Logistics

- I usually edit the slides a little bit after each lecture, either fixing typos/errors or better explaining things based on your feedback. So the slides might be slightly different from those in the video recordings.
- If you can’t make my office hours, schedule a different time.
- News flash sign-up sheet: https://docs.google.com/spreadsheets/d/1tEsOYMfvnLdxShdvxYMG7c4vp5BrFo60dx_vJRwVros/edit#gid=0
- Written assignment 1 is up and is due Sept. 7 11:30 AM.
- Course schedule: https://www.cs.rochester.edu/courses/572/fall2020/schedule.html. You will find reading assignments and slides.
- Start thinking and talking to me about your final project idea.
Scope of the Course

Physical Scene (Objects, lights) → Optics (Gather light) → Image Sensing (Optical to electrical signal transformation) → Image Signal Processing (Signal reconstruction) → Computer Vision (Semantics understanding)

Modeling (Scene, optics) → Computer Graphics (Simulating light transport/lenses/etc.) → Display (Generating lights) → Video/Image De/Compression → Cloud/Storage

Human Vision System (Eye, visual cortex)
What is Color?

- Unlike speed, shape, etc., color is completely subjective.

The Dress

https://en.wikipedia.org/wiki/The_dress

Color blind test

https://colormax.org/color-blind-test/
What is Color?

- What is the objective reality is light wavelength/frequency and power.
- Lights activates neural signals in your brain.
  - Different lights activate your brain differently.
- Color is how your brain *interprets* the neural signals.
- Understanding color starts from understanding human vision system.
Human Eye Anatomy

Control the focal length of the lens

Control the pupil size

Where light enters; similar to aperture

Focus light

https://garetina.com/patient-education/about-the-eye/
Contains photoreceptors, which generate neural signals from light.

Highest photoreceptor density and thus highest visual acuity.

No photoreceptor exists.

Transports neural signals to your brain, where “color” is interpreted.

https://garetina.com/patient-edication/about-the-eye/
Blind Spot

- Nerve fibers route before the retina in human eyes

https://www.reddit.com/r/interestingasfuck/comments/6ifcn6/our_similarity_with_cephalopod_eyes_despite_our/
Blind Spot: “See” it Yourself

‹ Cover your LEFT eye and stare at the cross with your RIGHT eye.
‹ SLOWLY move towards the computer screen while still staring at the cross with your RIGHT eye.
‹ At some point the black circle will disappear.
‹ Blind spot doesn’t appear dark because your brain “makes stuff up”

https://visionaryeyeare.wordpress.com/2008/08/04/eye-test-find-your-blind-spot-in-each-eye/
Photoreceptors: Cones and Rods

- A special kind of cell that **converts light to electrical signals** (a.k.a., *visual phototransduction*), which then stimulate biological processes.

  Primarily activated in high-light levels; ~6 million cones. Three kinds of cones (more later).

  Primarily activated in low-light levels; ~120 million rods in human eyes.

[Link to Ask a Biologist: Rods and Cones](https://askabiologist.asu.edu/rods-and-cones)
Photoreceptor Density

- Uneven distribution. The area on the retina with the highest cone density is called fovea.
- Fovea angle is about 2°. Peripheral vision has very low acuity.

Aside: Foveated Rendering

Aside: Foveated Rendering

- Idea: rendering fovea in high resolution while degrading rendering quality for peripheral vision.
- Reduce the rendering workload: important for mobile Virtual Reality.
- Mostly require precise eye-tracing, which is hard and power-hungry.

Tobii Spotlight’s foveated rendering can cut VR graphics load by 57%

Oculus Quest gets dynamic fixed foveated rendering

https://venturebeat.com/2019/12/22/oculus-quest-gets-dynamic-fixed-foveated-rendering/
Aside: Foveated Compression

- Most video/image compression techniques are “lossy”: they don’t preserve the original video/image content.
- Question: what information to preserve?
- Preserve foveate information while aggressively compressing the rest.
- Foveated rendering/compression are part of “perception-driven” techniques, as they exploit properties of human perception systems.
- Will get back to these later in the semester.
Light

- For the purpose of understanding color, we consider light as photons.
- Light sources emit photons. Materials reflect, transmit, absorb photons.
- Each photon has energy, which is related to its frequency/wavelength.

