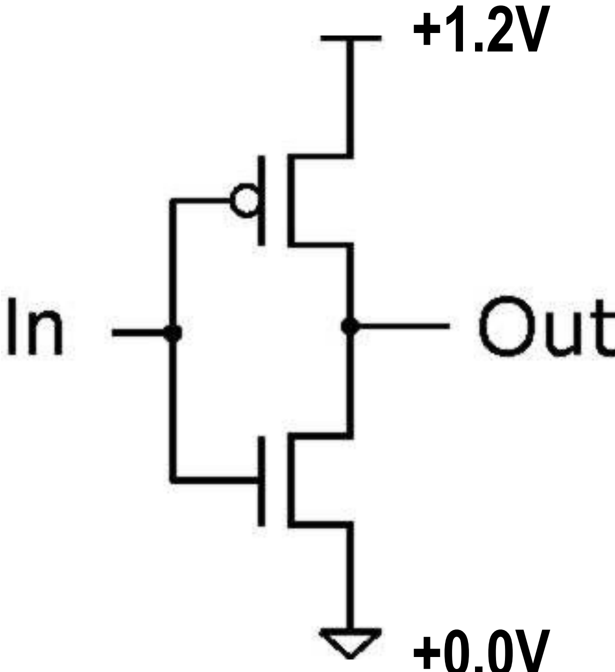
Is This Signed Addition an Overflow?



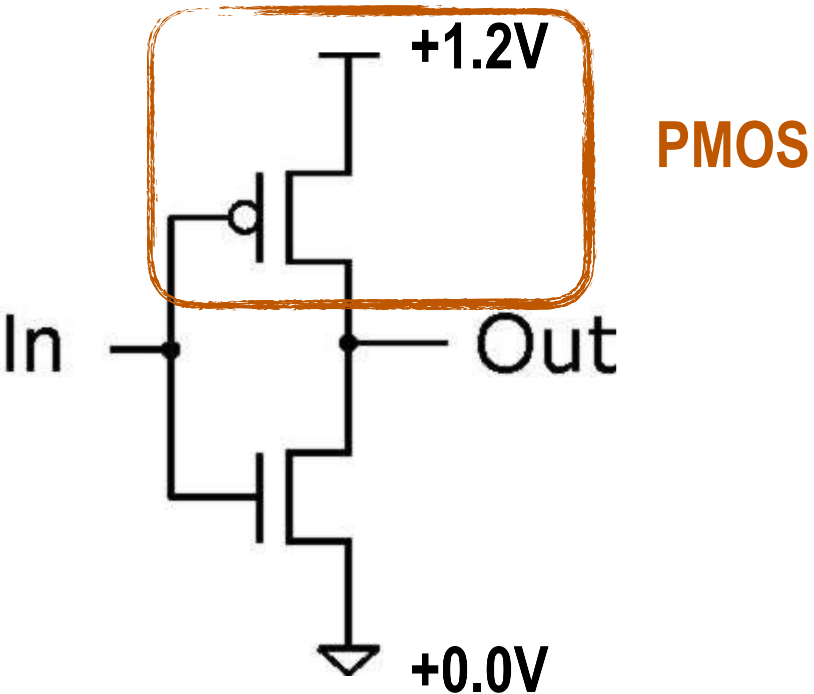
Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

- This is not an overflow by definition
- Because the actual result can be represented using the bit width of the datatype (3 bits here)

Inverter (NOT Gate)

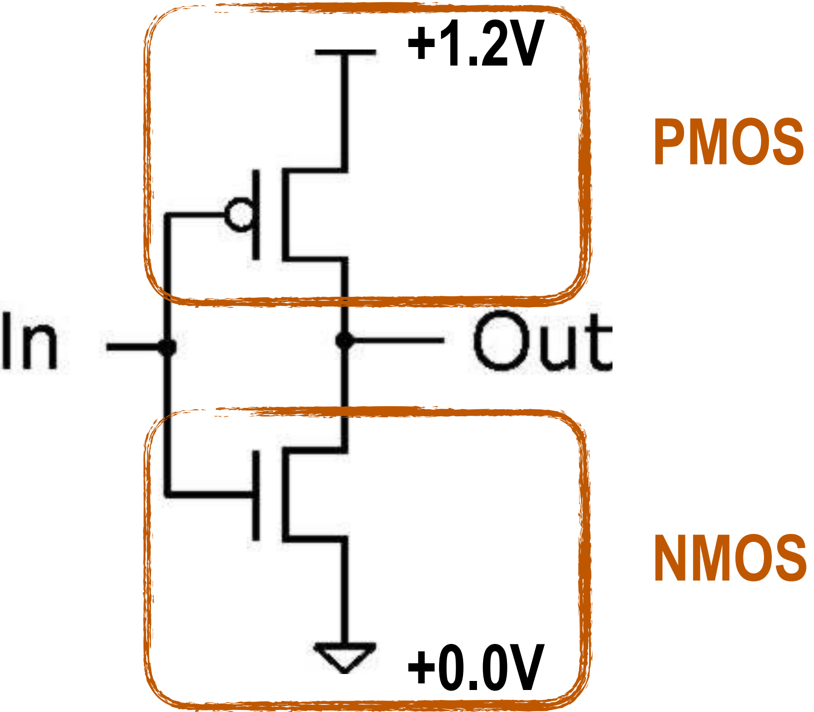


Inverter (NOT Gate)

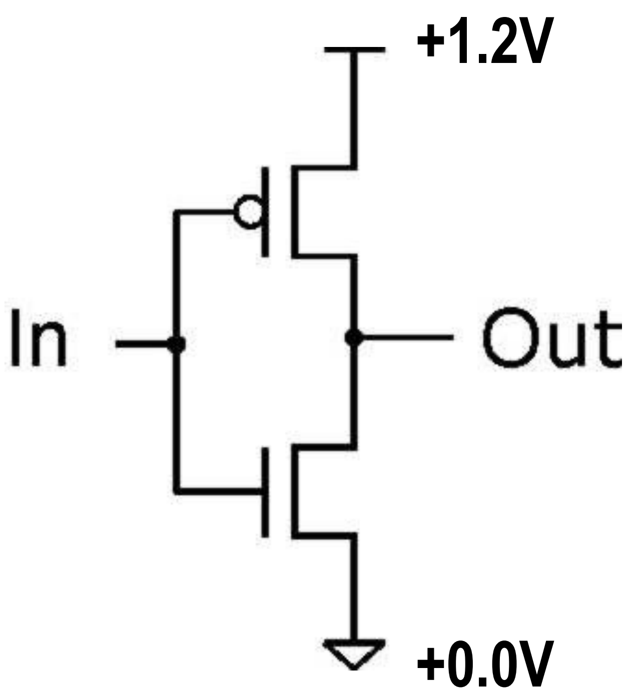


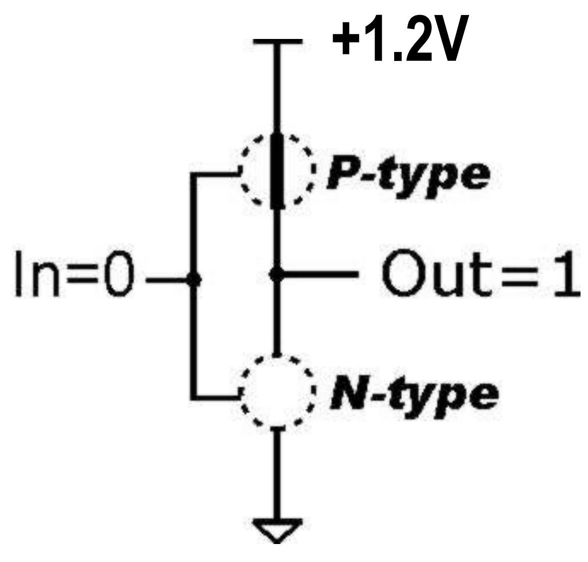
PMOS

Inverter (NOT Gate)

