CSC 252: Computer Organization
Spring 2023: Lecture 2

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Department of Computer Science
University of Rochester

Announcement

- Make sure you can access CSUG machines!!!
- Programming assignment 1 will be posted this week.
  - I will send an announcement when it's out.
  - It is in C language. Seek help from TAs.
  - TAs are best positioned to answer your questions about programming assignments!!!
- Programming assignments do NOT repeat the lecture materials. They ask you to synthesize what you have learned from the lectures and work out something new.

Previously in 252…

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

Microarchitecture

Circuit

Previously in 252…

Problem

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Instruction Set Architecture (ISA)

ISA is the contract between software and hardware.

Microarchitecture

Circuit

Previously in 252…

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Algorithm

	Renting
Service provider	Landlord
Service receiver	YOU
Contract	Lease
Contract's language	Natural language (e.g., English)

Circuit

;t
e and

Previously in 252…

Problem

Algorithm

	Renting	**Computing**
Service provider	Landlord	Hardware
Service receiver	YOU	Software
Contract	Lease	ISA
Contract's language	Natural language (e.g., English)	Assembly programming language

ct
e and

Circuit

Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

Microarchitecture

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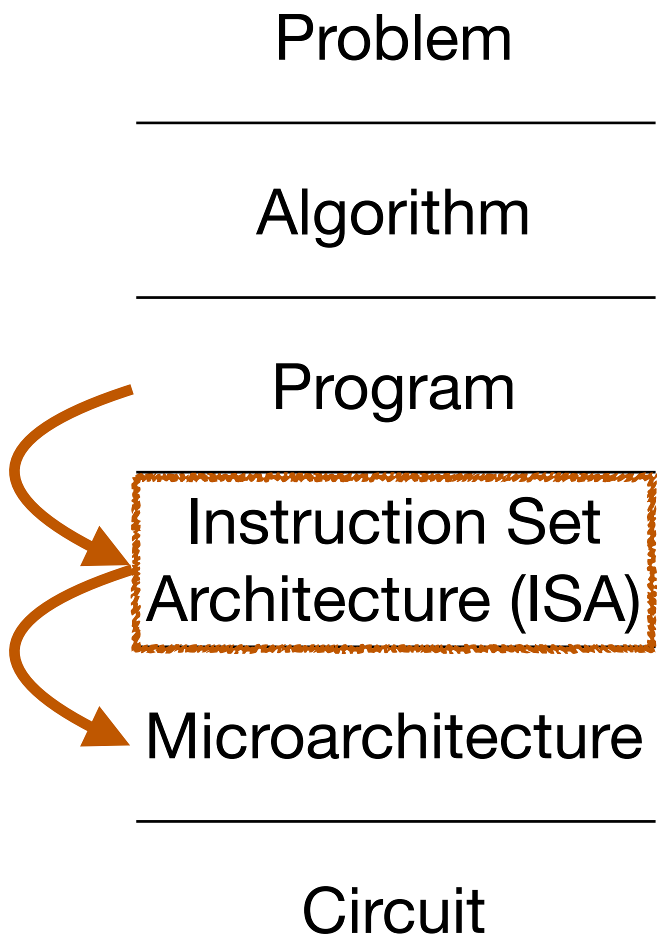
ISA is the contract between software and hardware.

Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?
- How does a modern computer execute that program?



ISA is the contract between software and hardware.

Previously in 252…

C Program

```
void add() {
  int a = 1;
  int b = 2;
  int c = a + b;
}
```

→

Assembly program

```
movl    $1, -4(%rbp)
movl    $2, -8(%rbp)
movl    -4(%rbp), %eax
addl    -8(%rbp), %eax
```

Previously in 252…

Assembly program

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movl    $1, -4(%rbp)
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addl    -8(%rbp), %eax
```

Executable Binary

```
00011001   …
01101010   …
11010101   …
01110001   …
```

Previously in 252…

Assembly program		*Executable Binary*
movl $1, -4(%rbp)		00011001 …
movl $2, -8(%rbp)	→	01101010 …
movl -4(%rbp), %eax		11010101 …
addl -8(%rbp), %eax		01110001 …

- What's the difference between an assembly program and an executable binary?
  - They refer to the same thing — a list of instructions that the software asks the hardware to perform
  - They are just different representations
- Instruction = Operator + Operand(s)

Previously in 252…

Assembly program

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Previously in 252…

Assembly program *Executable Binary*

```
movl   $1, -4(%rbp)          00011001  …
movl   $2, -8(%rbp)    →     01101010  …
movl   -4(%rbp), %eax        11010101  …
addl   -8(%rbp), %eax        01110001  …
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Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting

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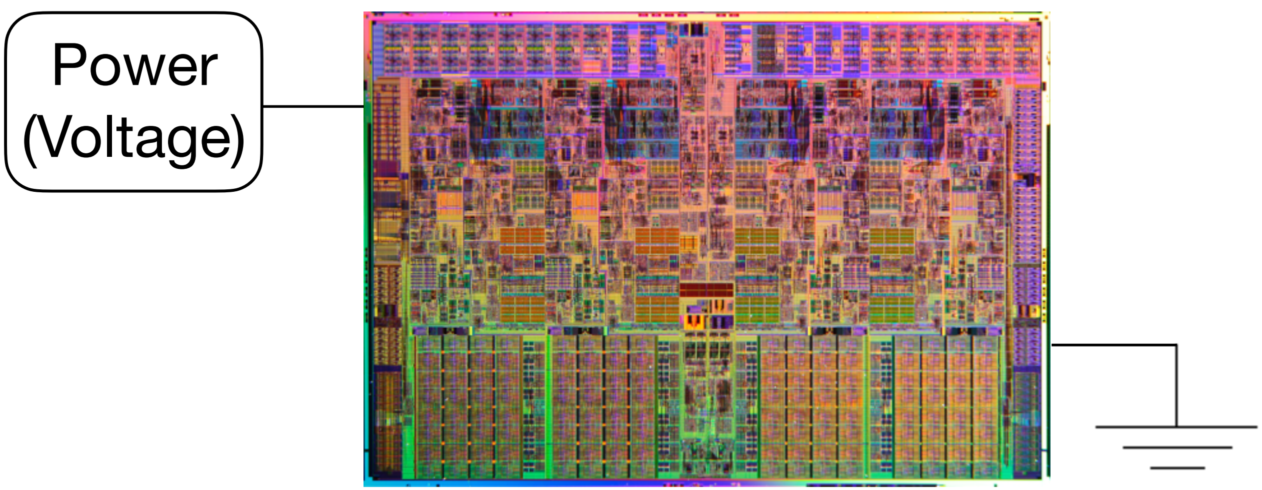

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  - Use high voltage to represent 1
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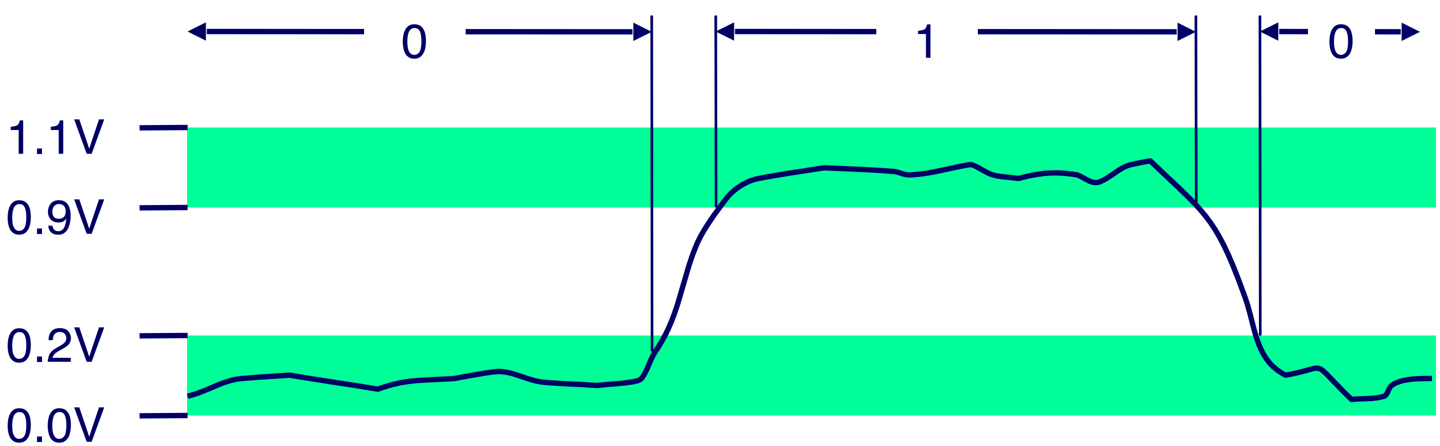

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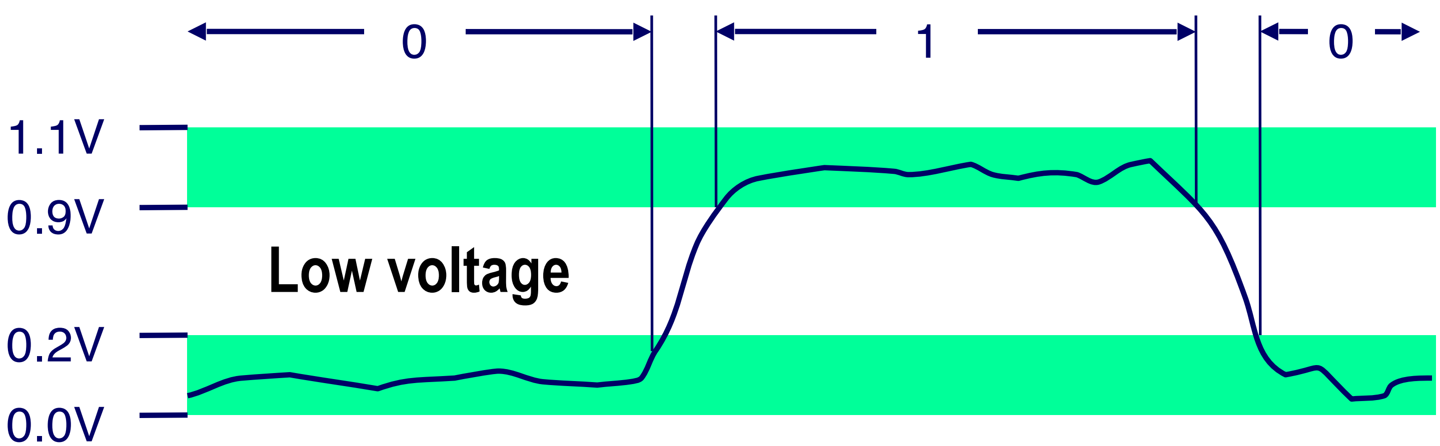

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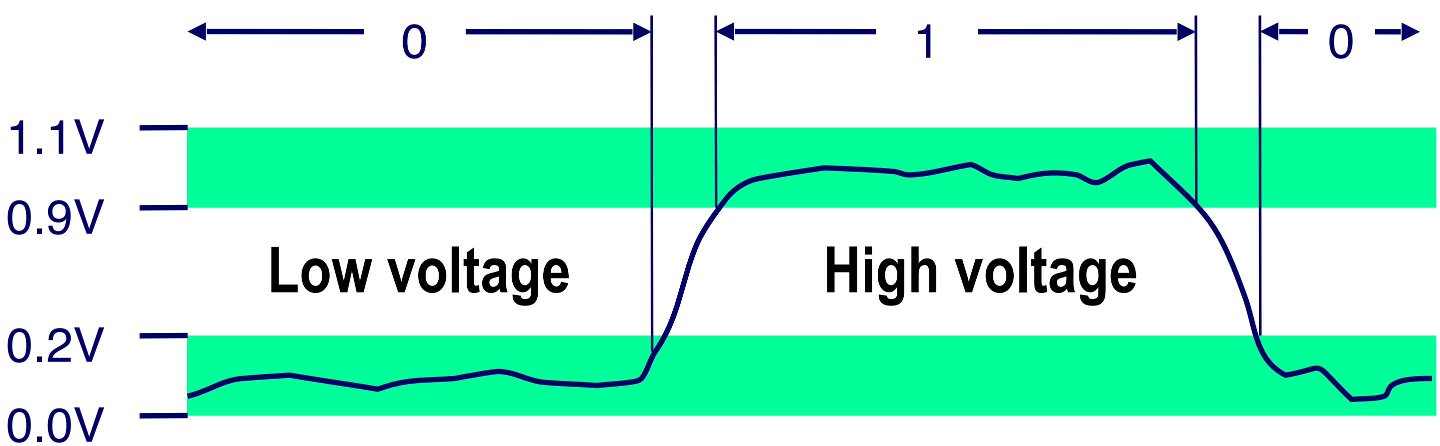

Why Bits?

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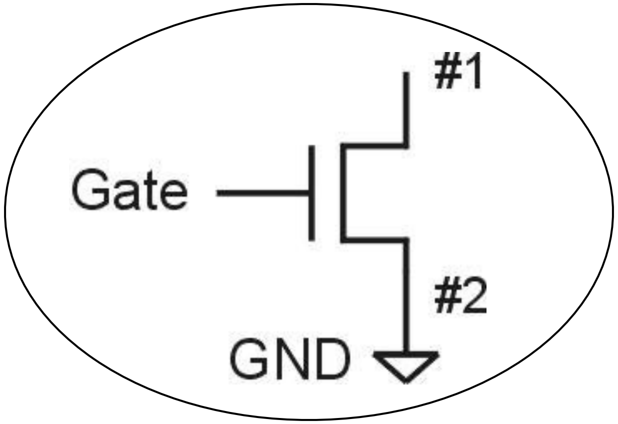

Terminal #2 must be connected to GND (0V).