Energy of a $\lambda = 500$ nm photon: $\sim 4 \times 10^{-34}$ J

Energy of a $\lambda = 750$ nm photon: $\sim 2.7 \times 10^{-34}$ J

$$E = hf = \frac{hc}{\lambda}$$

- Planck constant. $\sim 6.63 \times 10^{-34}$ J·s
- Light frequency
- Speed of light (m/s)
- Light wavelength
Light Spectrum

- Most light sources consist of a mix of photons of different wavelengths.
Light Spectrum

- Most light sources consist of a mix of photons of different wavelengths.

The *spectrum* of a light $\Phi(\lambda)$ is the amount of power available at each wavelength ($\lambda$). It is called the **spectral power distribution (SPD)** of the light.

[Image of light spectrum graphs for Daylight, Incandescent, Fluorescent, Halogen, Cool White LED, Warm White LED]
Light Spectrum

› Single-wavelength (monochromatic) light; mostly generated by lasers. Also called **spectral light**.

▷ SPDs of spectral lights are delta functions — very narrow spike-like functions.
Standard Illuminant

- The International Commission on Illumination (CIE, Commission internationale de l’éclairement in French) publishes a set of SPDs of theoretical sources of visible lights, which are called standard illuminants.

Spectral Response/Sensitivity of Cones

Optical Power ($\lambda$) → Cone → Electrical Power ($\lambda$)

Sensitivity ($\lambda$) $\propto \frac{\text{Electrical power} (\lambda)}{\text{Optical power} (\lambda)}$

https://www.handprint.com/HP/WCL/color1.html
Spectral Response/Sensitivity of Cones

Optical Power ($\lambda$) $\rightarrow$ Cone $\rightarrow$ Electrical Power ($\lambda$)

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Spectral Response/Sensitivity of Cones

Optical Power ($\lambda$) → Cone → Electrical Power ($\lambda$)

Sensitivity ($\lambda$) $\propto$ Electrical power ($\lambda$) / Optical power ($\lambda$)

**Short cone.** Sensitivity roughly peaks at blue.

**Medium cone.** Sensitivity roughly peaks at green.

**Long cone.** Sensitivity roughly peaks at red.

https://www.handprint.com/HP/WCL/color1.html
Spectral Response/Sensitivity of Cones

- There are three types of cones (L, M, S) whose sensitivities peak at different wavelengths.
- For cone type, the sensitivity varies with $\lambda$, hence the name **spectral response/sensitivity**.

\[
\text{Sensitivity} (\lambda) \propto \frac{\text{Electrical power} (\lambda)}{\text{Optical power} (\lambda)}
\]

- **Short cone.** Sensitivity roughly peaks at blue.
- **Medium cone.** Sensitivity roughly peaks at green.
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https://www.handprint.com/HP/WCL/color1.html
Spectral Response/Sensitivity of Cones

Each cone’s sensitivity in this figure is weighted by its number in the retina (at fovea: ~63% L, ~31% M, ~6% S)

**Short cone.**
Sensitivity roughly peaks at blue.

**Medium cone.**
Sensitivity roughly peaks at green.

**Long cone.**
Sensitivity roughly peaks at red.

[Link to Handprint website](https://www.handprint.com/HP/WCL/color1.html)
Luminous Efficiency Function

- LEF, $V(\lambda)$, describes the average spectral sensitivity of human eyes. Think of it as a weighted sum of the LMS cone response functions.
- Overall, human eyes under daylight are most sensitive to green-ish light (LEF peaks at 555 nm).

https://en.wikipedia.org/wiki/Luminous_efficacy
Perceived Power (~Brightness)

- $\Phi(\lambda)V(\lambda)$: “Human perceived” power of light, the product of light power and luminous efficiency.

https://en.wikipedia.org/wiki/Luminous_efficacy
Perceived Power (~Brightness)

- $\Phi(\lambda)V(\lambda)$: “Human perceived” power of light, the product of light power and luminous efficiency.

- If spectral light $\lambda=530$ (green-ish) and $\lambda=480$ (blue-ish) have the same power, the green-ish light looks “brighter”, as cones are more sensitive to 530 nm than to 480 nm.

https://en.wikipedia.org/wiki/Luminous_efficacy
Perceived Power (~Brightness)

- The perceived power of a light is measured by *luminous flux* \((Y)\), which integrates the product of power and luminous efficiency across the light spectrum.

\[
Y = 683 \frac{lm}{W} \int \Phi(\lambda)V(\lambda)d(\lambda)
\]

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- The perceived power of a light is measured by **luminous flux** ($Y$), which **integrates** the product of power and luminous efficiency across the light spectrum.
- Luminous flux’s unit is lumen (lm). 1 lumen is defined as the luminous flux of spectral light $\lambda=555$ nm emitting $1/683$ W power.