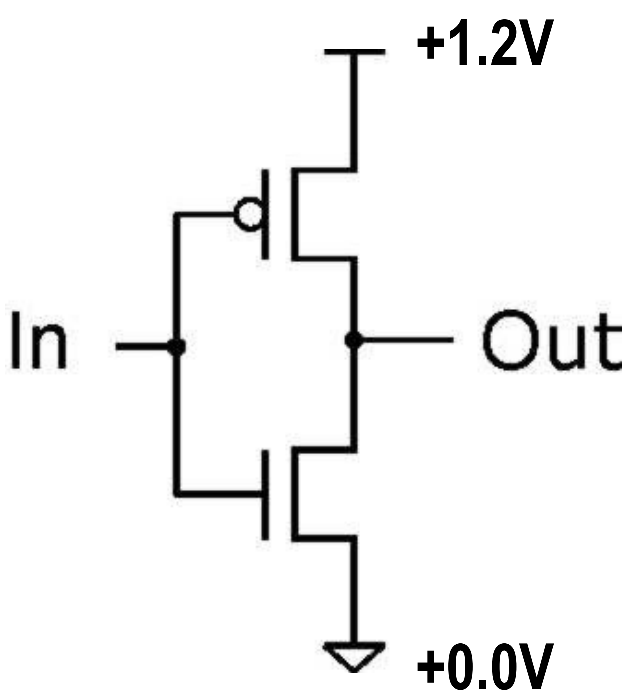


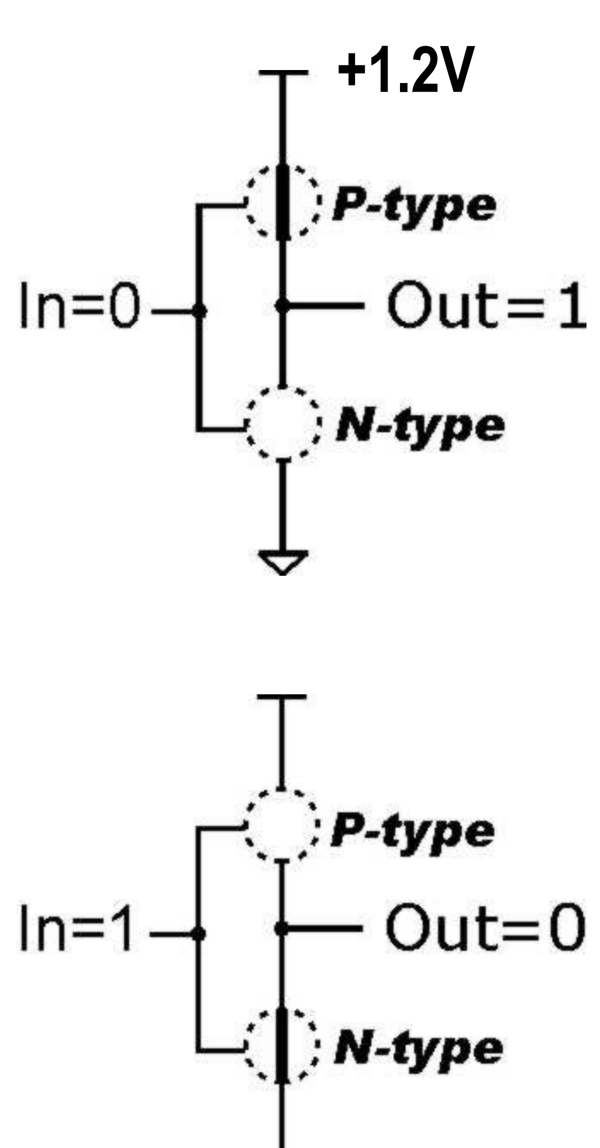
Inverter (NOT Gate)



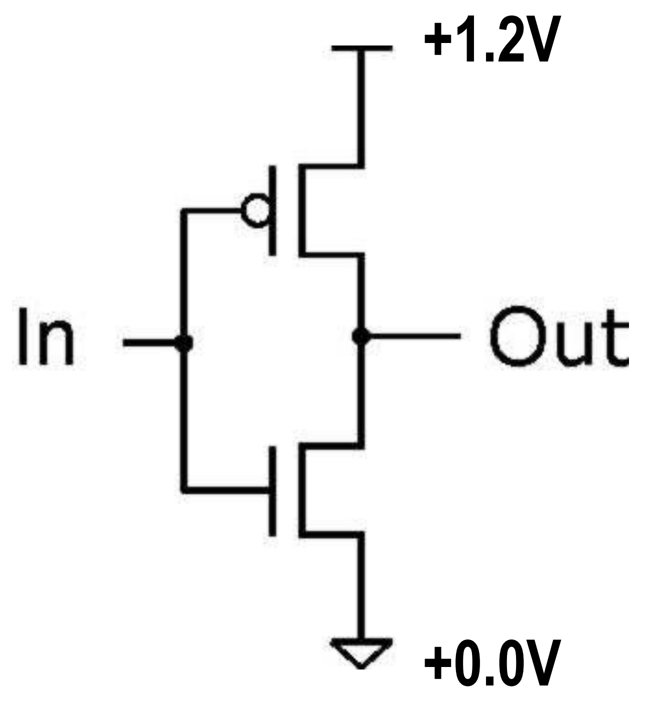


Inverter (NOT Gate)

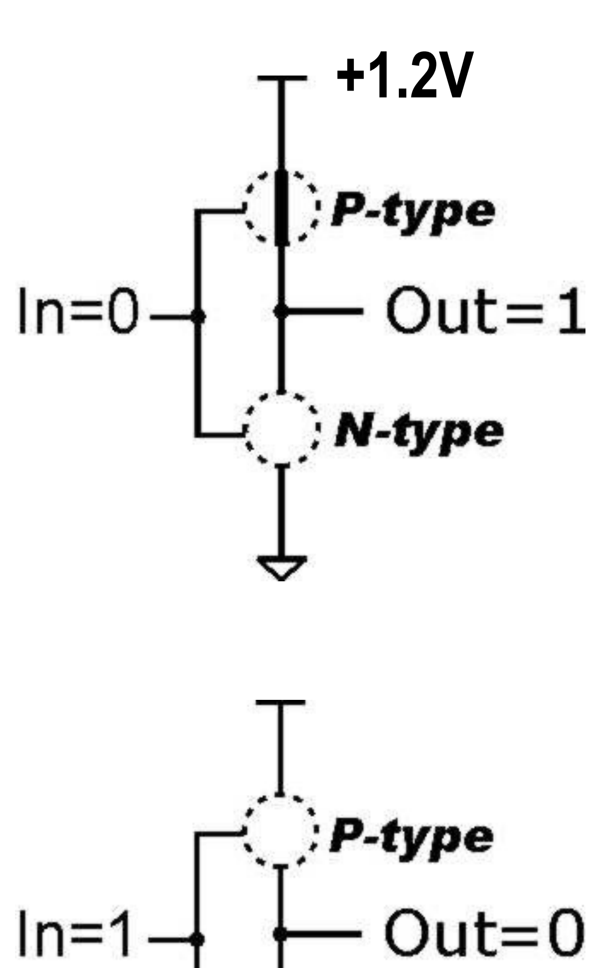


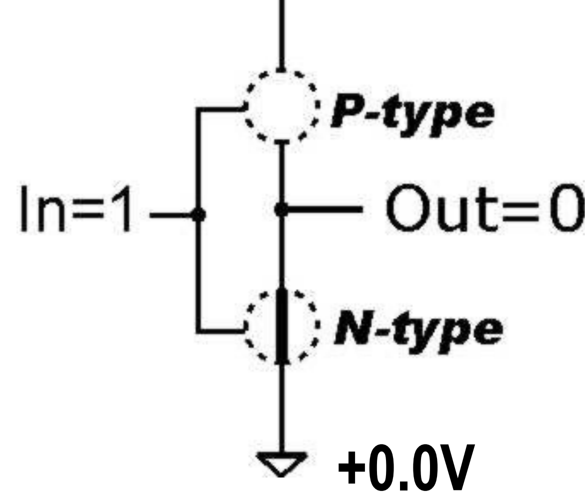


Inverter (NOT Gate)