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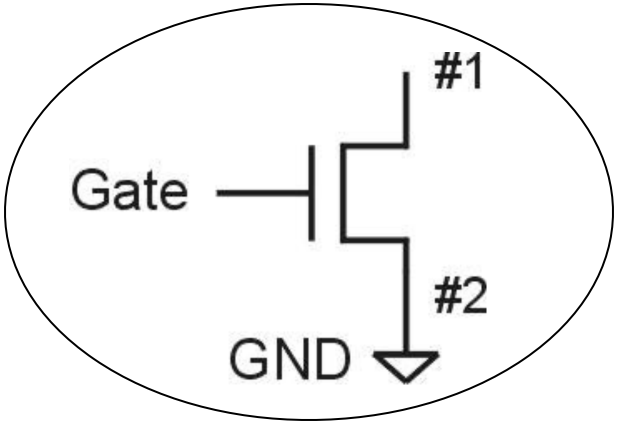
n-type (NMOS)

- when Gate has high voltage, short circuit between #1 and #2 (switch closed)



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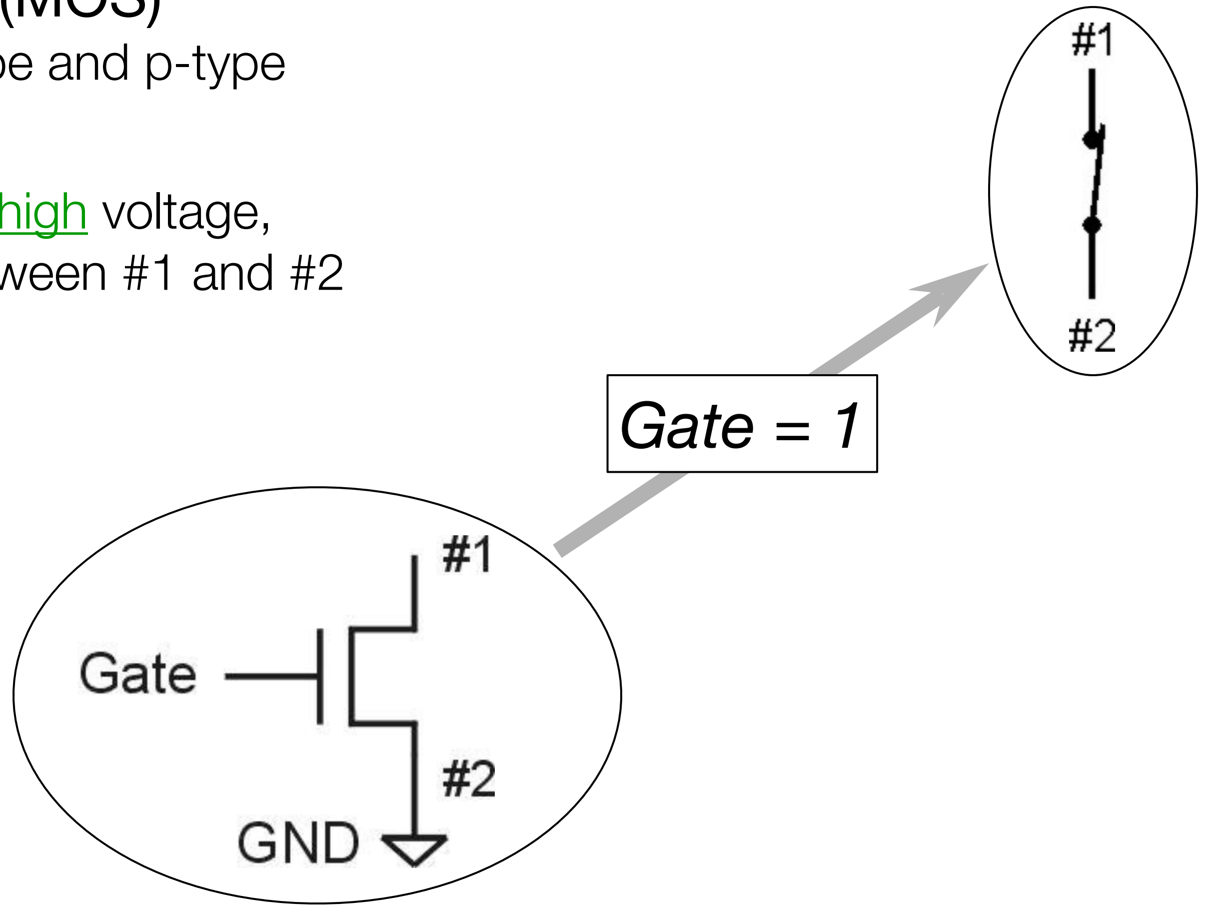

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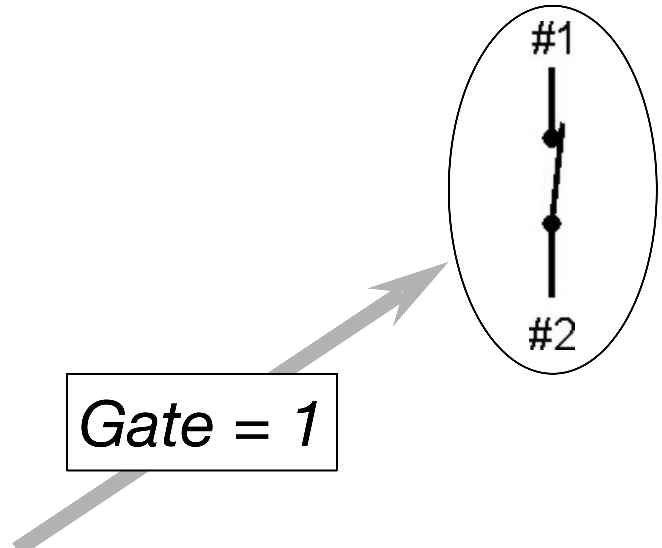

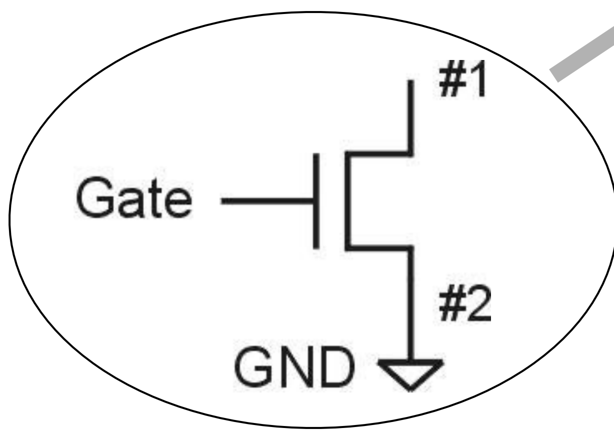

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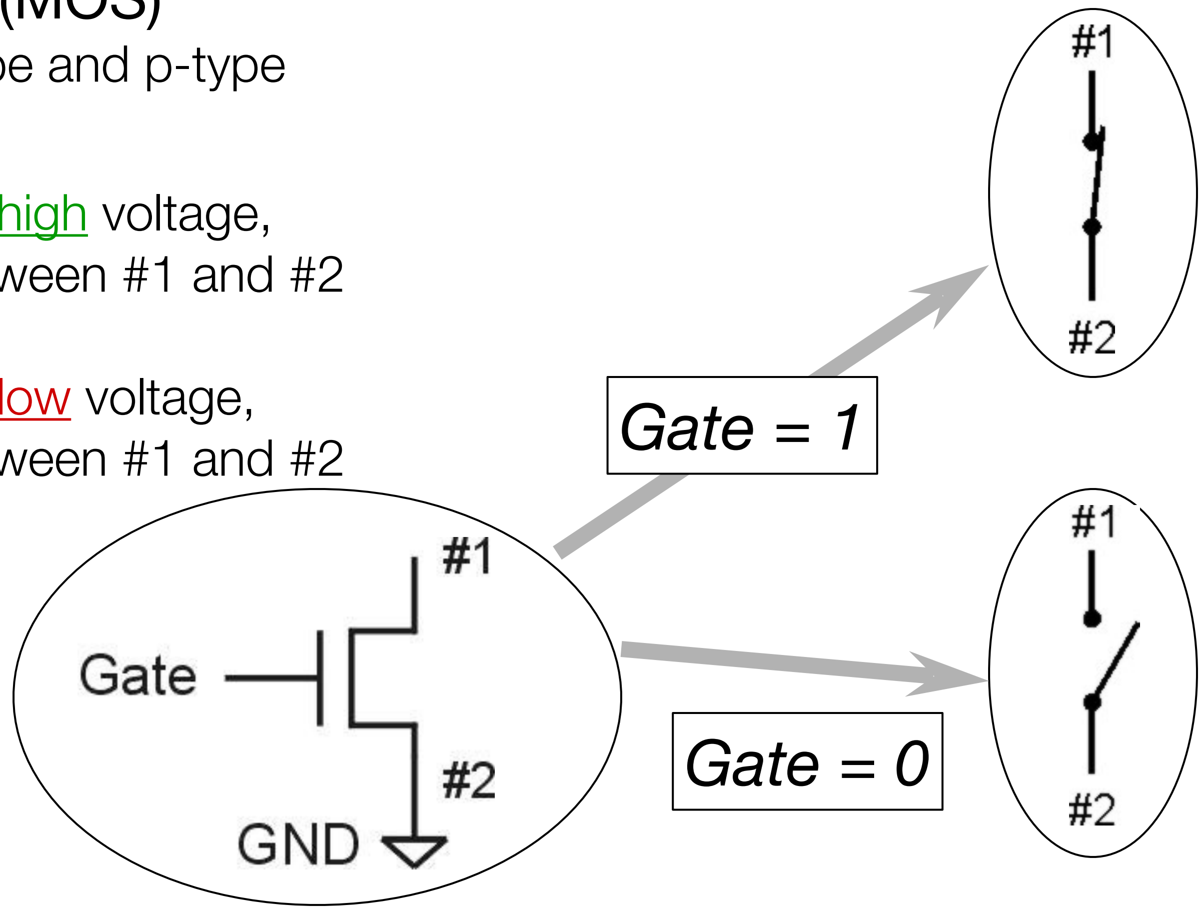

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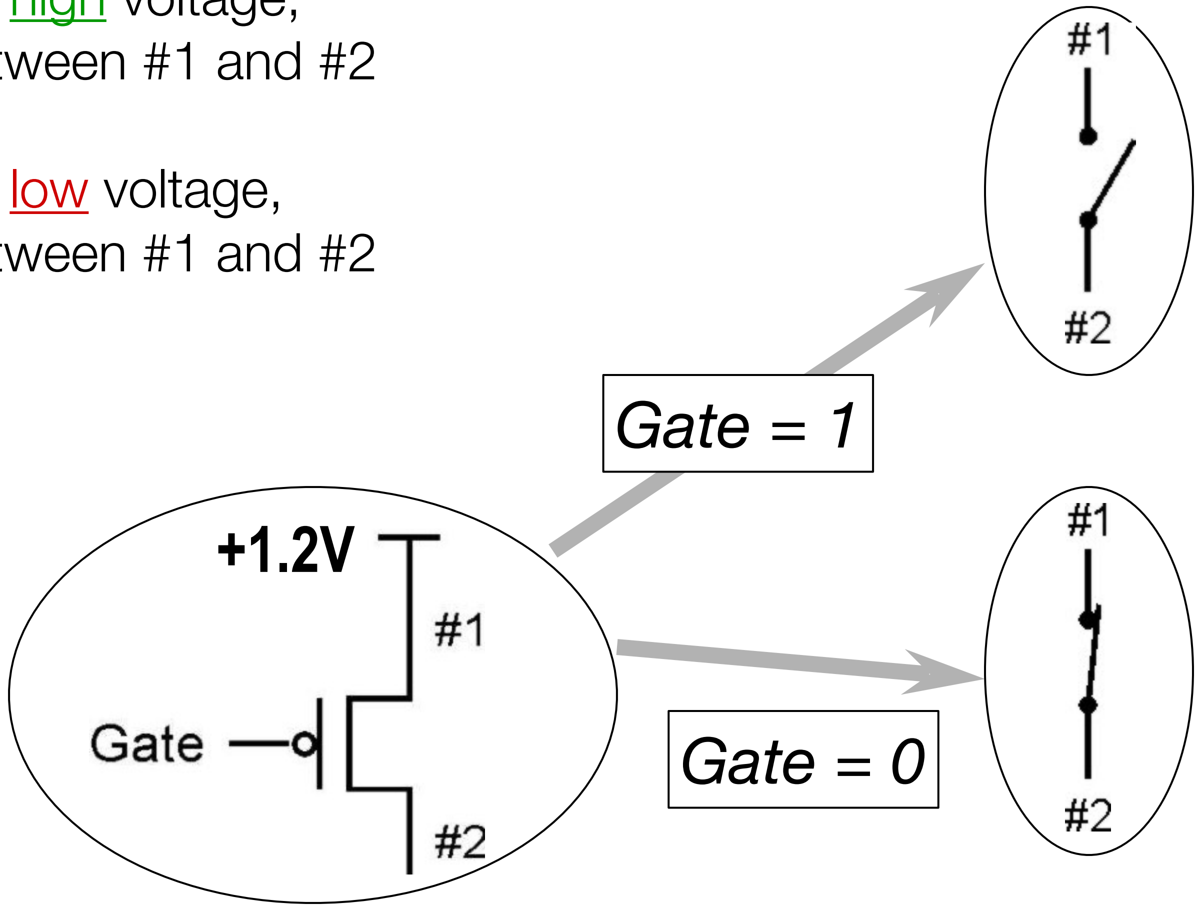
Why Bits?