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- Luminous flux’s unit is lumen (lm). 1 lumen is defined as the luminous flux of spectral light $\lambda=555$ nm emitting $1/683$ W power.
- Actual relationship between perceived brightness and luminance is non-linear. More on this later in gamma correction.

$$Y = 683 \frac{lm}{W} \int \Phi(\lambda)V(\lambda)d(\lambda)$$

https://en.wikipedia.org/wiki/Luminous_efficacy
## “Brightness”

![Brightness Table](https://uvebtech.com/articles/2018/why-is-there-no-standard-unit-similar-to-lighting-products-for-the-radiant-output-of-uv-lamps/)

<table>
<thead>
<tr>
<th>Type</th>
<th>250+</th>
<th>450+</th>
<th>800+</th>
<th>1100+</th>
<th>1600+</th>
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<tbody>
<tr>
<td>Standard</td>
<td>25W</td>
<td>40W</td>
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<td>75W</td>
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<td>29W</td>
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4-5 X increase in power but > 6 X increase in lumen. Why?

Spectral Reflectance

- Describes how much incident light power is reflected.

\[ \Phi(\lambda)R(\lambda) \]

Spectrum of light that enters eyes.

Combining Light Sources with Reflecting Materials

\[ Y = 683 \frac{lm}{W} \int \Phi(\lambda) R(\lambda) V(\lambda) d(\lambda) \]

Spectrum of light that enters eyes.
Color Blindness

- Color blind: certain cones missing/mutated

Fovea cone cell distribution in a normal person. Fovea cone cell distribution of a Protanopia person, who is insensitive to red.

Green: LEF of Protanopia. Yellow is normal LEF.

Color Blindness: The Perks

- the maze (at left) is recreated (at right) using subtle intensity differences, but overridden by stronger red-green color differences
- a deuteranope can easily see the maze at right

Nature

Explore our content → Journal information ↓
nature > news > article

Published: 05 December 2005
Colour blindness may have hidden advantages

Tom Simonite

Nature (2005) | Cite this article

3713 Accesses | 20 Altmetric | Metrics

People with red-green colour blindness are better at discerning shades of khaki.
Invisible Light

- Invisible light: light that doesn’t activate any cones cells.

https://commons.wikimedia.org/wiki/File:EM_spectrum.svg
Invisible Light

- Sunlight has invisible components, which don’t activate photoreceptors, but activate other cells.

http://www.psicorp.com/content/psis-applied-research-activities-0
Visualizing LMS Cone Responses

Any spectral light is represented by a \(<L, M, S>\) vector in the 3D space.

Play with an interactive applet here: https://graphics.stanford.edu/courses/cs178/applets/locus.html
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Total Cone Response

- Given an input spectrum $\Phi(\lambda)$, we can calculate the total response of the L, M, and S cone cells.

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Given an input spectrum $\Phi(\lambda)$, we can calculate the total response of the L, M, and S cone cells.

Intuition: $\Phi_{675} \times L_{675}$ can be seen as the L cone responses contributed by the $\lambda=675$ nm photons in the incident light.

Total Cone Response

- Given an input spectrum $\Phi(\lambda)$, we can calculate the **total response** of the L, M, and S cone cells.

\[
L = \int \Phi(\lambda) L(\lambda) d\lambda
\]

Intuition: $\Phi_{675} \times L_{675}$ can be seen as the L cone responses contributed by the $\lambda=675$ nm photons in the incident light.

Total Cone Response

- Any light could be expressed as \(<L, M, S>\) signals, a.k.a., the *tristimulus values* of the spectrum *in the LMS space*.

\[
L = \int_{\lambda} \Phi(\lambda)L(\lambda)d\lambda
\]

\[
M = \int_{\lambda} \Phi(\lambda)M(\lambda)d\lambda
\]

\[
S = \int_{\lambda} \Phi(\lambda)S(\lambda)d\lambda
\]

Brain doesn’t see light; it receives the LMS signals.