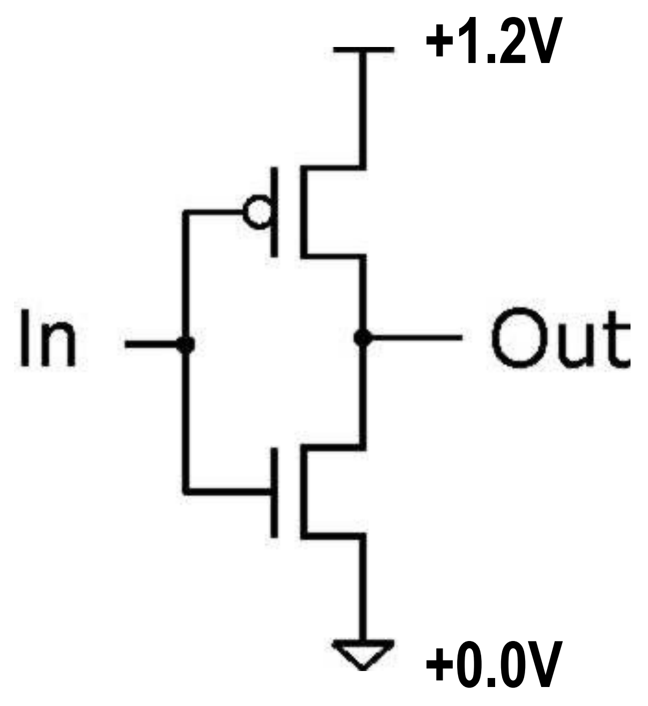


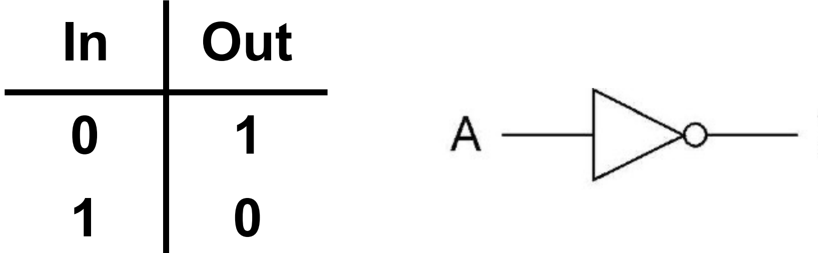
In	Out
0	1
1	0



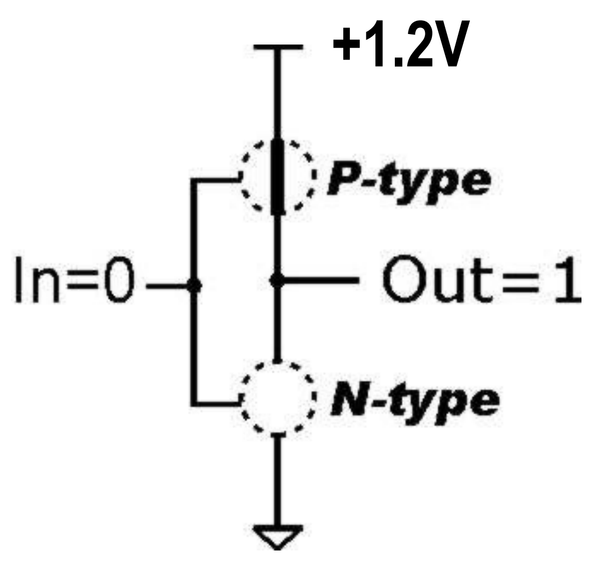


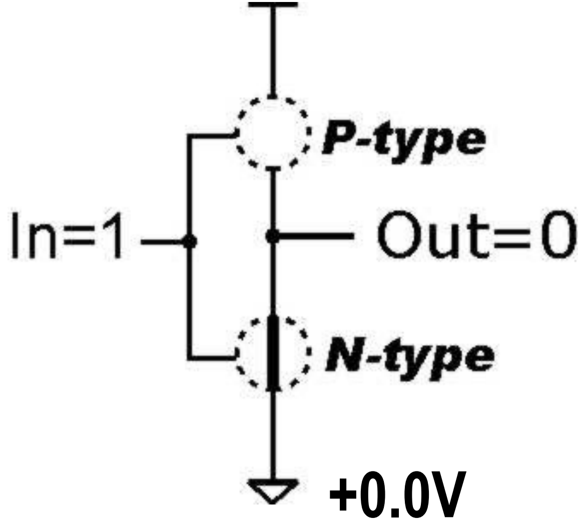
Inverter (NOT Gate)