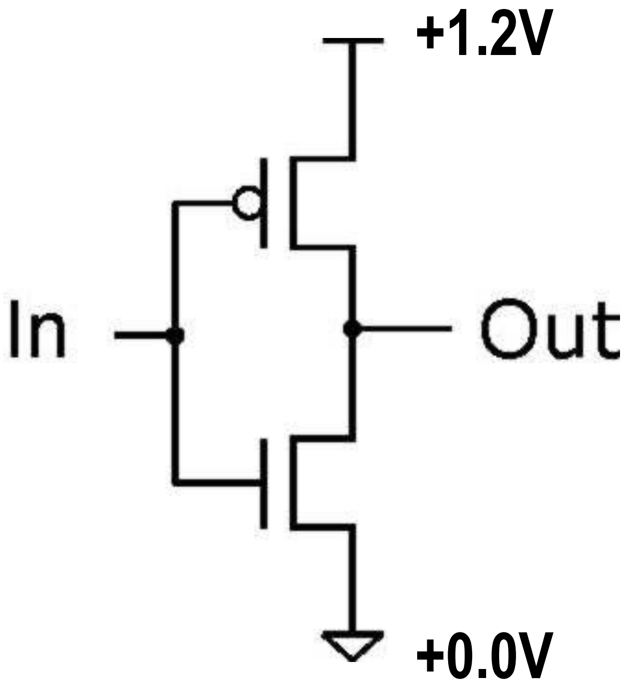
p-type is *complementary* to n-type (PMOS)

- when Gate has high voltage, open circuit between #1 and #2 (switch open)
- when Gate has low voltage, short circuit between #1 and #2 (switch closed)



Terminal #1 must be connected to +1.2V

Inverter



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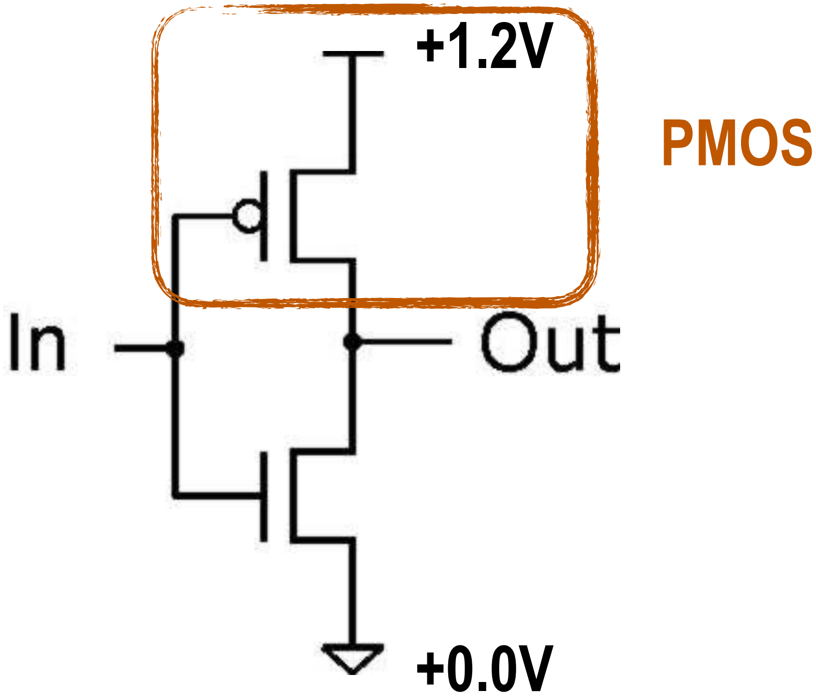

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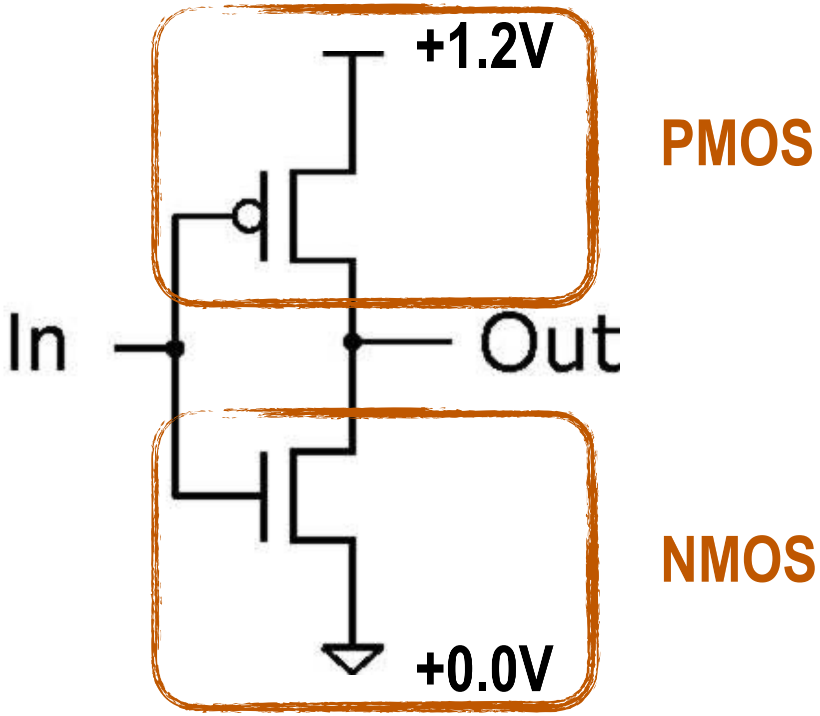

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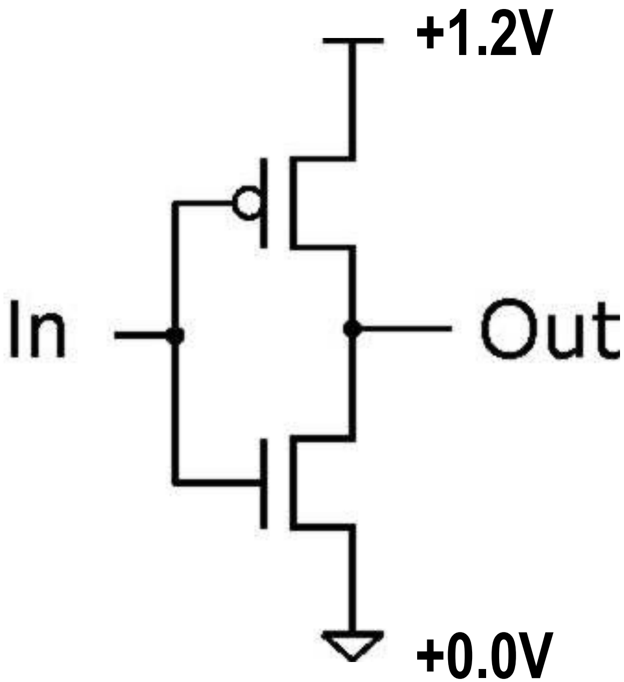
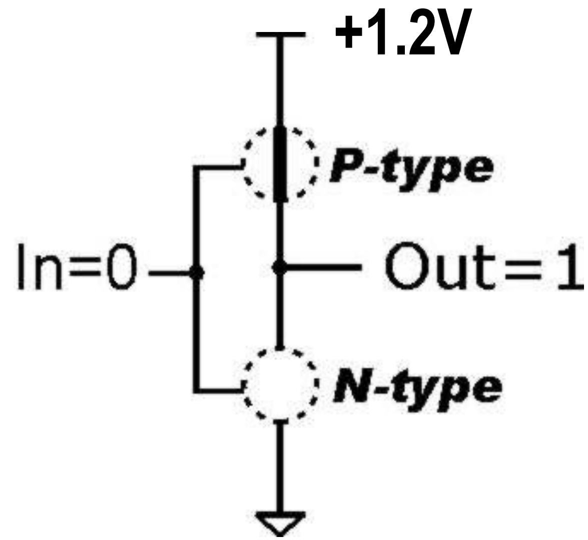
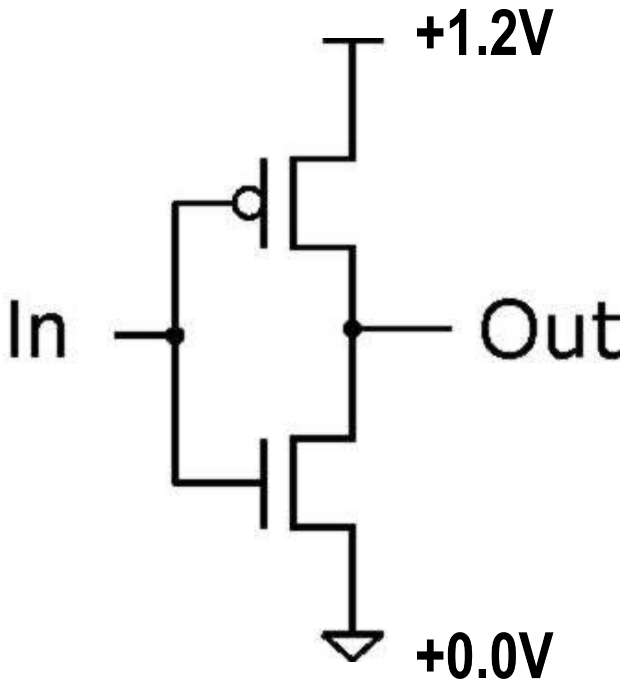
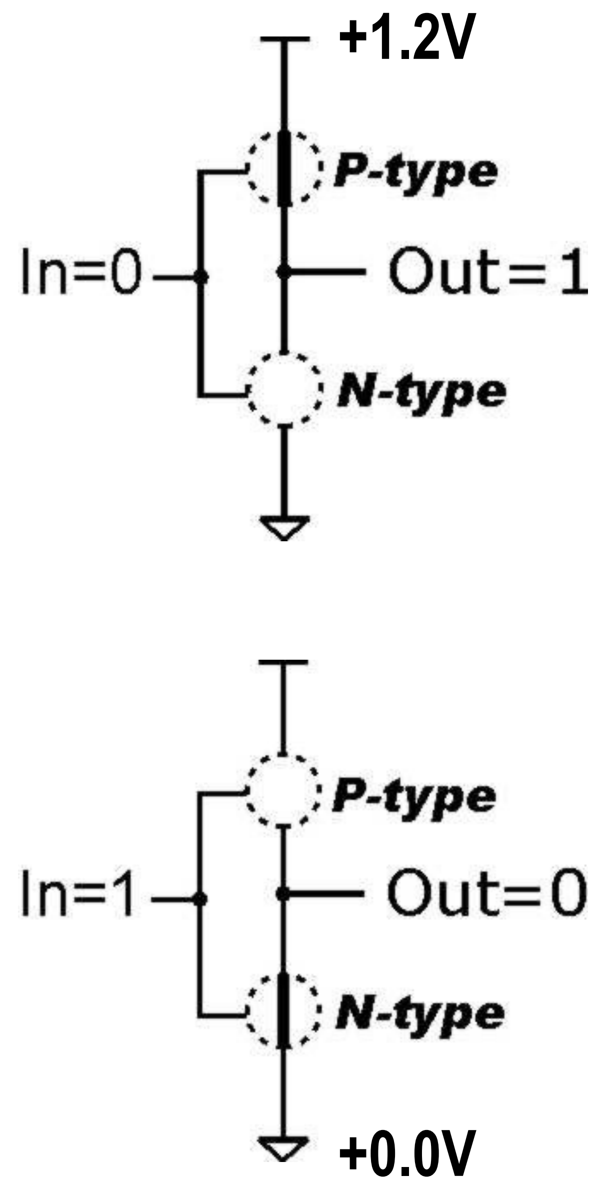