What this mean: lights that produce the same LMS signals are **perceived** by the brain as the same “color”!

https://www.eyeworld.org/research-finds-human-visual-cortex-continues-development-mid-30s
Metamerism

- Two lights have different spectrums but are perceived as the same color, as their LMS values match.

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\[
\int_{\lambda} \Phi_1(\lambda)L(\lambda)d\lambda = \int_{\lambda} \Phi_2(\lambda)L(\lambda)d\lambda
\]

\[
\int_{\lambda} \Phi_1(\lambda)M(\lambda)d\lambda = \int_{\lambda} \Phi_2(\lambda)M(\lambda)d\lambda
\]

\[
\int_{\lambda} \Phi_1(\lambda)S(\lambda)d\lambda = \int_{\lambda} \Phi_2(\lambda)S(\lambda)d\lambda
\]

Metamerism

- Does metamerism exist? That is, given a spectrum $\Phi$, can we always find at least one $\Phi'$ that is $\Phi$'s metamer? Can we find multiple?
Metamerism

- Does metamerism exist? That is, given a spectrum \( \Phi \), can we always find at least one \( \Phi' \) that is \( \Phi \)'s metamer? Can we find multiple?
- We will later show how to do so, but it requires knowing \( L(\lambda) \), \( M(\lambda) \), and \( S(\lambda) \), which requires physiological experiments and was historically not known until late 1980s.
Metamerism

- Does metamerism exist? That is, given a spectrum $\Phi$, can we always find at least one $\Phi'$ that is $\Phi$'s metamer? Can we find multiple?
- We will later show how to do so, but it requires knowing $L(\lambda)$, $M(\lambda)$, and $S(\lambda)$, which requires physiological experiments and was historically not known until late 1980s.
- But people had been able to find metamer, as early as late 19th century, through a clever color matching experiment.
Color Matching Experiments

- You are given three monochromatic lights pointing to the same point. You tune each light’s power so that the perceived color of the point matches the perceived color of the target light.
- The three tunable light sources are called **primary lights**.
- First invented in about 1853.
Color Matching Experiments

- Commission internationale de l’éclairage (CIE) 1931 standard chose the three primary lights as $\lambda = 435.8$ nm, 546.1 nm, 700 nm, known as blue, green, and red.
- It limited the viewing angle to be $2^\circ$ so that only the fovea gets light.
- It swept all the monochromatic lights in the visible spectrum as the target light (5-nm interval).
Color Matching Experiments

Play with an interactive color matching “game” here: https://graphics.stanford.edu/courses/cs178/applets/colormatching.html
Color Matching Experiments

Record the units of primary lights and normalize them so that they add up to 1.
Color Matching Experiments

Record the units of primary lights and normalize them so that they add up to 1.

Scale the RGB values at each \( \lambda \) such that the areas under curve are the same.

\[
\int_{\lambda} \bar{r}(\lambda) d\lambda = \int_{\lambda} \bar{g}(\lambda) d\lambda = \int_{\lambda} \bar{b}(\lambda) d\lambda
\]
The resulting curves after scaling is called the color matching functions.

Color Matching Functions

- For **one unit power** (technically it’s one unit radiance) of each target light, how many units of primary light is needed to match the color.

- The RGB values are called the **tristimulus values** of the target color **in the RGB space**.

https://en.wikipedia.org/wiki/CIE_1931_color_space
Color Matching Functions

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If the target light $\lambda$ has a power of $\Phi_{580}$, the amount of RGB we need is:

$$\Phi_{580} \times R_{580}, \Phi_{580} \times G_{580}, \Phi_{580} \times B_{580}$$

https://en.wikipedia.org/wiki/CIE_1931_color_space
Color Matching Functions

To match color of $\lambda=540$ nm, we need a bit of negative red???
To match color of $\lambda=540$ nm, we need a bit of negative red...
Color Matching Functions

- No combination of RGB matches the target light $\lambda = 540$ nm, but adding some R with the target light matches the color of some combination of G and B.

- Color of the light $\lambda = 540$ nm can’t be reproduced by R, G, B. Doesn’t mean $\lambda = 540$ nm doesn’t exist; it just can’t be generated from RGB.

https://en.wikipedia.org/wiki/CIE_1931_color_space
Conjectured by Thomas Young as early as 1801, and expanded by Helmholtz and Maxwell in 1860.

Color matching experiments empirically verified it.