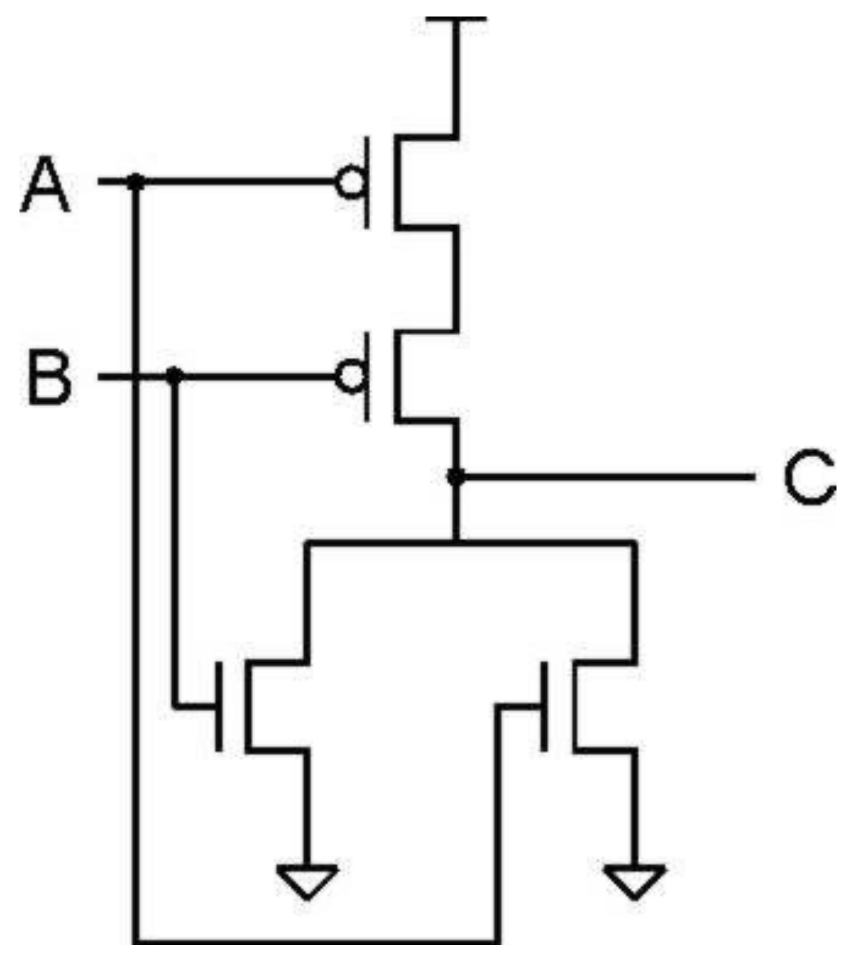


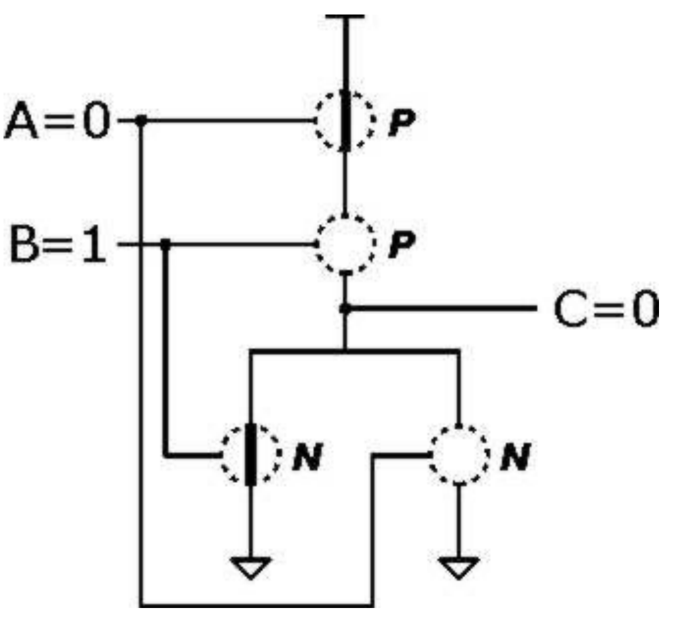
In	Out
0	1
1	0





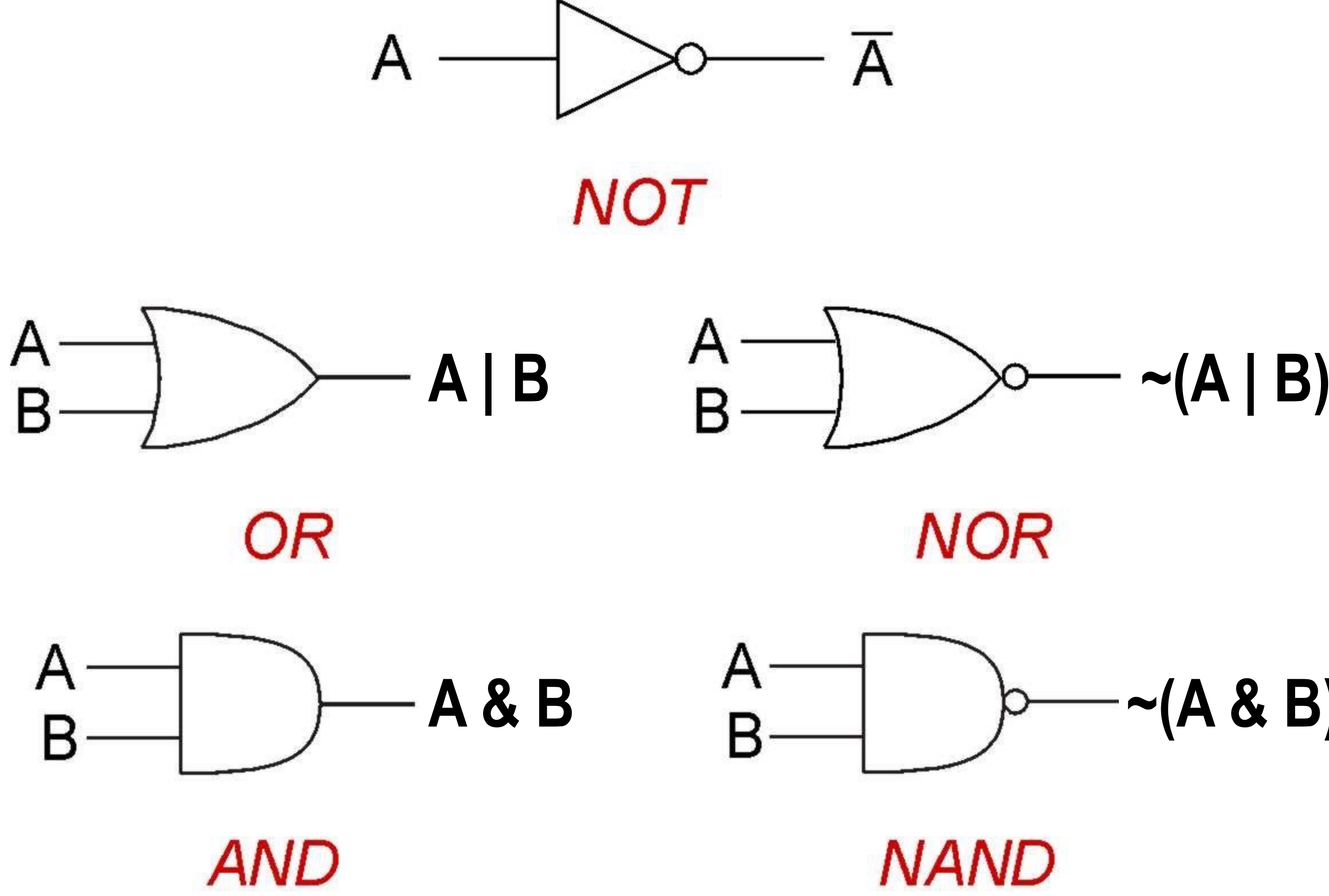
NOR Gate (NOT + OR)



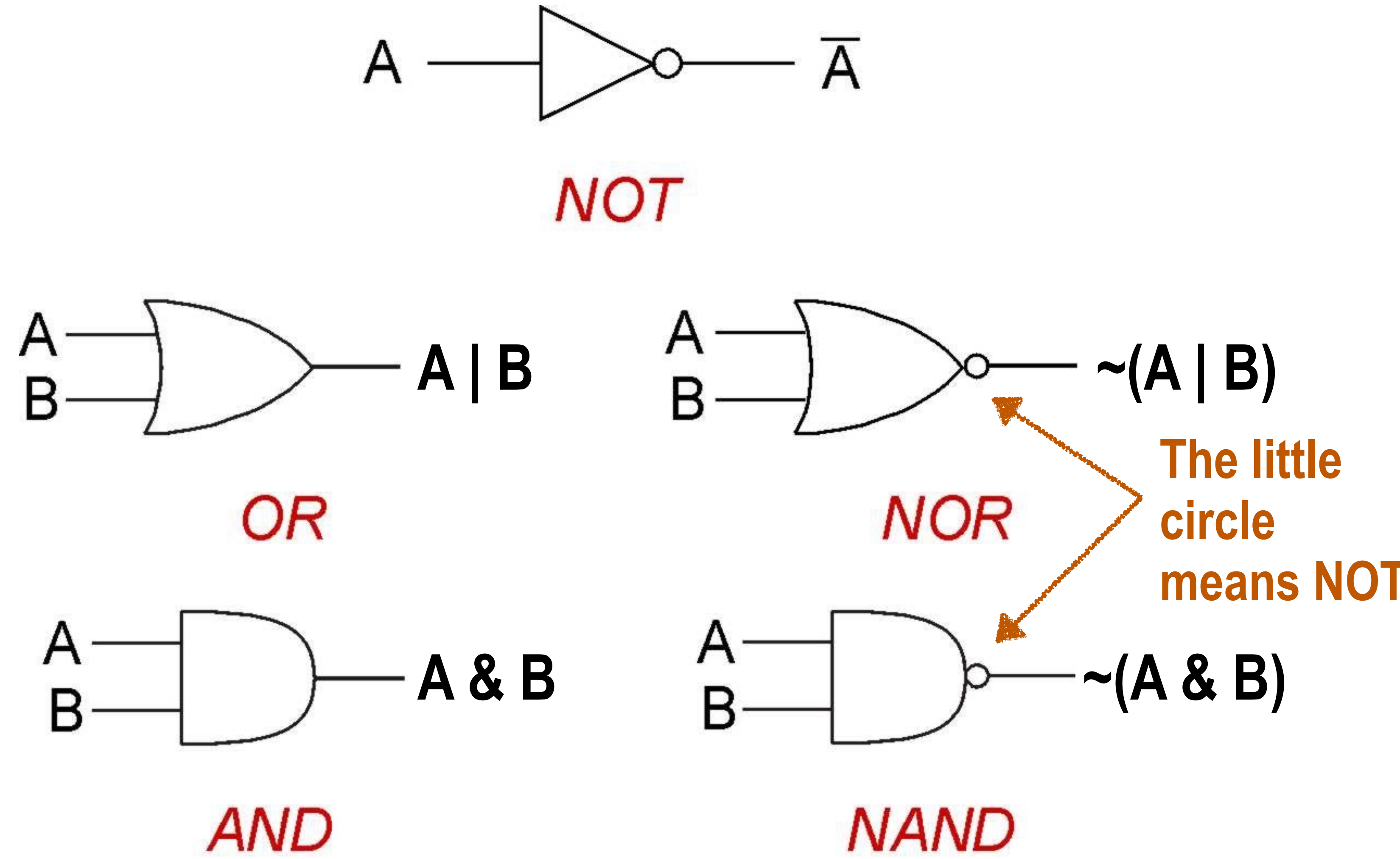


A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Basic Logic Gates



Basic Logic Gates



Full (1-bit) Adder

Add two bits and carry-in,
produce one-bit sum and carry-out.

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full (1-bit) Adder

Add two bits and carry-in,
produce one-bit sum and carry-out.