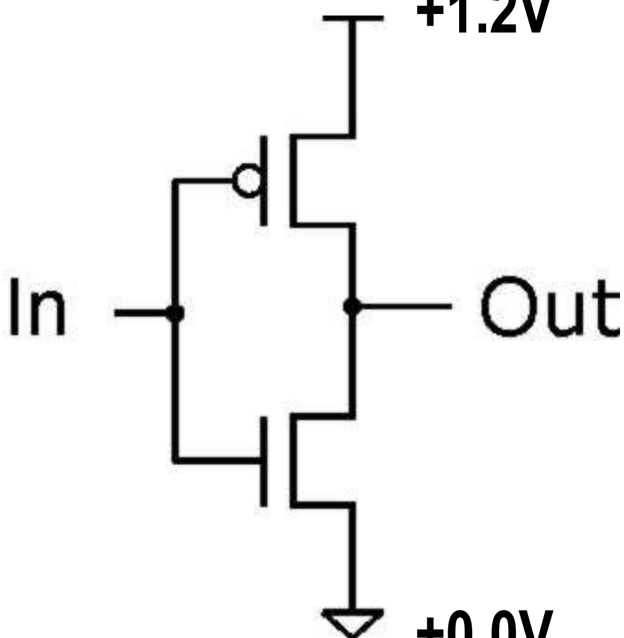


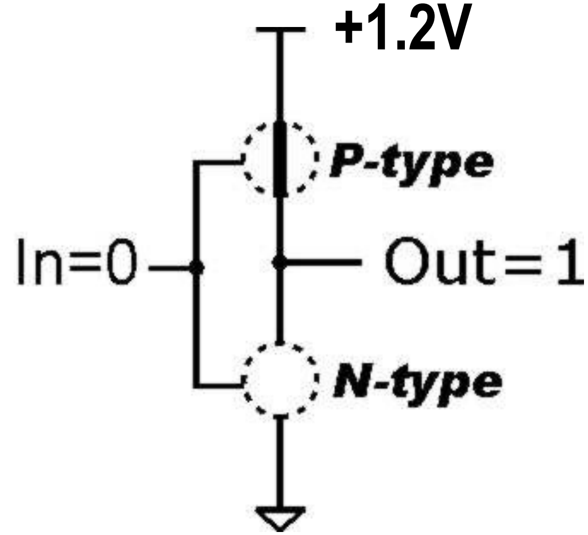
Inverter

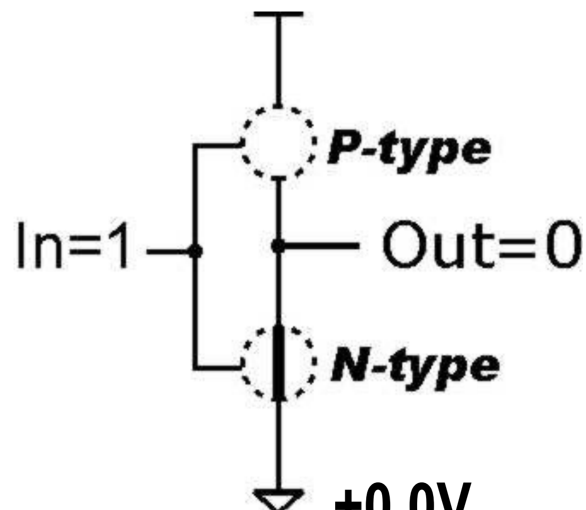

+1.2V
In
Out
+0.0V


+1.2V
P-type
In=0
Out=1
N-type
P-type
In=1
Out=0
N-type
+0.0V

Inverter

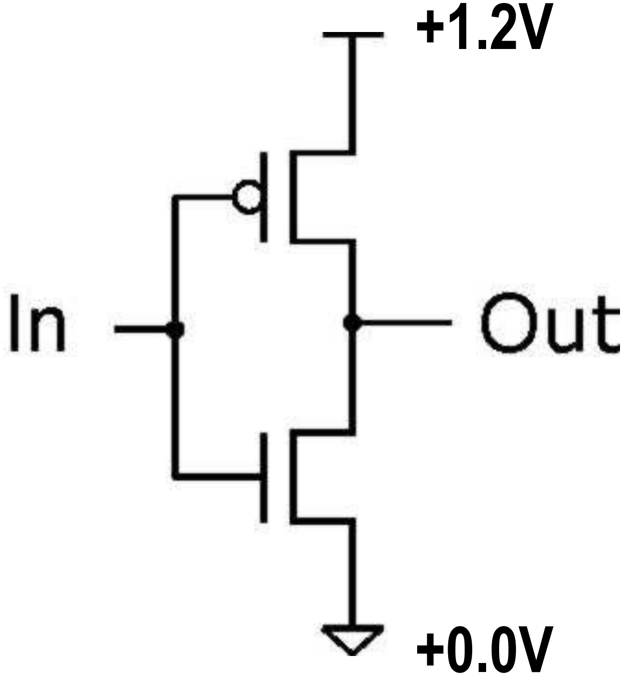


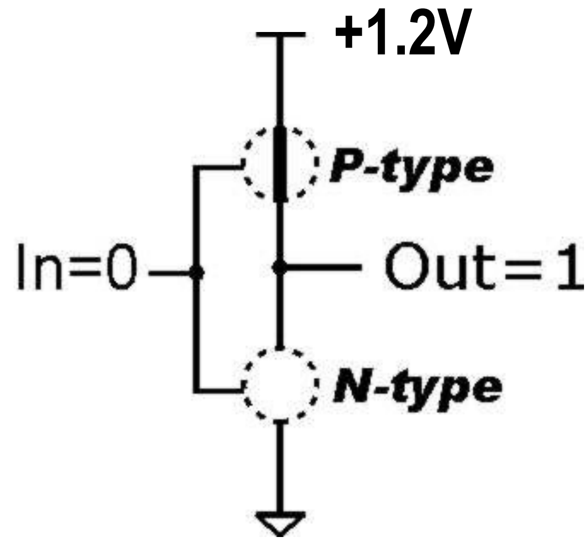


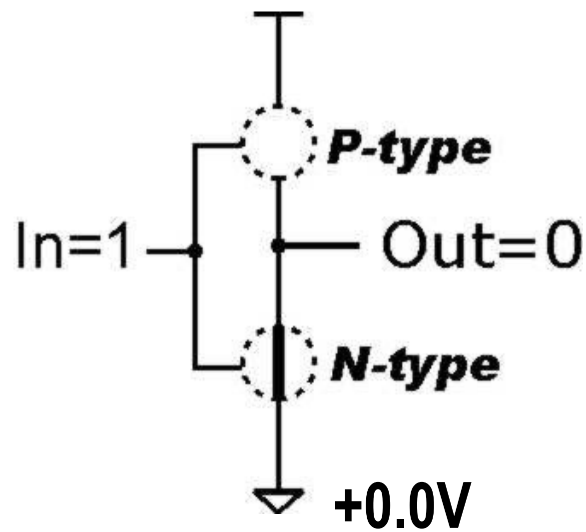


In	Out
0	1
1	0

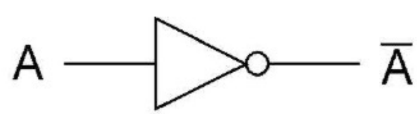
Inverter



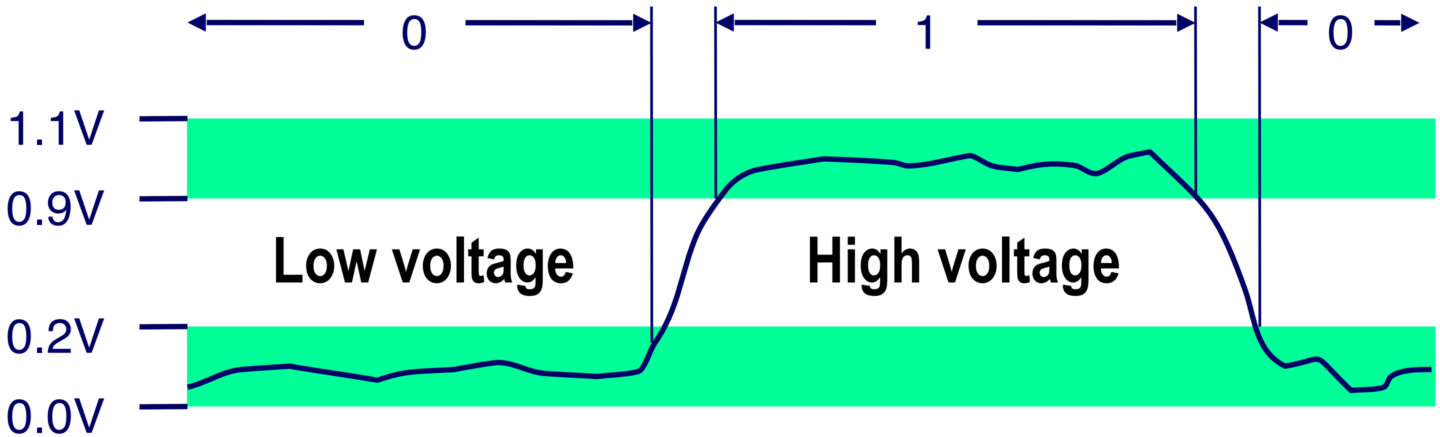


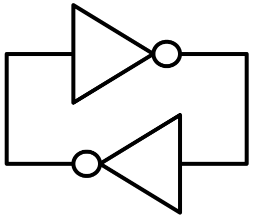


In	Out
0	1
1	0



Store/Access Data





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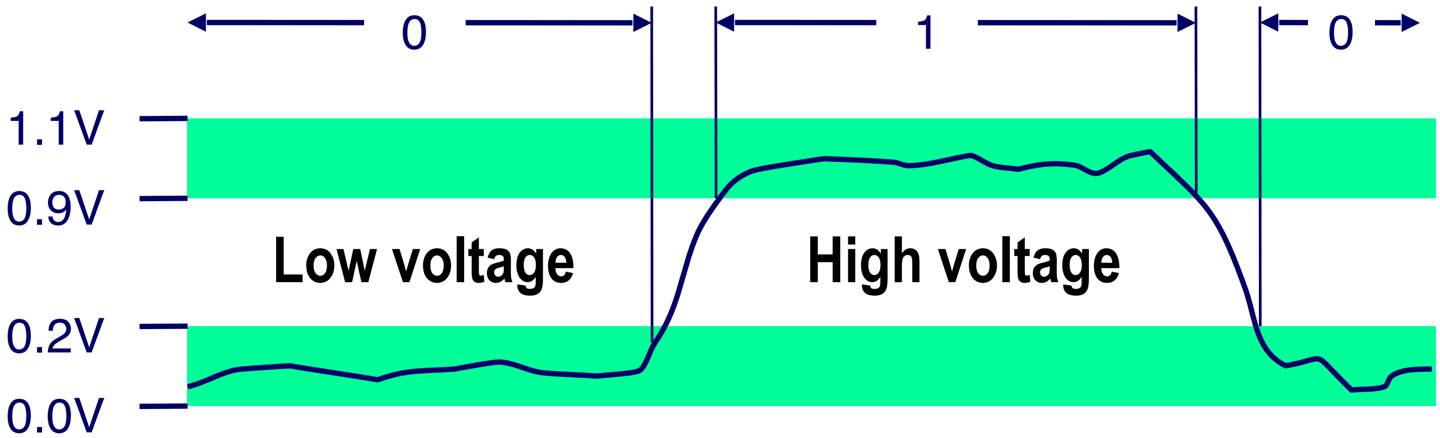

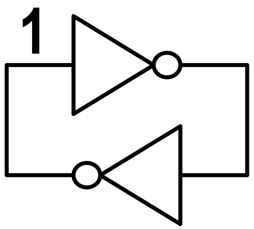

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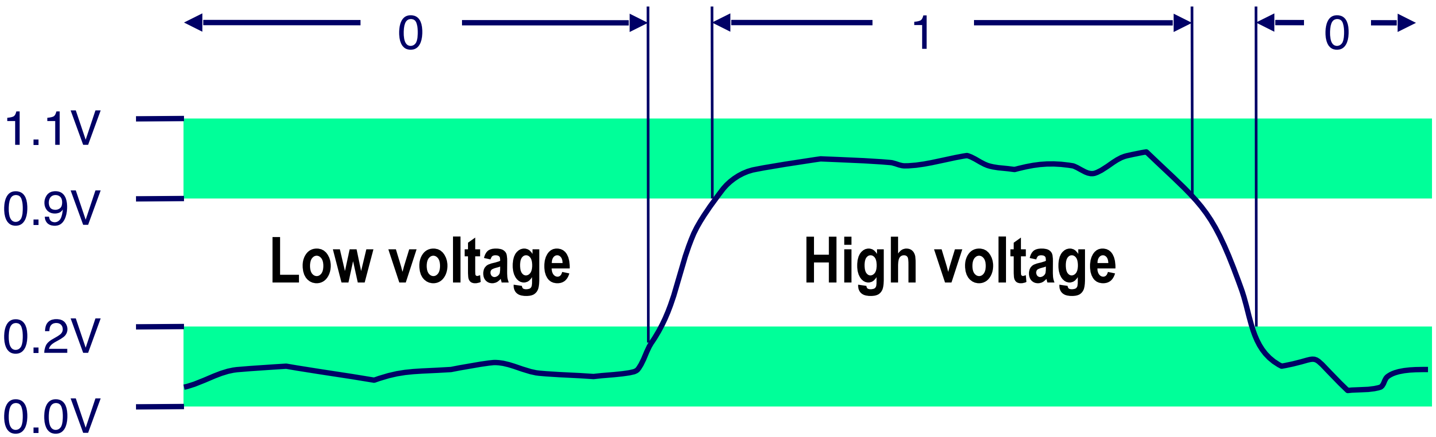

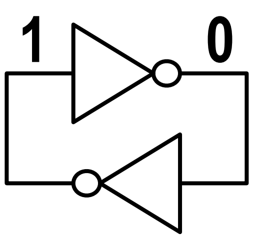

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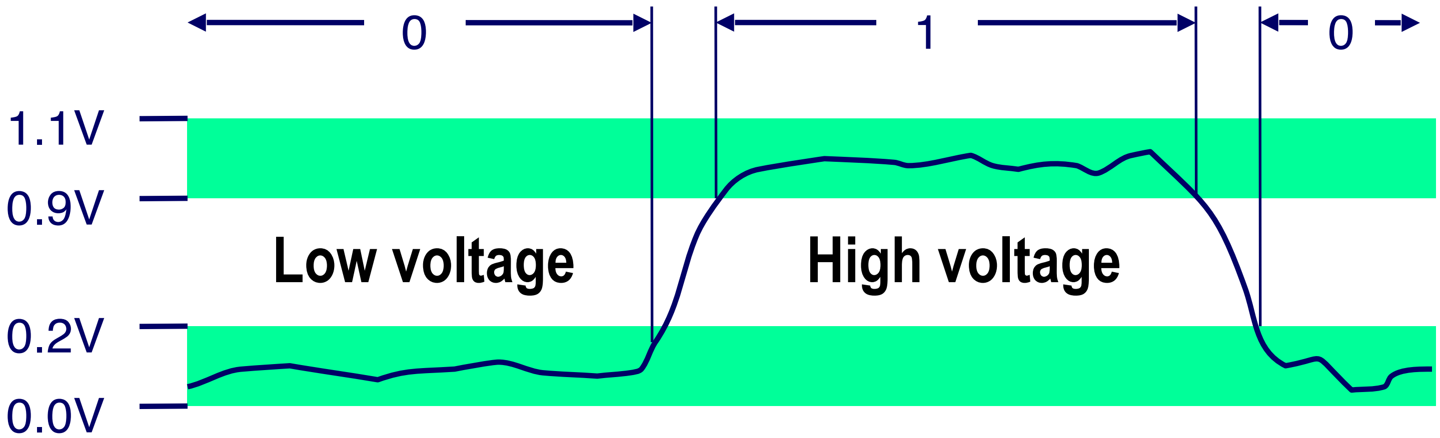

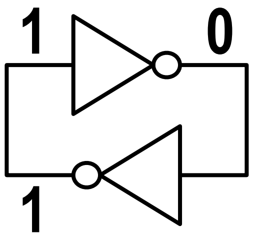

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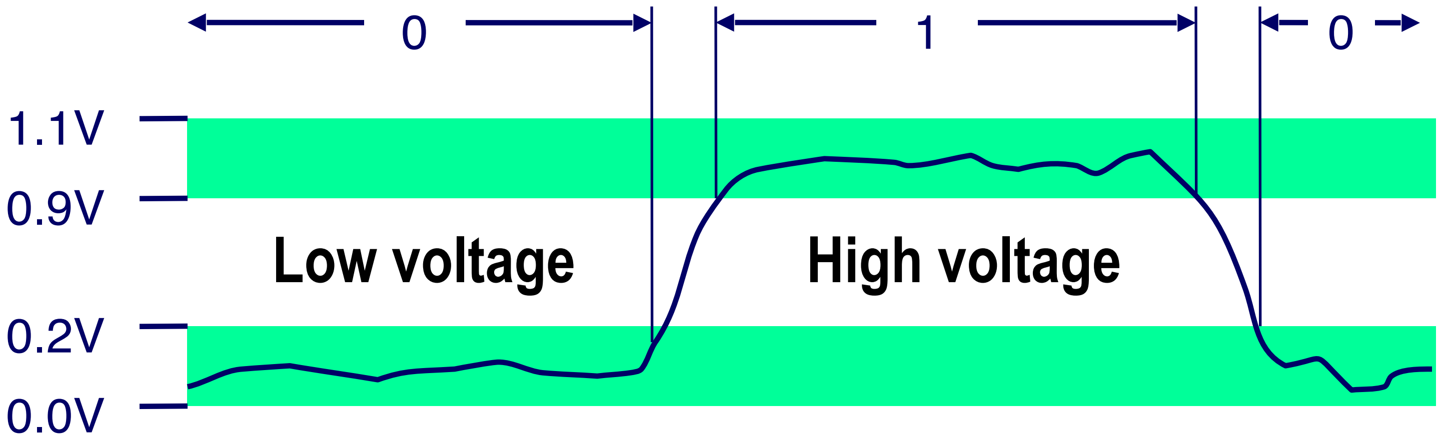
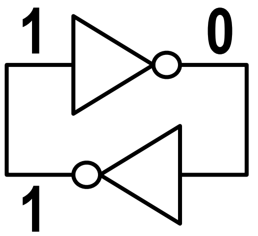


- Two cross coupled inverters store a single bit
  - Feedback path persists the value in the “cell”