Experimental measurement of cones as late as 1983.
Explaining Color Matching Using LMS Cones

Ignoring some constant scaling factors here for simplicity.
Explaining Color Matching Using LMS Cones

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Explaining Color Matching Using LMS Cones

http://www.math.ubc.ca/~cass/courses/m309-03a/m309_projects/bajwa/images/scan0012.jpg

Target light \((\lambda=580)\)

Ignoring some constant scaling factors here for simplicity.
Explaining Color Matching Using LMS Cones

Target light
($\lambda=580$)

$\mathbf{L}_{580} \times \Phi_{580}$
The L cone response of the target light

Ignoring some constant scaling factors here for simplicity.
Explaining Color Matching Using LMS Cones

The L cone response of the target light $L_{580} \times \Phi_{580}$

Ignoring some constant scaling factors here for simplicity.
Explaining Color Matching Using LMS Cones

\[ L_{700} \times (R_{580} \times \Phi_{580}) + L_{546} \times (G_{580} \times \Phi_{580}) \]

\[ L_{580} \times \Phi_{580} \]

The L cone response of the target light

Ignoring some constant scaling factors here for simplicity.

http://www.math.ubc.ca/~cass/courses/m309-03a/m309_projects/bajwa/images/scan0012.jpg
Explaining Color Matching Using LMS Cones

The L cone response of the target light

Ignoring some constant scaling factors here for simplicity.
Explaining Color Matching Using LMS Cones

The L cone response of the three primary lights:

\[ L_{700} \times (R_{580} \times \Phi_{580}) + L_{546} \times (G_{580} \times \Phi_{580}) + L_{435} \times (B_{580} \times \Phi_{580}) = L_{580} \times \Phi_{580} \]

The L cone response of the target light:

\[ L_{580} \times \Phi_{580} \]

Ignoring some constant scaling factors here for simplicity.
Explaining Color Matching Using LMS Cones

\[ M_{700} \times (R_{580} \times \Phi_{580}) + M_{546} \times (G_{580} \times \Phi_{580}) + M_{435} \times (B_{580} \times \Phi_{580}) = M_{580} \times \Phi_{580} \]

The M cone response of the three primary lights

The M cone response of the target light
Explaining Color Matching Using LMS Cones

The S cone response of the target light

$$S_{700} \times (R_{580} \times \Phi_{580}) + S_{546} \times (G_{580} \times \Phi_{580}) + S_{435} \times (B_{580} \times \Phi_{580}) = S_{580} \times \Phi_{580}$$

The S cone response of the three primary lights

http://www.math.ubc.ca/~cass/courses/m309-03a/m309_projects/bajwa/images/scan0012.jpg
Explaining Color Matching Using LMS Cones

\[ L_{700} \times (R \times \Phi_\lambda) + L_{546} \times (G \times \Phi_\lambda) + L_{435} \times (B \times \Phi_\lambda) = L_\lambda \times \Phi_\lambda \]

\[ M_{700} \times (R \times \Phi_\lambda) + M_{546} \times (G \times \Phi_\lambda) + M_{435} \times (B \times \Phi_\lambda) = M_\lambda \times \Phi_\lambda \]

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Explaining Color Matching Using LMS Cones

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L_{700} (R \times \Phi_\lambda) + L_{546} (G \times \Phi_\lambda) + L_{435} (B \times \Phi_\lambda) = L_\lambda \times \Phi_\lambda
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S_{700} (R \times \Phi_\lambda) + S_{546} (G \times \Phi_\lambda) + S_{435} (B \times \Phi_\lambda) = S_\lambda \times \Phi_\lambda
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\[
\begin{bmatrix}
L_{700} & L_{546} & L_{435} \\
M_{700} & M_{546} & M_{435} \\
S_{700} & S_{546} & S_{435}
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
= 
\begin{bmatrix}
L_\lambda \\
M_\lambda \\
S_\lambda
\end{bmatrix}
\]
Explaining Color Matching Using LMS Cones

\[
\begin{align*}
L_{700} \times (R \times \Phi_\lambda) + L_{546} \times (G \times \Phi_\lambda) + L_{435} \times (B \times \Phi_\lambda) &= L_\lambda \times \Phi_\lambda \\
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S_{700} \times (R \times \Phi_\lambda) + S_{546} \times (G \times \Phi_\lambda) + S_{435} \times (B \times \Phi_\lambda) &= S_\lambda \times \Phi_\lambda
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\]