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full (1-bit) Adder

Add two bits and carry-in,
produce one-bit sum and carry-out.

$$S = (\sim A \ \& \sim B \ \& \ C_{in})$$

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full (1-bit) Adder

Add two bits and carry-in,
produce one-bit sum and carry-out.

$$S = (\sim A \ \& \ \sim B \ \& \ C_{in})$$
$$| \ (\sim A \ \& \ B \ \& \ \sim C_{in})$$

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full (1-bit) Adder

Add two bits and carry-in,
produce one-bit sum and carry-out.

$$S = (\sim A \ \& \ \sim B \ \& \ C_{in})$$
$$| \ (\sim A \ \& \ B \ \& \ \sim C_{in})$$
$$| \ (A \ \& \ \sim B \ \& \ \sim C_{in})$$

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

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$$|\ (\sim A \,\&\, B \,\&\, \sim C_{in})$$
$$|\ (A \,\&\, \sim B \,\&\, \sim C_{in})$$
$$|\ (A \,\&\, B \,\&\, C_{in})$$

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
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$$C_{ou} = (\sim A \ \& \ B \ \& \ C_{in}) \ | \ (A \ \& \ \sim B \ \& \ C_{in}) \ | \ (A \ \& \ B \ \& \ \sim C_{in}) \ | \ (A \ \& \ B \ \& \ C_{in})$$

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full (1-bit) Adder

Add two bits and carry-in,
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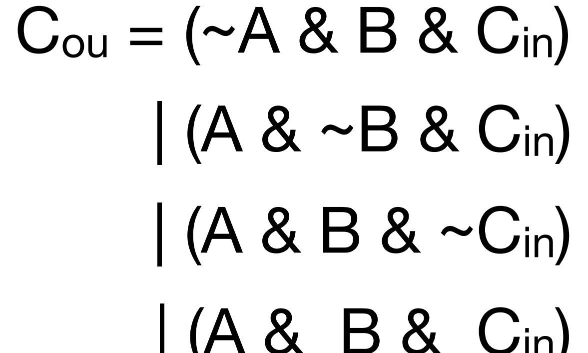
$$C_{ou} = (\sim A \ \& \ B \ \& \ C_{in})$$
$$| \ (A \ \& \ \sim B \ \& \ C_{in})$$
$$| \ (A \ \& \ B \ \& \ \sim C_{in})$$
$$| \ (A \ \& \ B \ \& \ C_{in})$$

Full (1-bit) Adder



Add two bits and carry-in,
produce one-bit sum and carry-out.

A B

C_{in}

C_{out} S

Full (1-bit) Adder

$C_{ou} = (\sim A \ \& \ B \ \& \ C_{in})$
$| \ (A \ \& \ \sim B \ \& \ C_{in})$
$| \ (A \ \& \ B \ \& \ \sim C_{in})$
$| \ (A \ \& \ B \ \& \ C_{in})$

Add two bits and carry-in,
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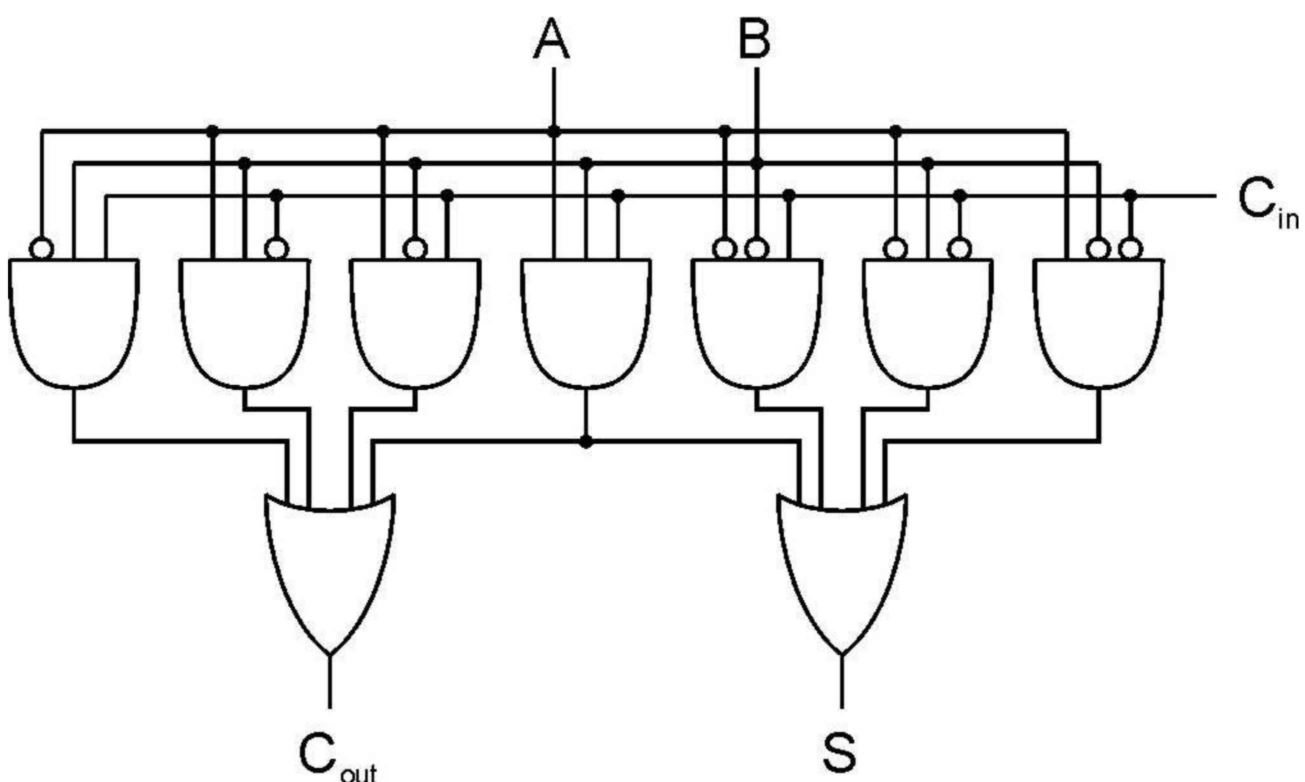
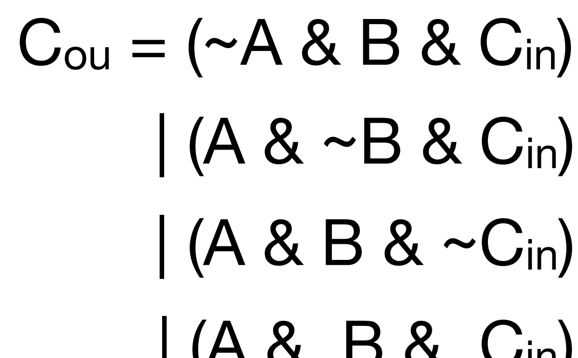


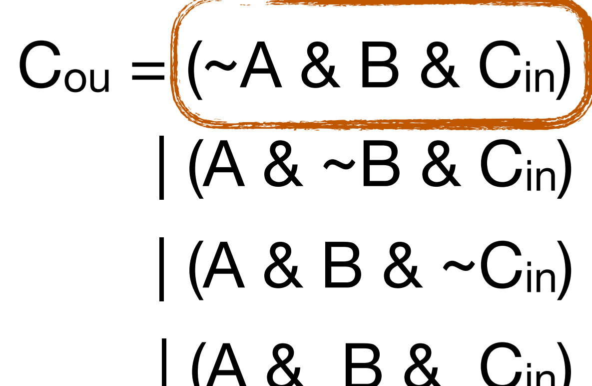
Full (1-bit) Adder



Add two bits and carry-in,
produce one-bit sum and carry-out.

A
B
C_{in}
AND Gates
OR Gates
C_{out}
S

Full (1-bit) Adder



Add two bits and carry-in,
produce one-bit sum and carry-out.