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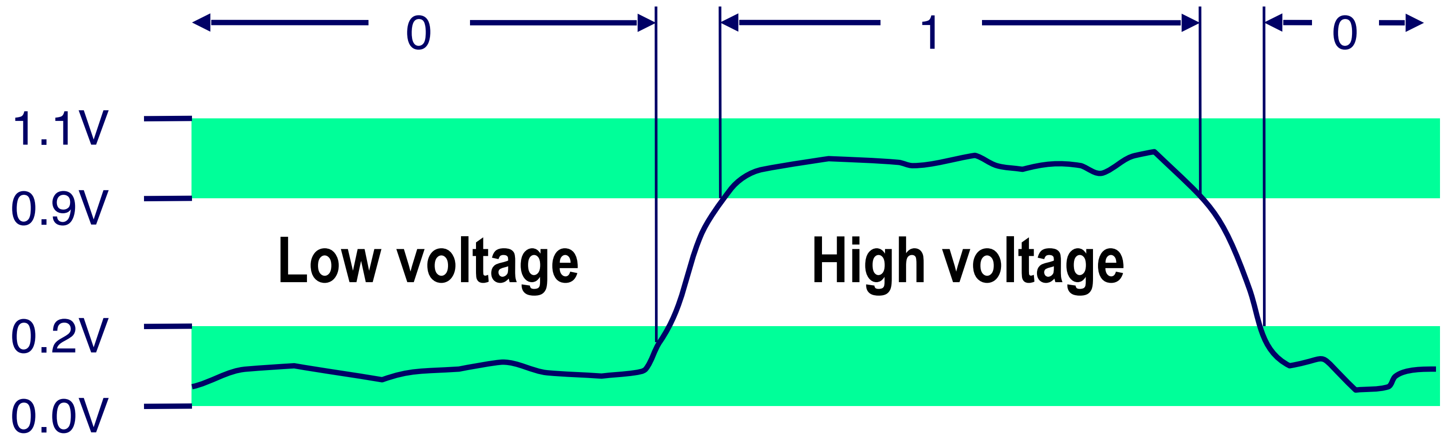

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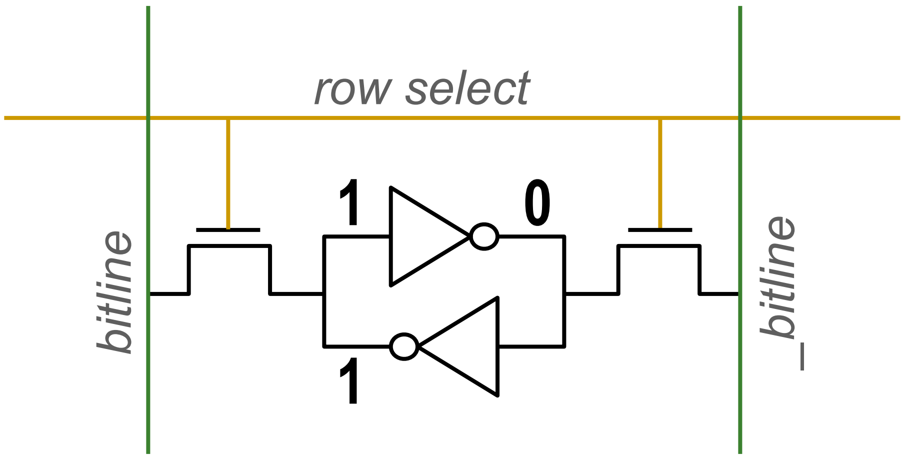

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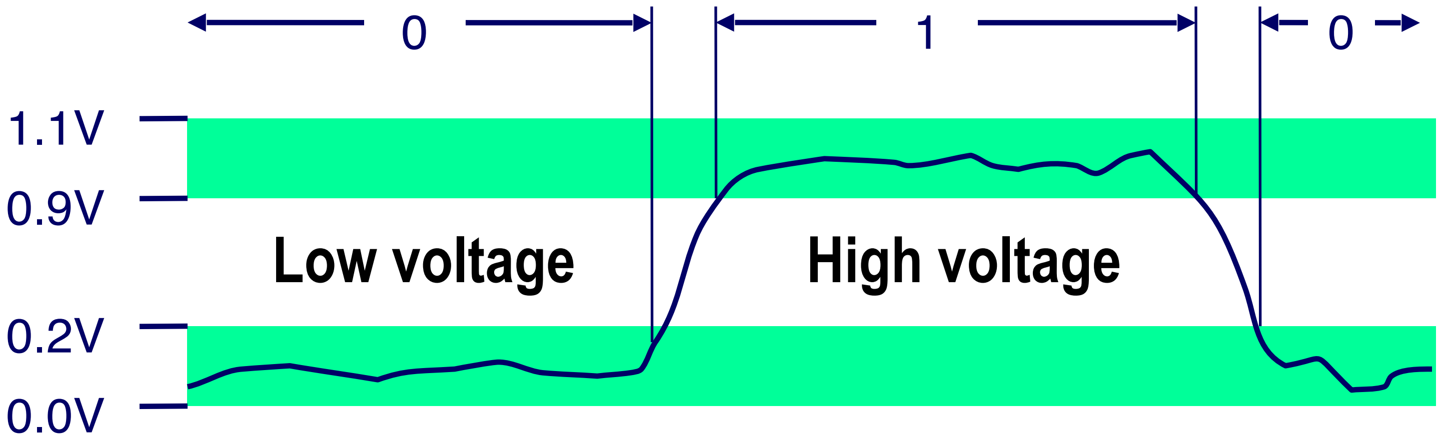

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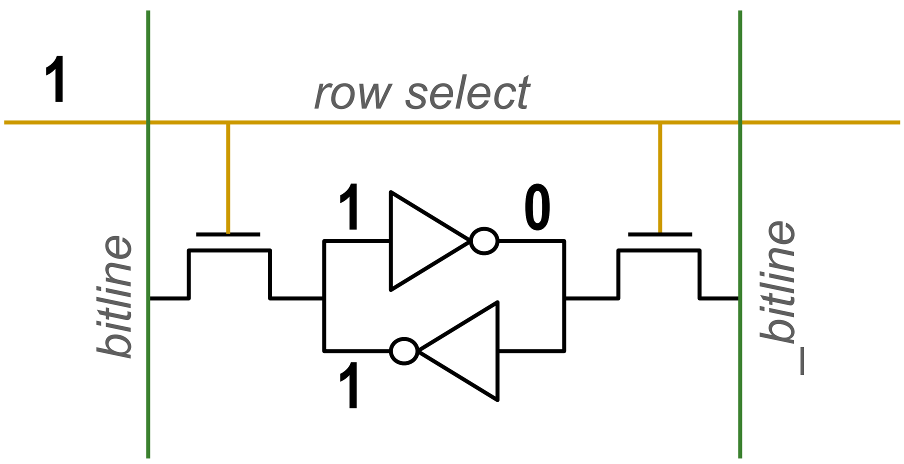

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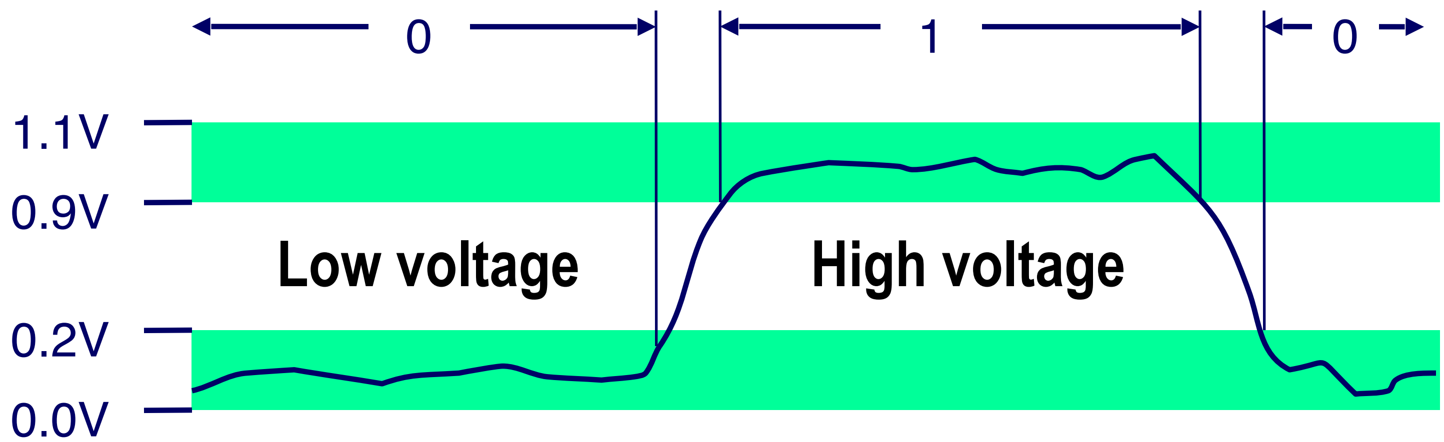

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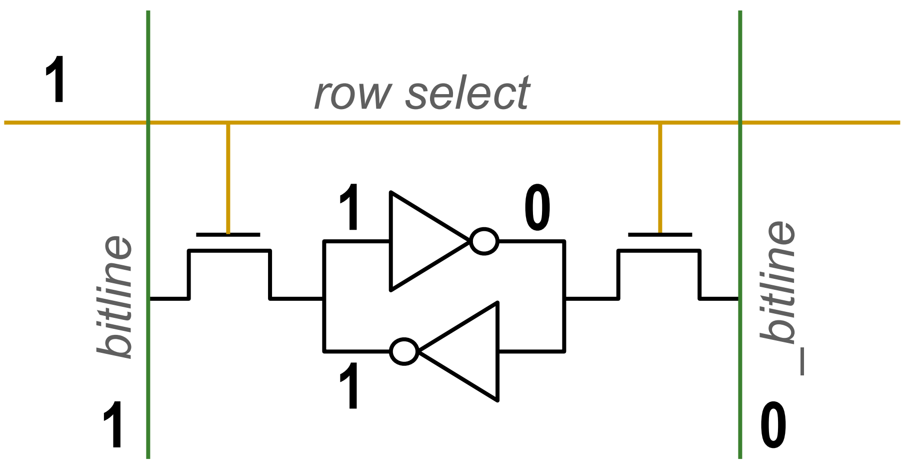

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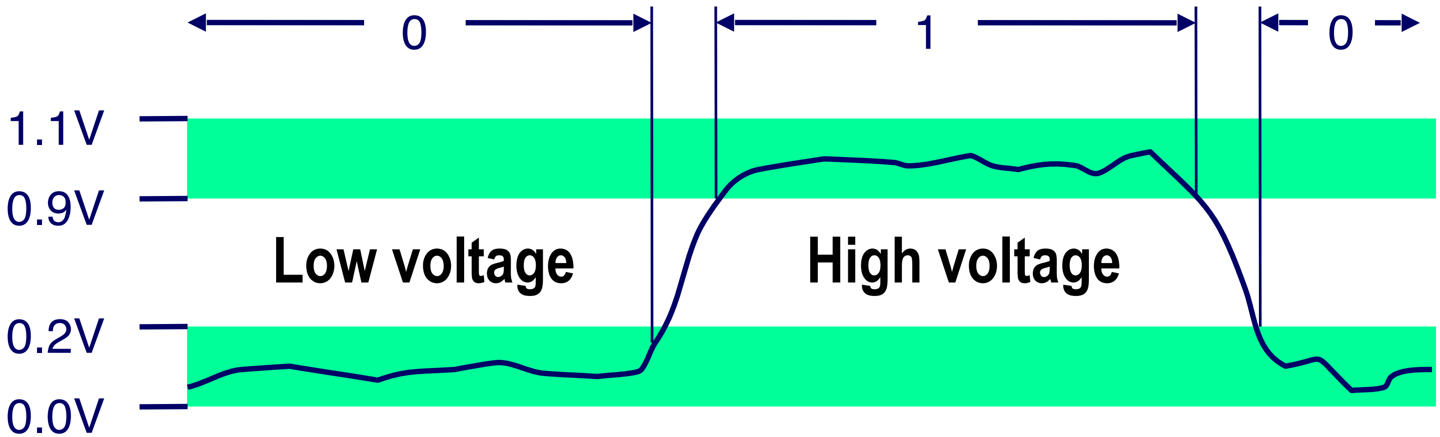

- Two cross coupled inverters store a single bit
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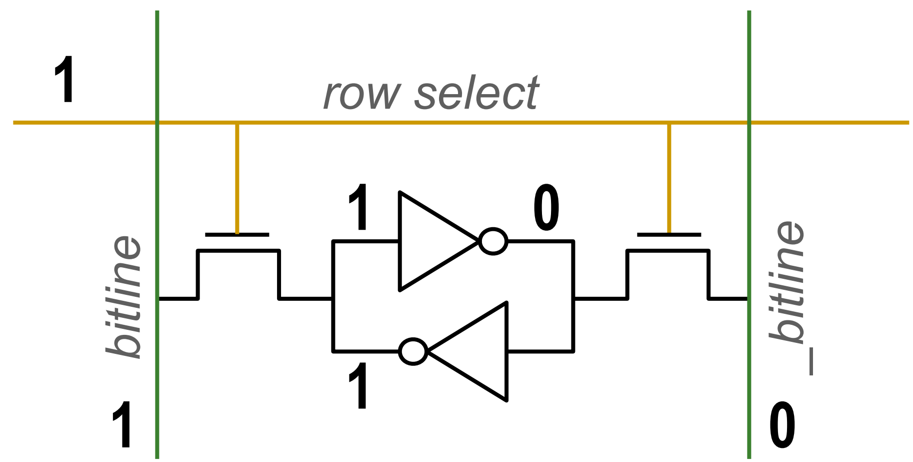
  - 4 transistors for storage