A B

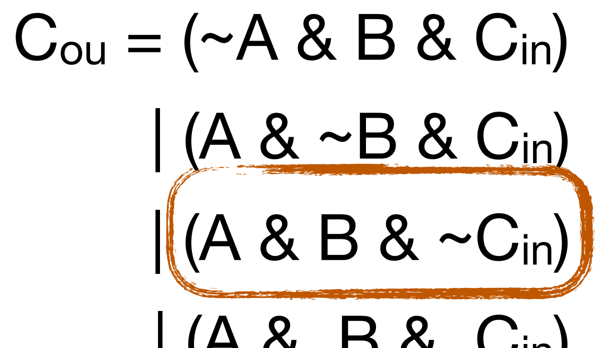
C_{in}

AND Gates

OR Gates

C_{out} S

Full (1-bit) Adder



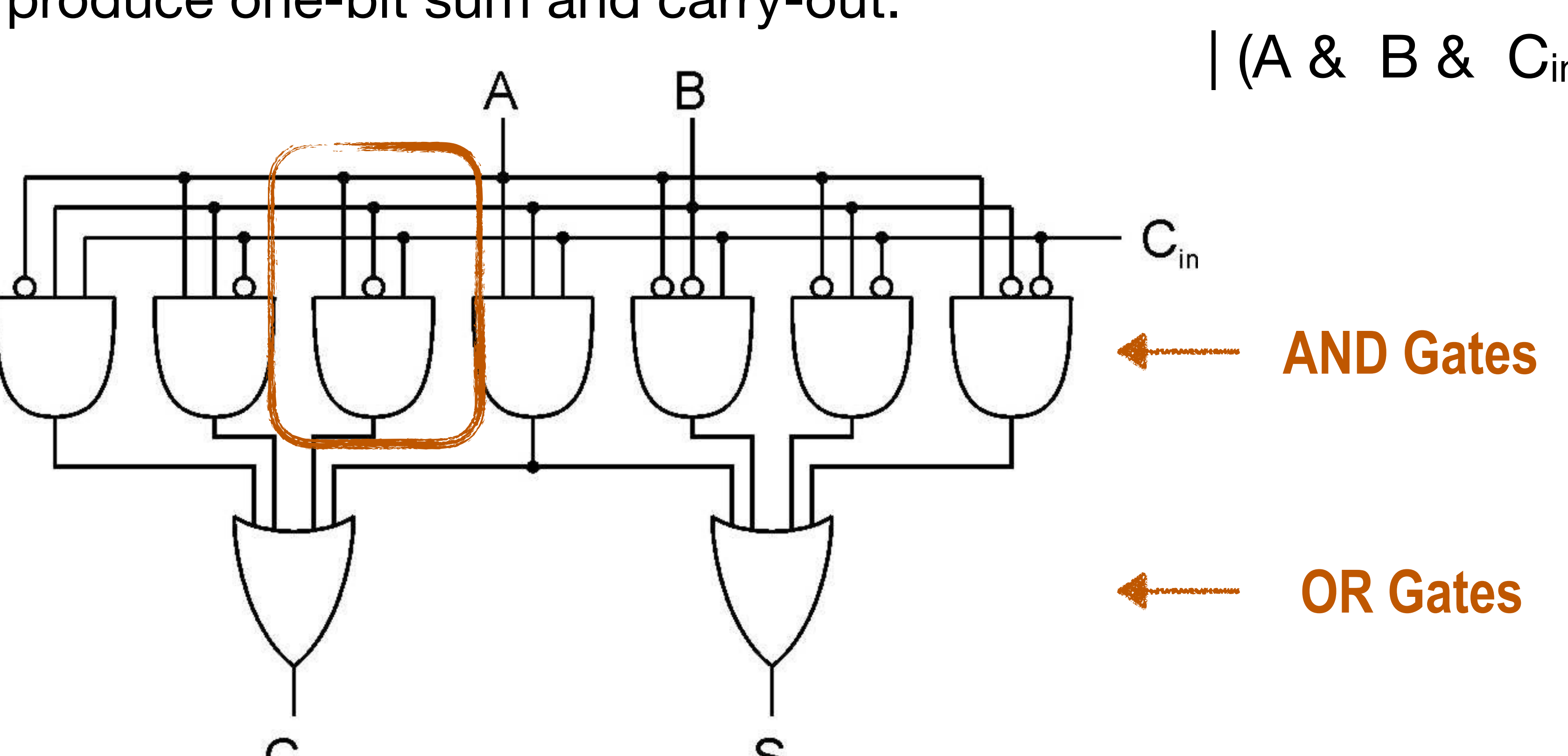
Add two bits and carry-in,
produce one-bit sum and carry-out.

A
B
C_{in}
AND Gates
OR Gates
C_{out}
S

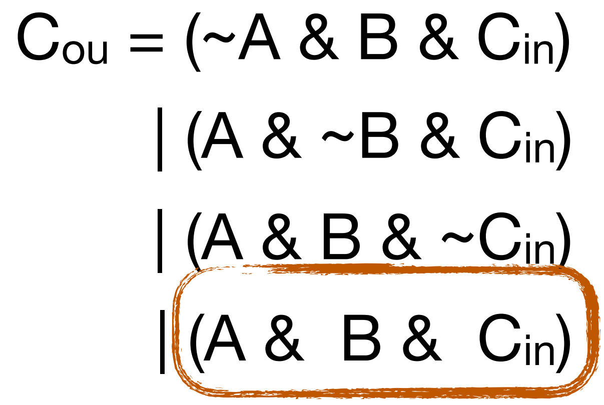
Full (1-bit) Adder

Add two bits and carry-in,
produce one-bit sum and carry-out.

$$
\begin{aligned}
C_{ou} = &(\sim A \,\&\, B \,\&\, C_{in}) \\
| &(A \,\&\, \sim B \,\&\, C_{in}) \\
| &(A \,\&\, B \,\&\, \sim C_{in}) \\
| &(A \,\&\, B \,\&\, C_{in})
\end{aligned}
$$



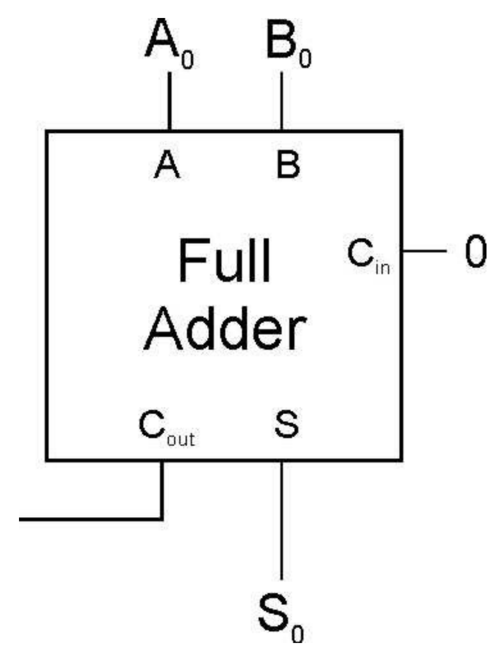
Full (1-bit) Adder



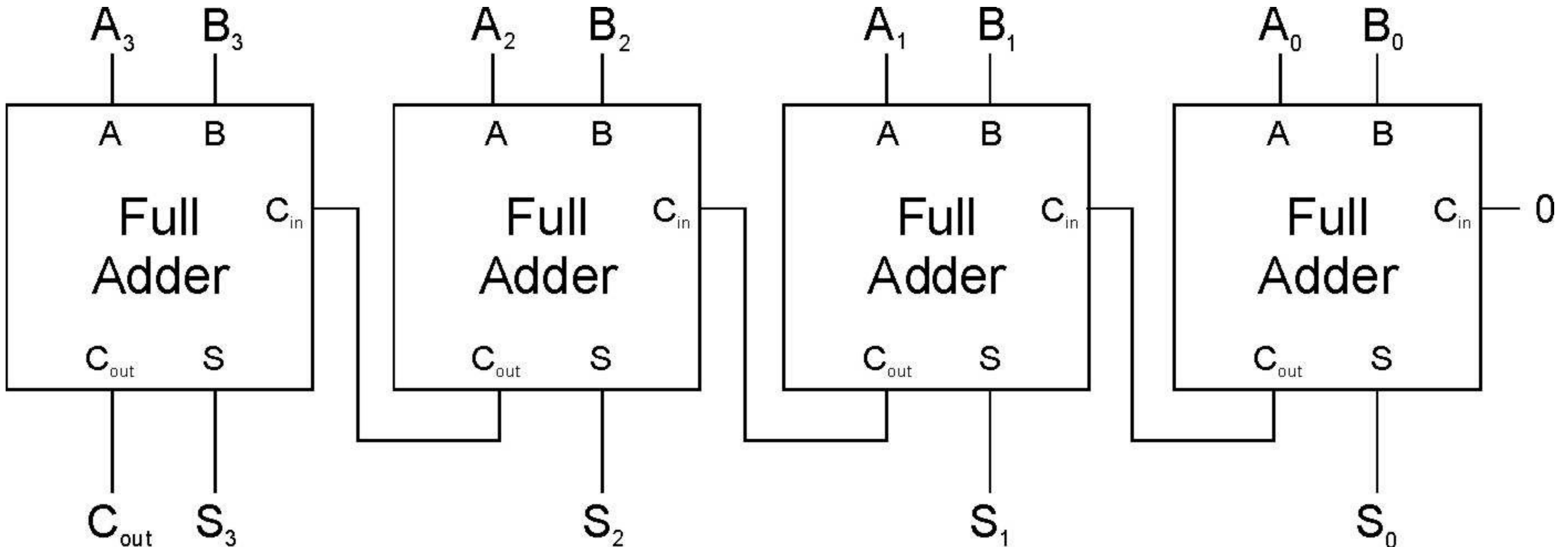
Add two bits and carry-in,
produce one-bit sum and carry-out.