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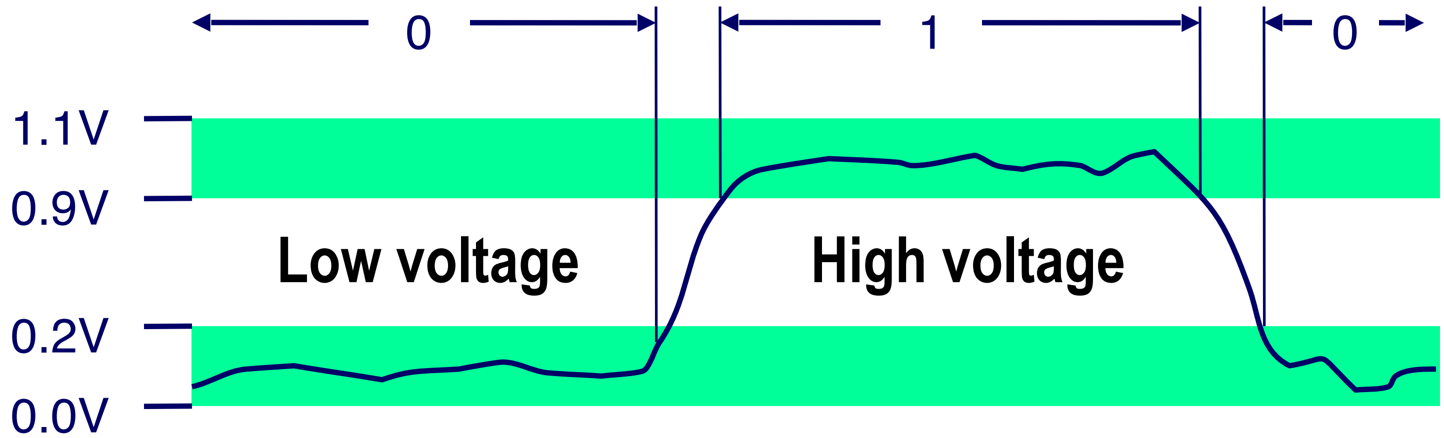

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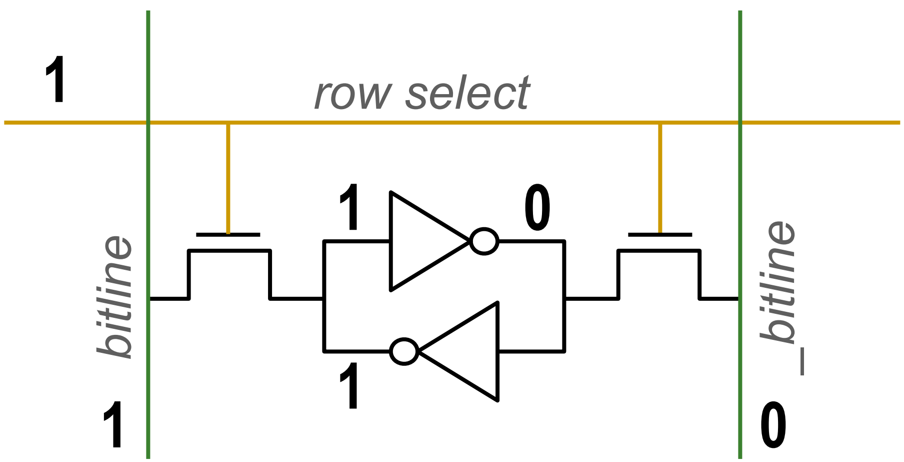

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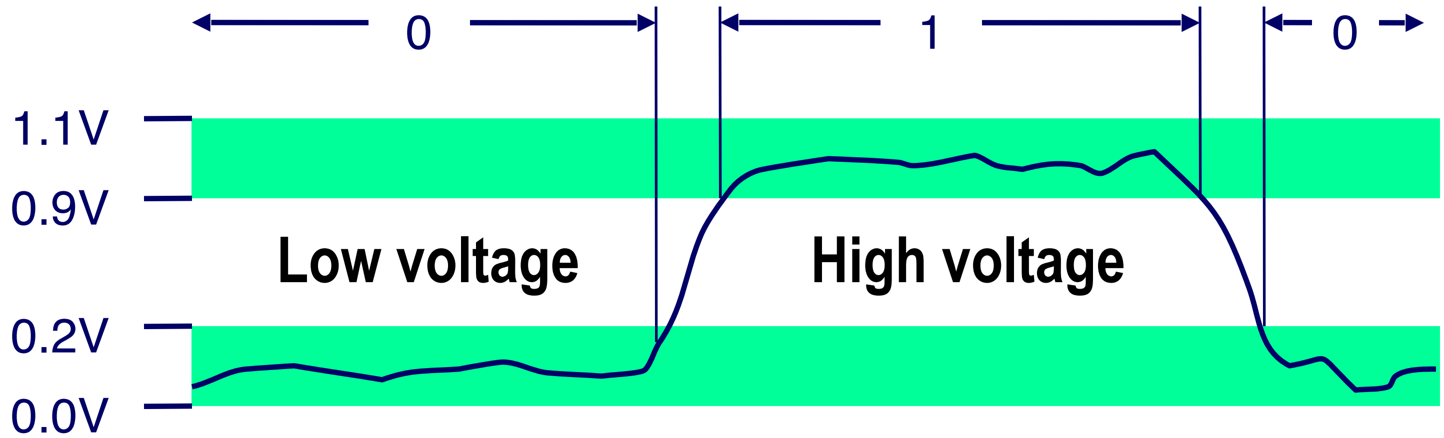
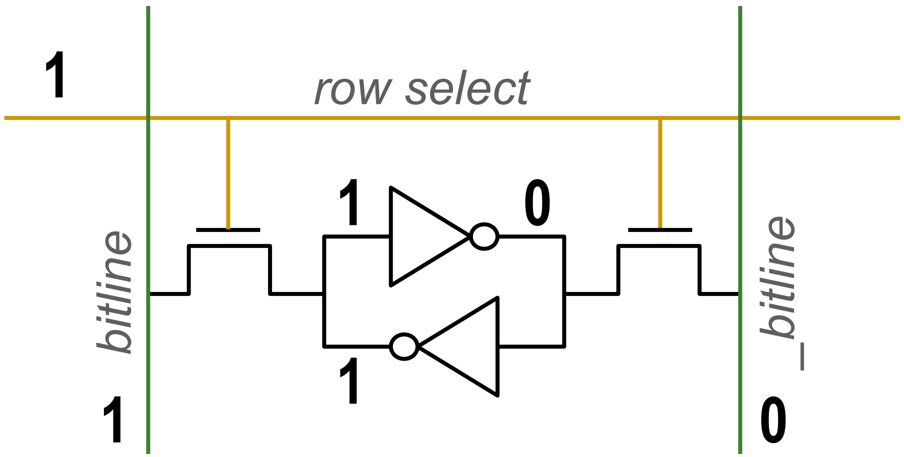
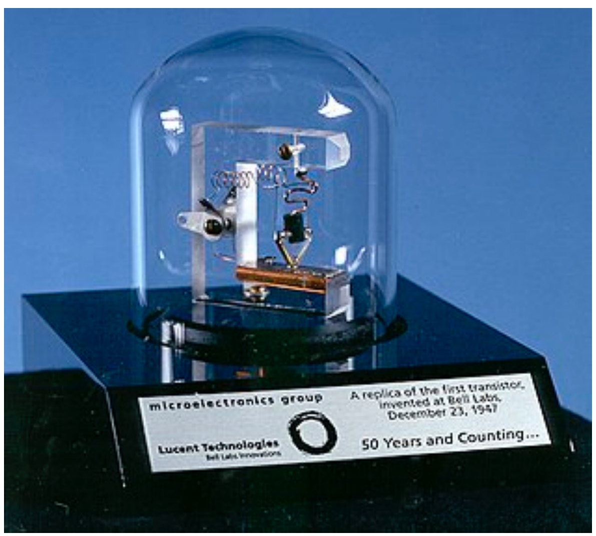


- Two cross coupled inverters store a single bit
  - Feedback path persists the value in the "cell"
  - 4 transistors for storage
  - 2 transistors for access
  - A "6T" cell



Transistors

- Computers are made of transistors
- Transistors have become smaller over the years
    - Not so much anymore…



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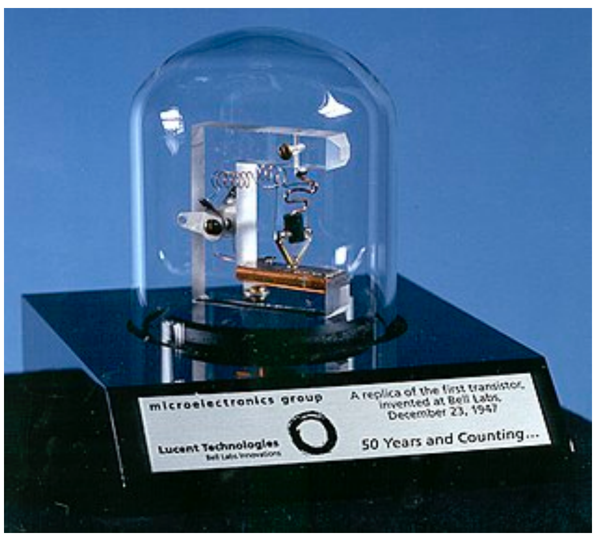
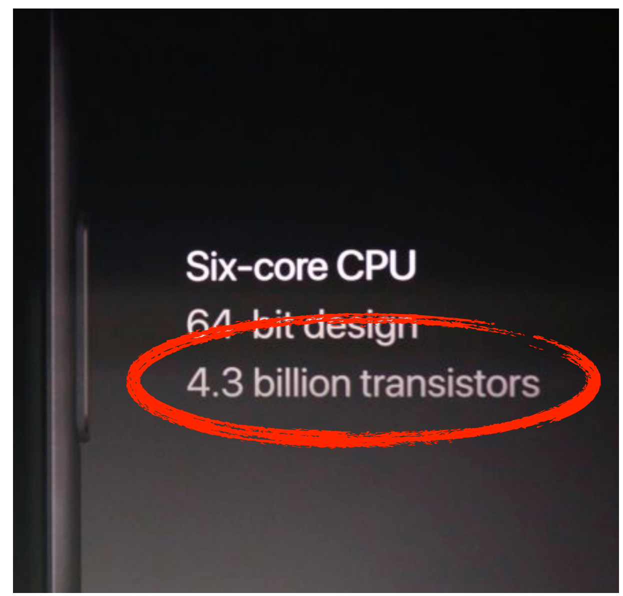
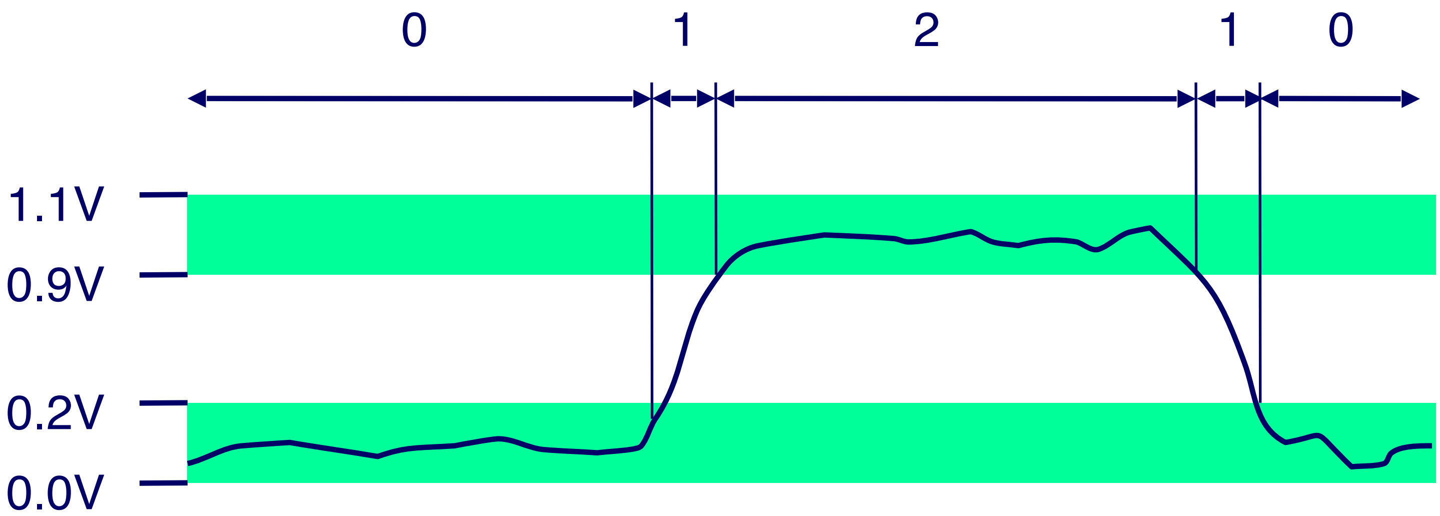




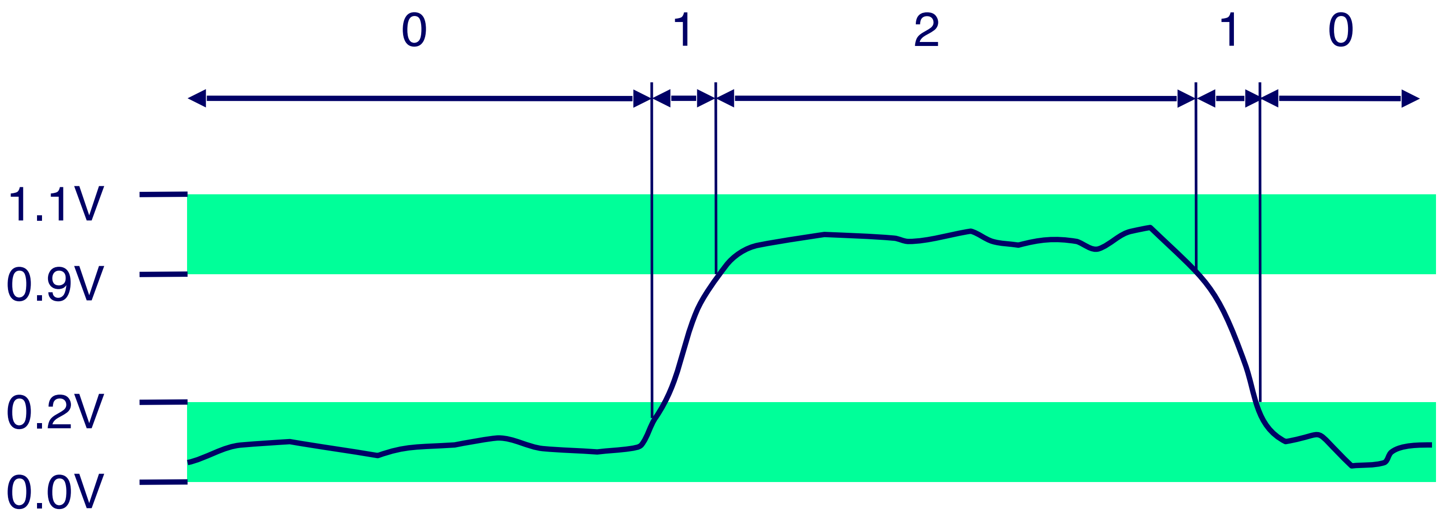
Why Limit Ourselves Only to Binary?

- Voltage is continuous. Why interpret it only as 0s and 1s?



Why Limit Ourselves Only to Binary?

- Voltage is continuous. Why interpret it only as 0s and 1s?
- Answer: Noise



Can We Tolerate the Noise?

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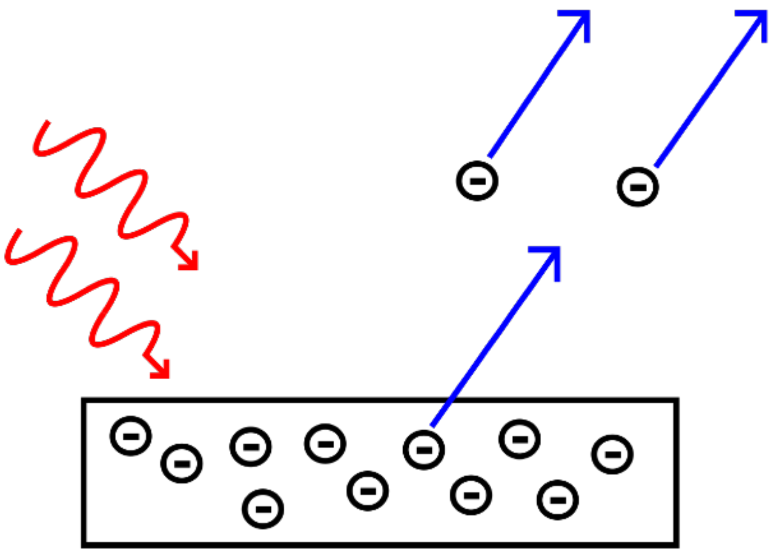
- Classic Example: Camera Sensor
  - Photoelectric Effect



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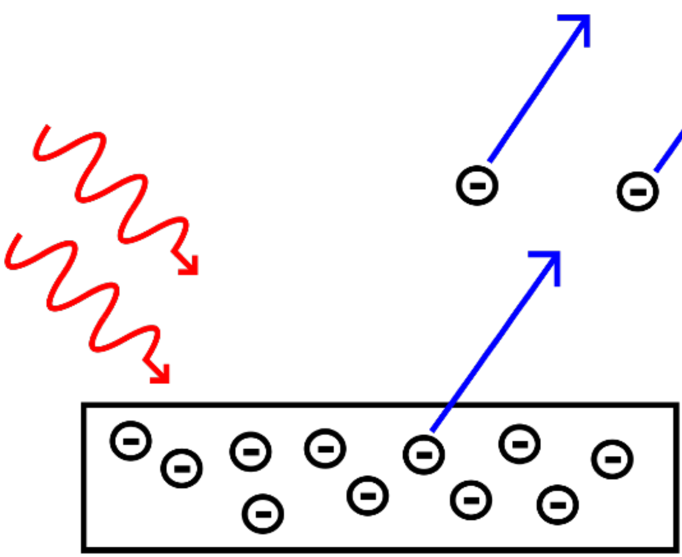

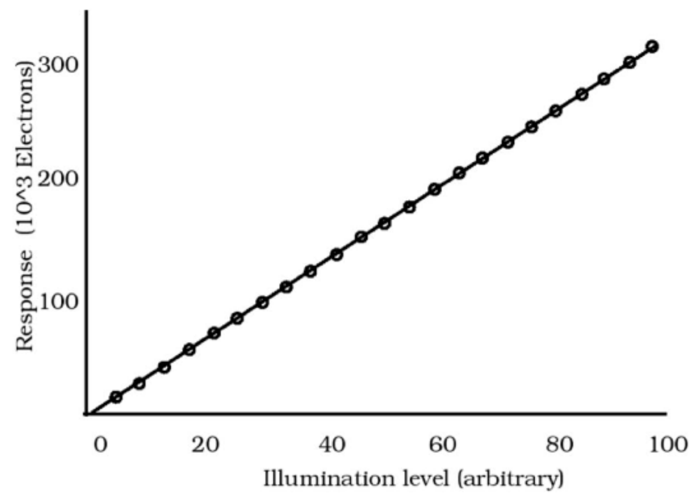

(Epperson, P.M. et al. Electro-optical characterization of the Tektronix TK5 ...., Opt Eng., 25, 1987)

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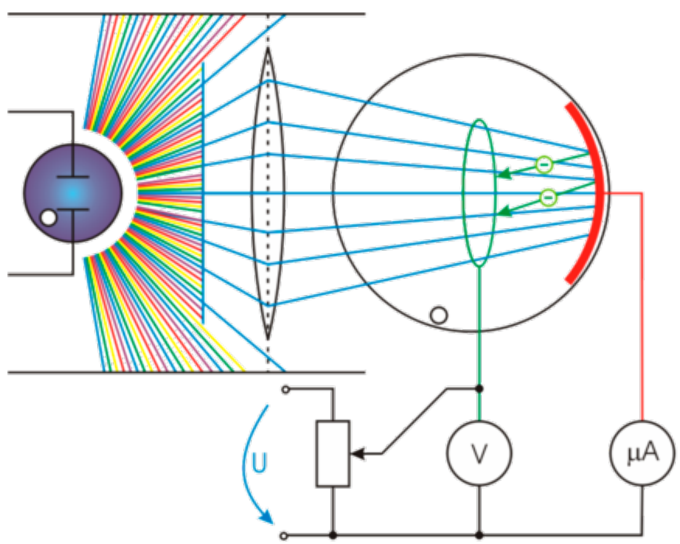

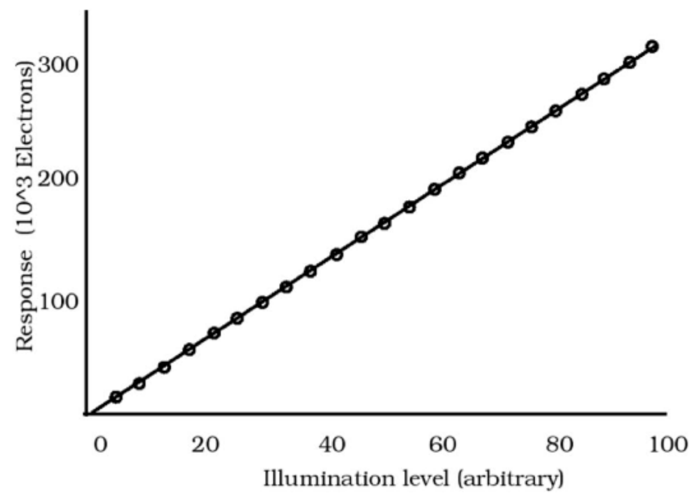

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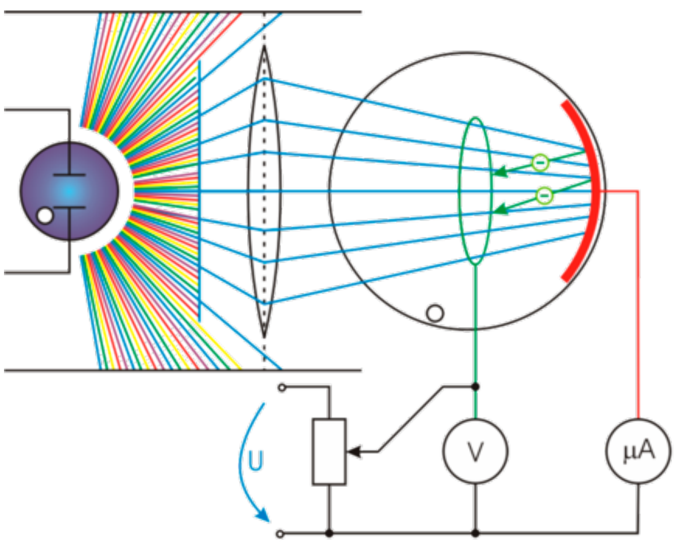

Binary Notation

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- Base 2 Number Representation (Binary)

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Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

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```
  0110
+ 0101
------
  1011
```

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
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5	0101
6	0110
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- Binary Arithmetic

```
   0110          6
 + 0101        + 5
 ------        ----
   1011         11
```

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Hexdecimal (Hex) Notation

- Base 16 Number Representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Four bits per Hex digit
  - $11111110_2 = FE_{16}$
- Write $FA1D37B_{16}$ in C as
  - 0xFA1D37B
  - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit, Byte, Word

- Byte = 8 bits
  - Binary 00000000_2 to 11111111_2; Decimal: 0_{10} to 255_{10}; Hex: 00_{16} to FF_{16}
  - Least Significant Bit (LSb) vs. Most Significant Bit (MSb)

$$\boxed{\mathbf{1}}\mathbf{011011}\boxed{\mathbf{1}}$$

MSb **LSb**

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  - Least Significant Bit (LSb) vs. Most Significant Bit (MSb)

$$\boxed{1}011011\boxed{1}$$

MSb **LSb**

- Word = 4 Bytes (32-bit machine) / 8 Bytes (64-bit machine)
  - Least Significant Byte (LSB) vs. Most Significant Byte (MSB)

Today: Representing Information in Binary

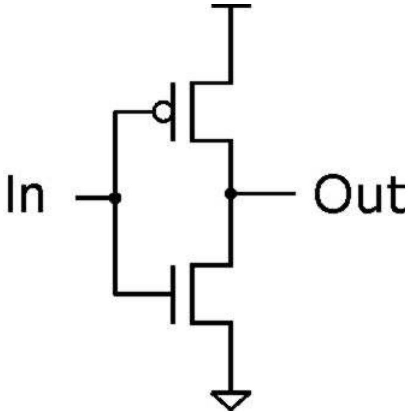
- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting

Bit-level manipulations

Not

- **~A = 1 when A=0**

~	
0	1
1	0



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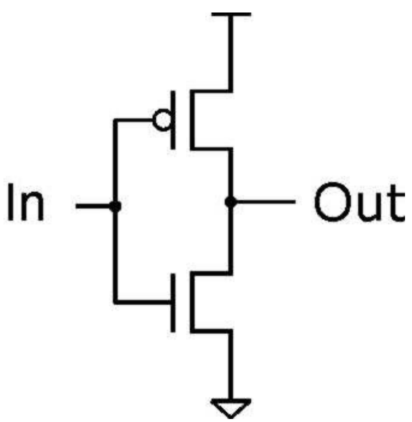

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Bit-level manipulations

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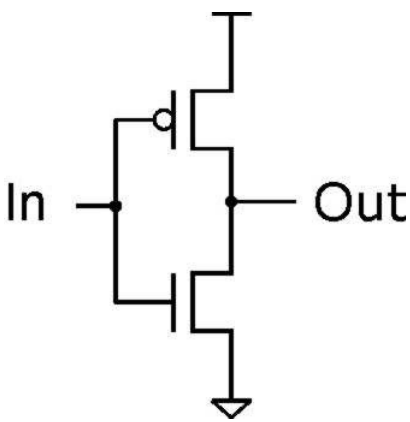

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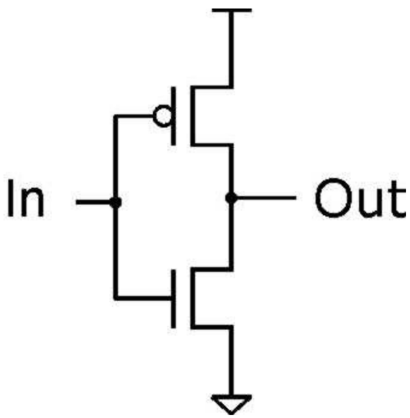

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&	0	1
0	0	0
1	0	1

Exclusive-Or (Xor)

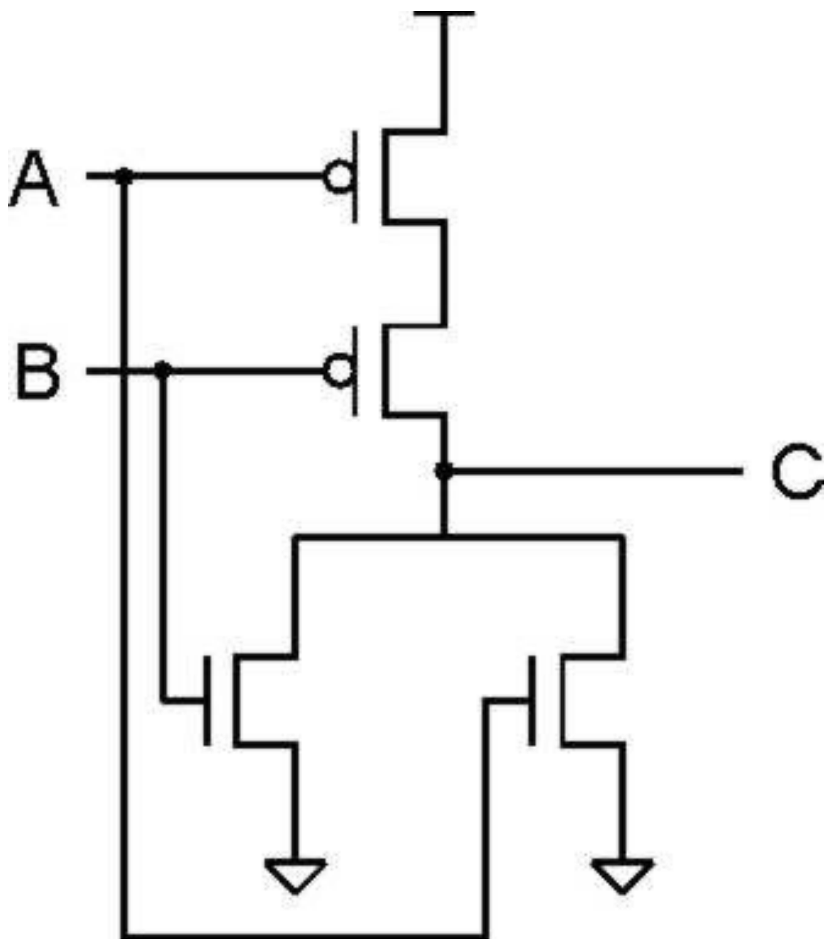
- A^B = 1 when either A=1 or B=1, but not both

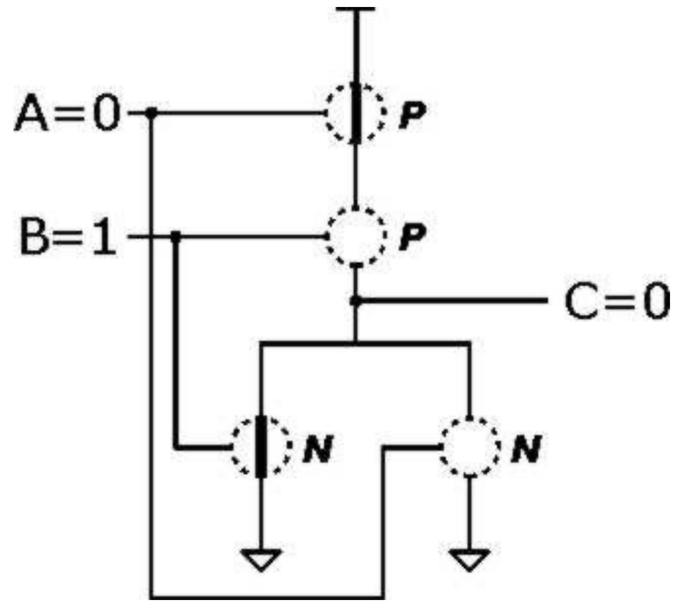
^	0	1
0	0	1
1	1	0

NOR (OR + NOT)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

NOR (OR + NOT)





A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Bit Vector Operations

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
__________    __________    __________    __________
```