A
B
C_{in}
AND Gates
OR Gates
C_{out}
S

Four-bit Adder

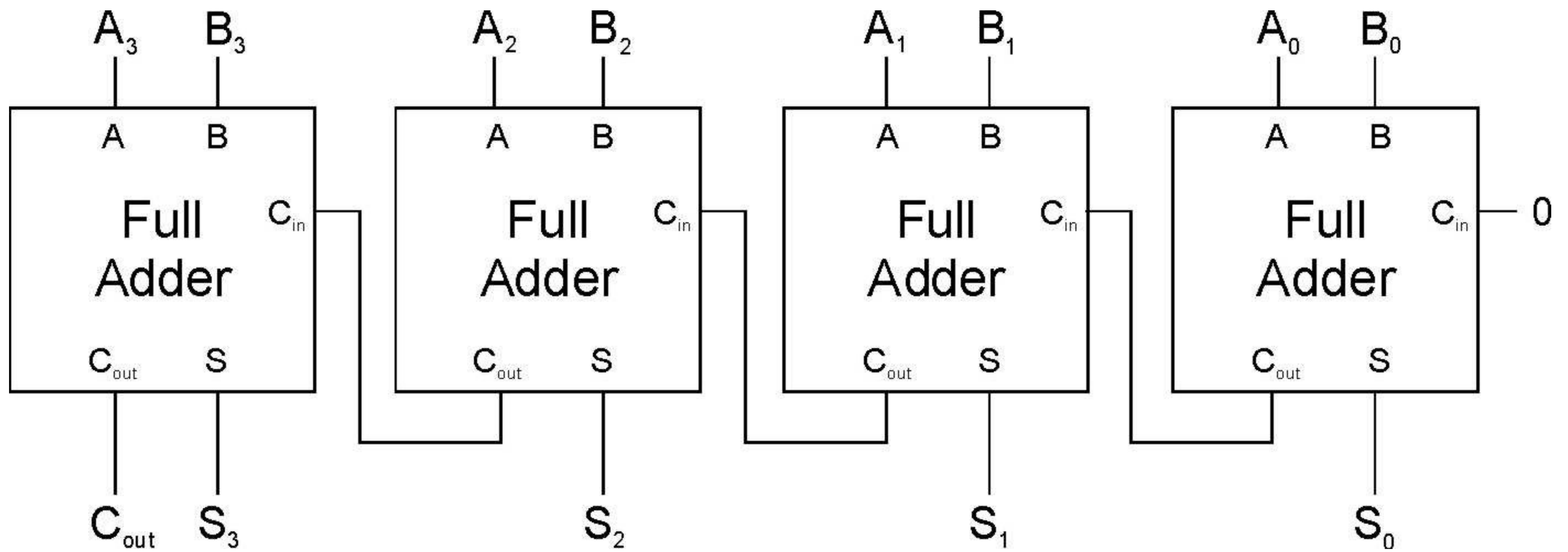


Four-bit Adder



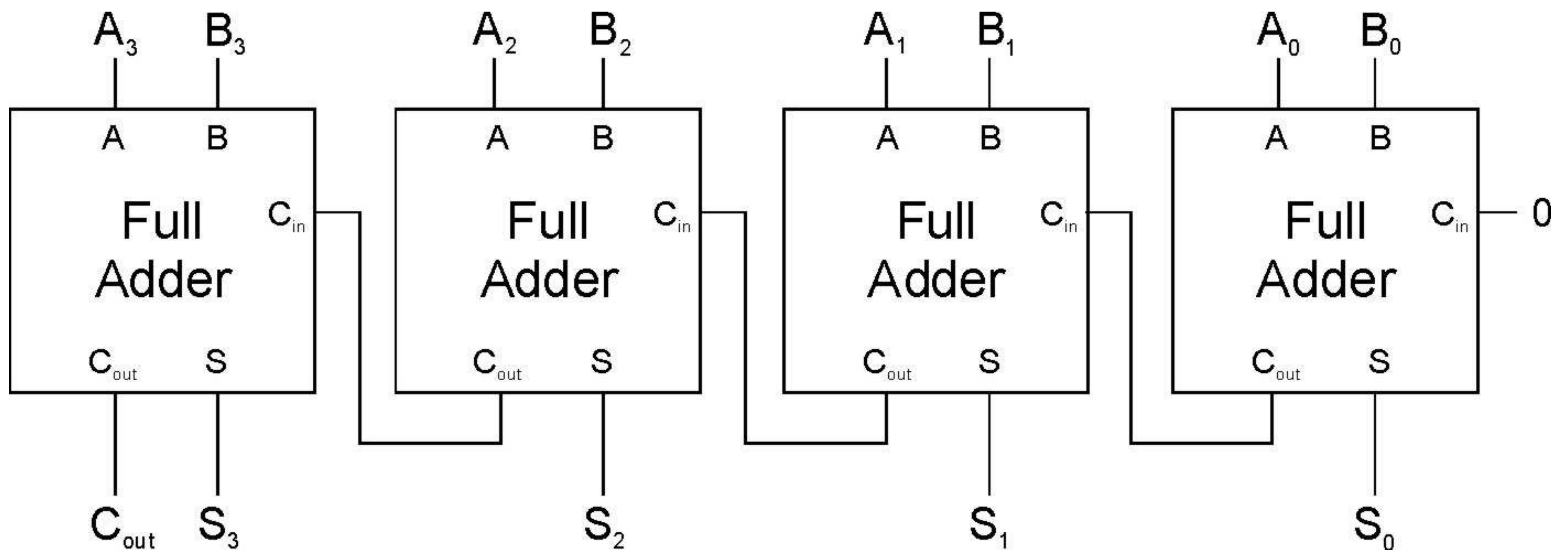
Four-bit Adder

- Ripple-carry Adder
  - Simple, but performance linear to bit width



Four-bit Adder

- Ripple-carry Adder
  - Simple, but performance linear to bit width
- Carry look-ahead adder (CLA)
  - Generate all carriers simultaneously



Logic Design

- Design digital components from basic logic gates

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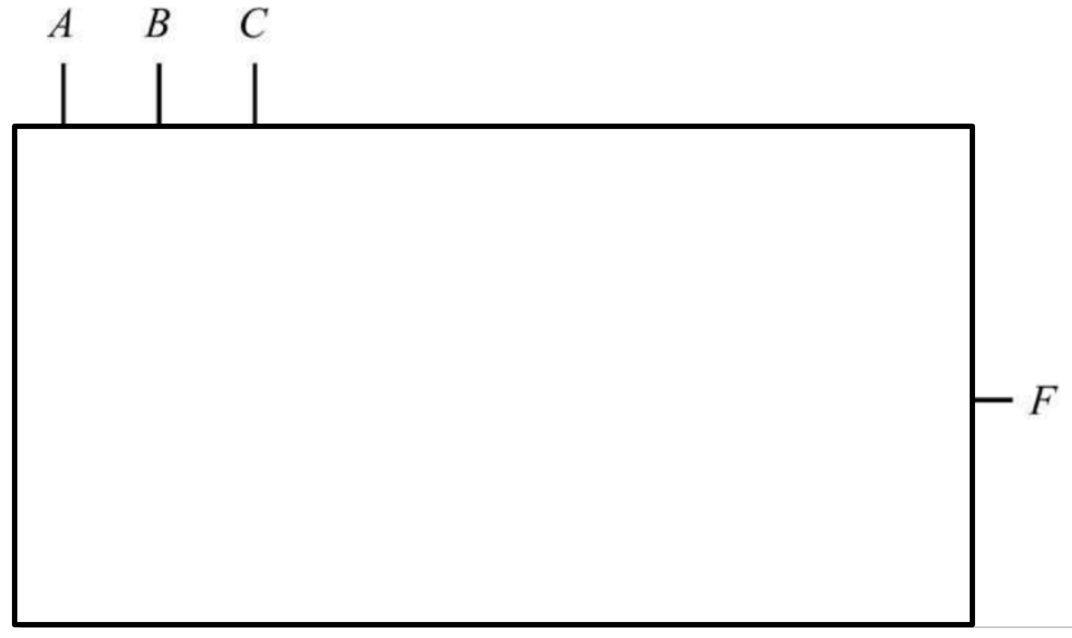