Bit Vector Operations

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```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  --------      --------      --------      --------
  01000001
```

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```
   01101001     01101001     01101001
&  01010101   | 01010101   ^ 01010101   ~ 01010101
-----------   -----------  -----------  -----------
   01000001     01111101
```

Bit Vector Operations

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Bit Vector Operations

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  - Operations applied bitwise

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  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------     --------
  01000001     01111101     00111100     10101010
```

Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any “integral” data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - `~0x41` → `0xBE`
    - $\sim 01000001_2 \rightarrow 10111110_2$
  - `~0x00` → `0xFF`
    - $\sim 00000000_2 \rightarrow 11111111_2$
  - `0x69 & 0x55` → `0x41`
    - $01101001_2 \,\&\, 01010101_2 \rightarrow 01000001_2$
  - `0x69 | 0x55` → `0x7D`
    - $01101001_2 \mid 01010101_2 \rightarrow 01111101_2$

Aside: Logic Operations in C

- Contrast to Logical Operators
  - `&&, ||, !`
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination (e.g., 0 && 1 && 1)
- Examples (char data type)
  - `!0x41` → `0x00`
  - `!0x00` → `0x01`
  - `!!0x41` → `0x01`

  - `0x69 && 0x55` → `0x01`
  - `0x69 || 0x55` → `0x01`
  - `p && *p` (avoids null pointer access)

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
      - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
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  - Arithmetic shift
    - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount < 0 or ≥ total amount of bits

Argument x	01100010
<< 3	
Log. >> 2	
Arith. >> 2	

Argument x	10100010
<< 3	
Log. >> 2	
Arith. >> 2	

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Argument **x**	01100010
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Log. >> 2	101000
Arith. >> 2	

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Log. >> 2	*00*011000
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Log. >> 2	*00*101000
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Argument x	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Argument x	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	101000

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<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

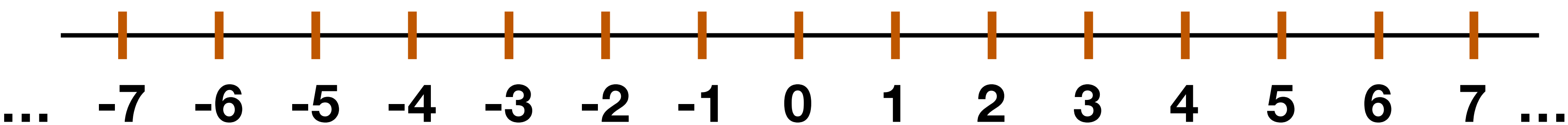
Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting

Representing Numbers in Binary

- Different types of number
  - Integer (Negative and Non-negative)
  - Fractions
  - Irrationals



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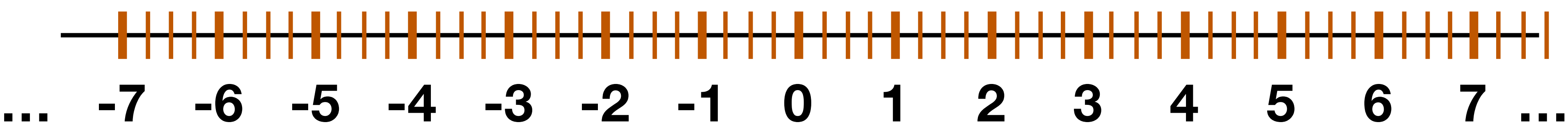
  - Fractions
  - Irrationals



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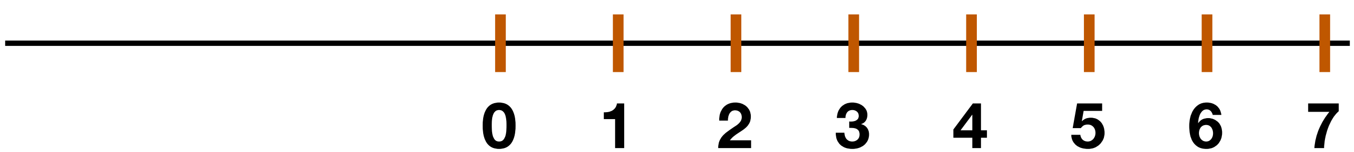

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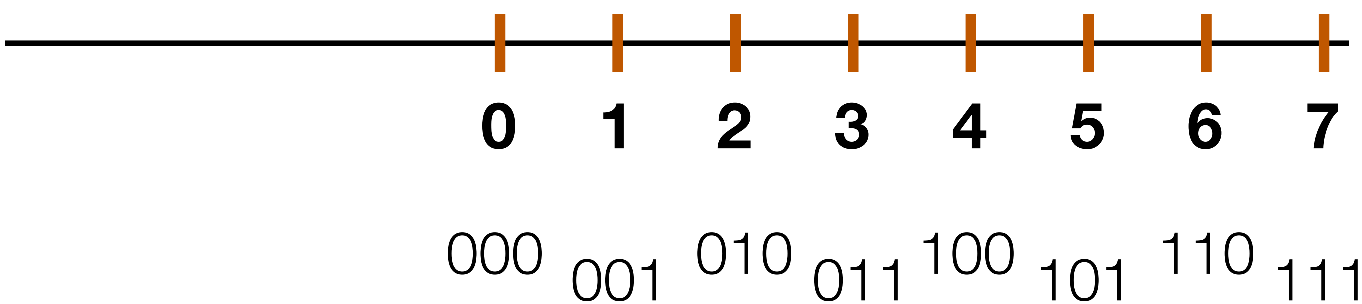

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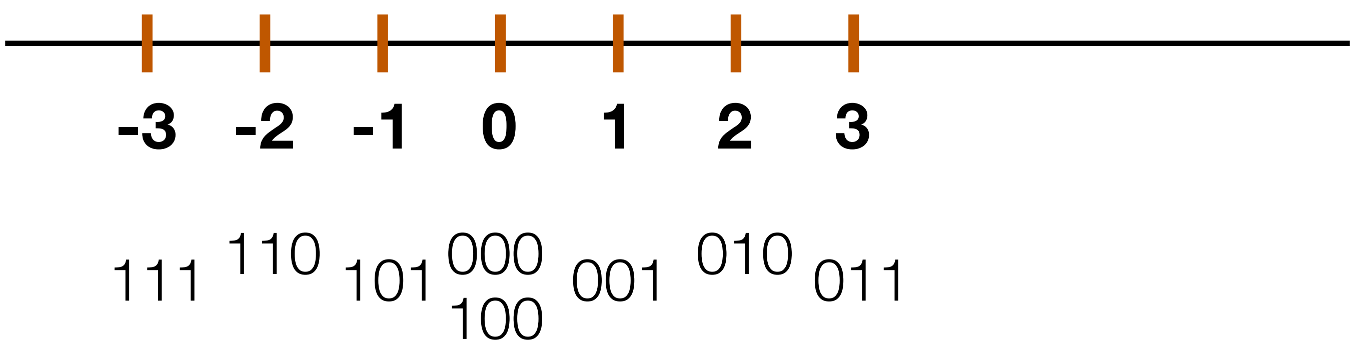

Sign-Magnitude Implications

- Bits have different semantics
  - Two zeros…
  - Normal arithmetic doesn't work
  - Make hardware design harder

Signed Value	**Binary**
0	000
1	001
2	010
3	011
-0	100
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- Solution 2: Two’s Complement

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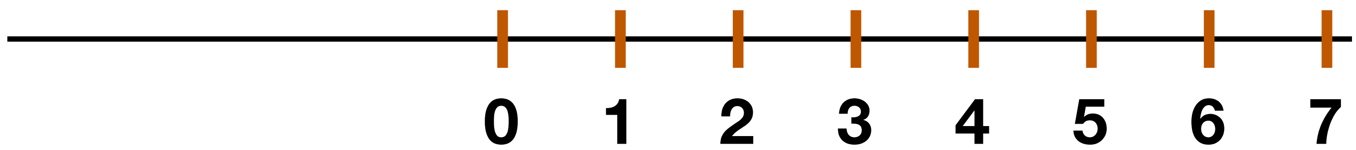

Unsigned	Binary
0	000
1	001
2	010
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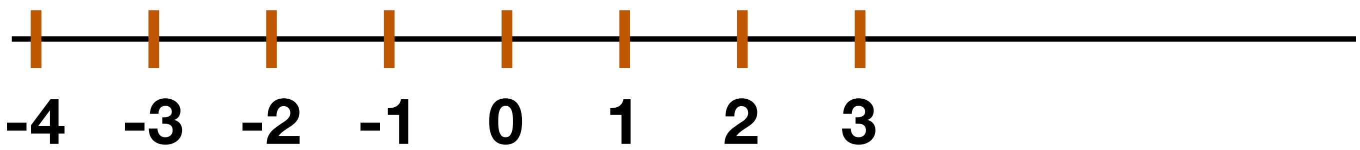
4	100
5	101
6	110
7	111

Encoding Negative Numbers

- Solution 2: Two's Complement



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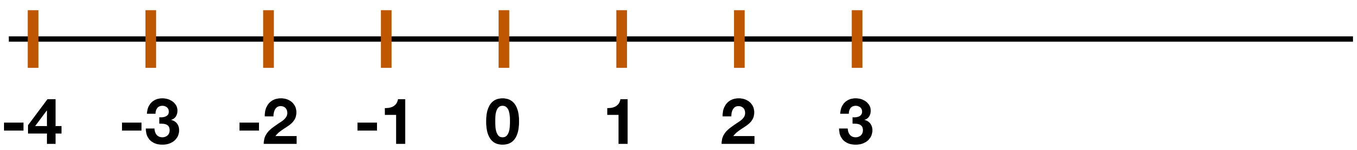

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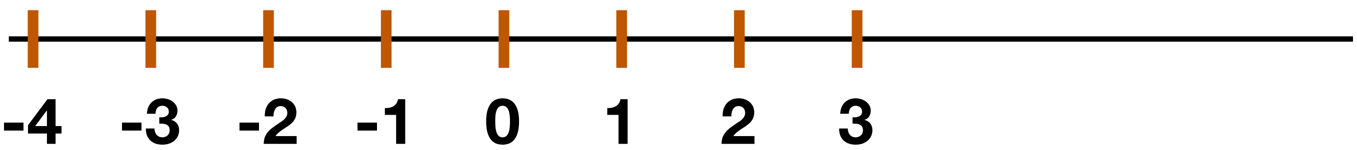

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2^0	2^0	0
2^1	2^1	1
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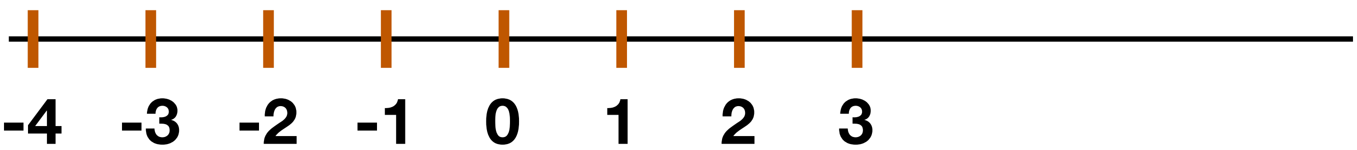

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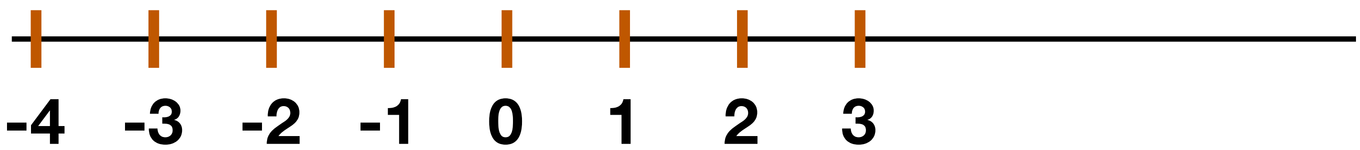

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$$101_2 = 1*2^0 + 0*2^1 - 1*2^2 = -3_{10}$$

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Two-Complement Encoding Example

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Two-Complement Implications

- Only 1 zero
- Usual arithmetic still works
- There is a bit that represents the sign!
- Most widely used in today’s machines

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Values for *W* = 16

	Decimal	Hex	Binary
`UMax`	65535	`FF FF`	`11111111 11111111`
`TMax`	32767	`7F FF`	`01111111 11111111`
`TMin`	-32768	`80 00`	`10000000 00000000`
`-1`	-1	`FF FF`	`11111111 11111111`
`0`	0	`00 00`	`00000000 00000000`

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    100…0
  - *TMax* = $2^{w-1} - 1$

    011…1
- Other Values
  - Minus 1

    111…1

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-1	**-1**	**FF FF**	**11111111 11111111**
0	**0**	**00 00**	**00000000 00000000**

Data Representations in C (in Bytes)

- By default variables are signed
- Unless explicitly declared as unsigned (e.g., `unsigned int`)
- Signed variables use two-complement encoding

C Data Type	32-bit	64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8

Data Representations in C (in Bytes)

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

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- C Language
  - `#include <limits.h>`
  - Declares constants, e.g.,
    - `ULONG_MAX`
    - `LONG_MAX`
    - `LONG_MIN`
  - Values platform specific