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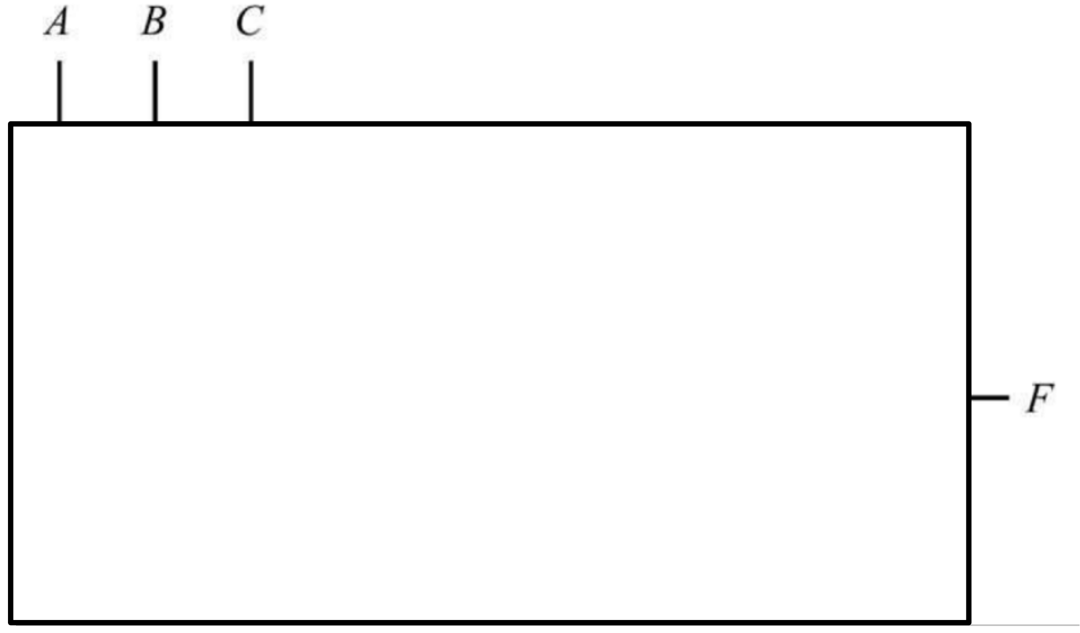

A	B	C	F
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

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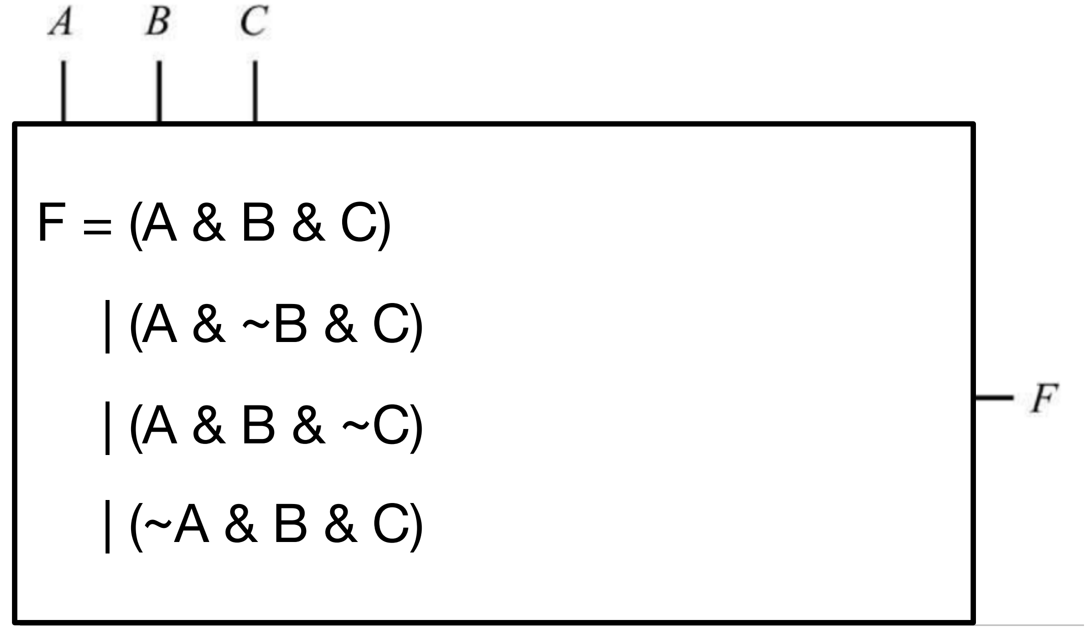

A	B	C	F
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
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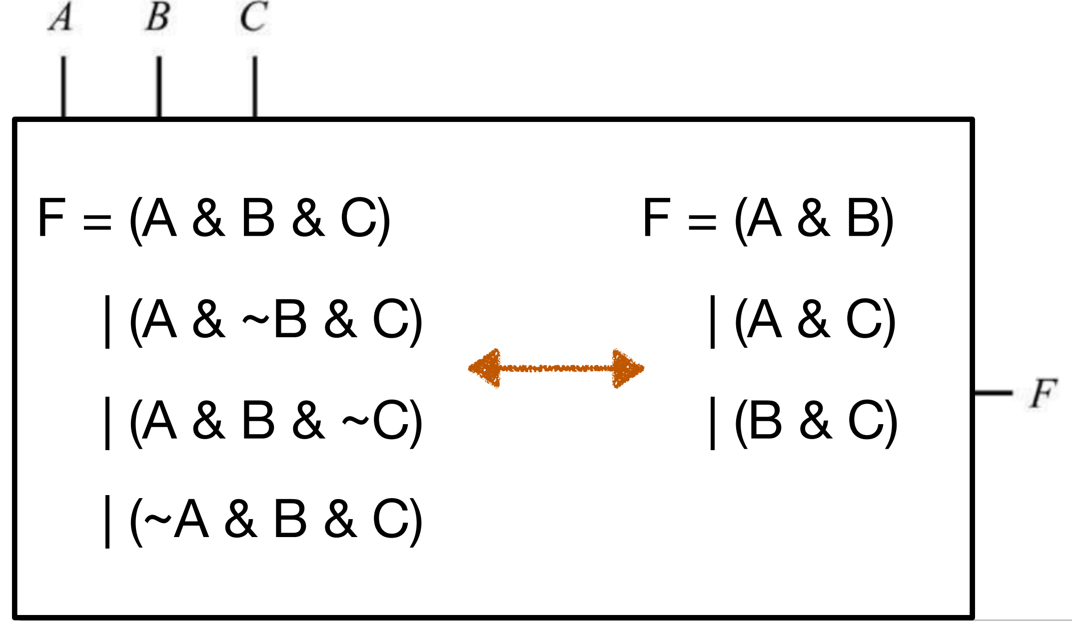

A	B	C	F
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

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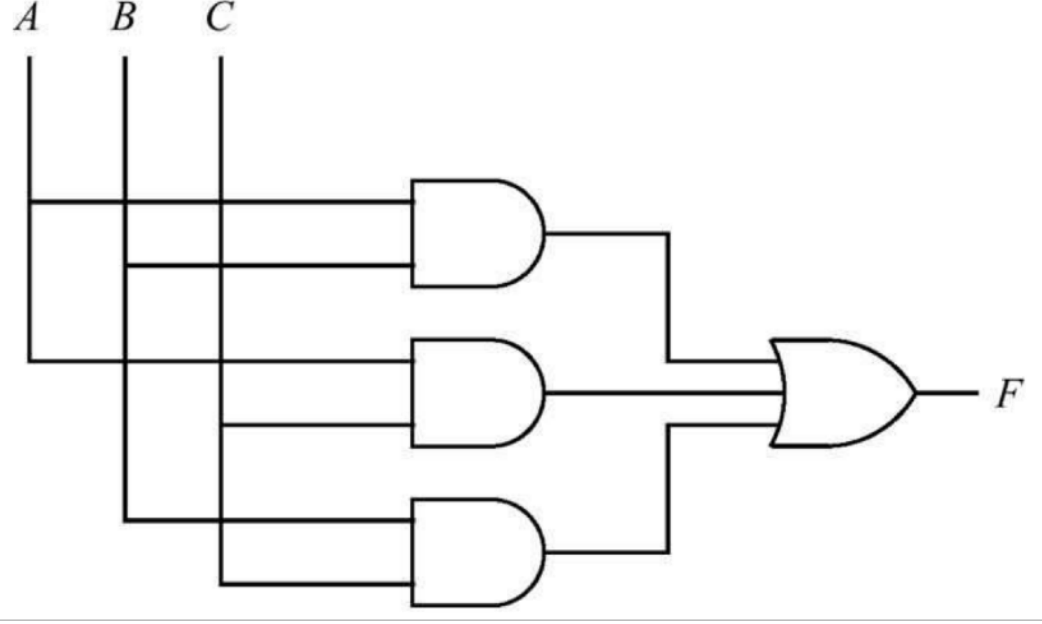

A	B	C	F
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**