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\end{bmatrix}
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M_\lambda \\
S_\lambda
\end{bmatrix}
\]

CIE 1931 standard uses \( \lambda = 700 \) for red, \( \lambda = 546.1 \) for green, and \( \lambda = 435.8 \) for blue. But any other choice of RGB will work. In fact, primaries need not be monochromatic as we will see later.
Explaining Color Matching Using LMS Cones

\[ L_{700} \times (R \times \Phi_\lambda) + L_{546} \times (G \times \Phi_\lambda) + L_{435} \times (B \times \Phi_\lambda) = L_\lambda \times \Phi_\lambda \]

\[ M_{700} \times (R \times \Phi_\lambda) + M_{546} \times (G \times \Phi_\lambda) + M_{435} \times (B \times \Phi_\lambda) = M_\lambda \times \Phi_\lambda \]

\[ S_{700} \times (R \times \Phi_\lambda) + S_{546} \times (G \times \Phi_\lambda) + S_{435} \times (B \times \Phi_\lambda) = S_\lambda \times \Phi_\lambda \]

\[ \begin{bmatrix} L_R & L_G & L_B \\ M_R & M_G & M_B \\ S_R & S_G & S_B \end{bmatrix} \times \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} L_\lambda \\ M_\lambda \\ S_\lambda \end{bmatrix} \]

CIE 1931 standard uses \( \lambda = 700 \) for red, \( \lambda = 546.1 \) for green, and \( \lambda = 435.8 \) for blue. But any other choice of RGB will work. In fact, primaries need not be monochromatic as we will see later.
Color Matching Without Experiments

- Given any target wavelength $\lambda$ and the three primary lights, can we calculate the corresponding tristimulus values **without** the color matching experiment?
- Equivalent to solving the system of linear equations with 3 equations and 3 variables ($R$, $G$, $B$). The system has a unique solution (generally).

\[
\begin{bmatrix}
L_R & L_G & L_B \\
M_R & M_G & M_B \\
S_R & S_G & S_B
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
= 
\begin{bmatrix}
L_\lambda \\
M_\lambda \\
S_\lambda
\end{bmatrix}
\]
Why Three Primary Colors?

- If only have two primary colors, say \( P \) (\( \lambda = p \)) and \( Q \) (\( \lambda = q \)), the system of equations has two variables with three equations, which generally doesn’t have a solution!

- More than three primaries: more variables than equations, so there are many solutions to the system of equation, i.e., there are multiple ways of combining the primary lights to match the target color.

\[
\begin{align*}
L_p \times P + L_q \times Q &= L_\lambda \\
M_p \times P + M_q \times Q &= M_\lambda \\
S_p \times P + S_q \times Q &= S_\lambda
\end{align*}
\]
Linear Transformation Perspective

- Matrix representation of the system of linear equation.

\[
\begin{bmatrix}
L_R & L_G & L_B \\
M_R & M_G & M_B \\
S_R & S_G & S_B \\
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix}
= 
\begin{bmatrix}
L_\lambda \\
M_\lambda \\
S_\lambda \\
\end{bmatrix}
\]

Transformation matrix from the RGB space to the LMS space
(constant scaling factors omitted)
Plot all the spectral lights in the 3D space according to its \([R, G, B]\) tristimulus values.

Plot all the spectral lights in the 3D space according to its \([L, M, S]\) tristimulus values.

Spectral locus of the RGB space


Spectral locus of the LMS space

Metamers in RGB space are metamers in LMS space too.

https://graphics.stanford.edu/courses/cs178/applets/threedgamut.html

https://graphics.stanford.edu/courses/cs178/applets/locus.html
Reference Materials

- Two great “popular science” articles about color:
  - https://medium.com/hipster-color-science/a-beginners-guide-to-colorimetry-401f1830b65a
  - https://blog.soimort.org/cv/colors/

- A more rigorous and comprehensive article:

- Trichromatic theory and cone/rod measurements:
  - Human visual pigments: microspectrophotometric results from the eyes of seven persons.
  - The development of Thomas Young's theory of color vision.
Reference Materials

‣ How Wright-Guild initial data was transformed to CIE 1931 RGB:
  ▶ A Critical Review of the Development of the CIE1931 RGB Color-Matching Functions
  ▶ Calculation from the original experimental data of the CIE 1931 RGB standard observer spectral chromaticity co-ordinates and color matching functions, same content as above, but with (I think) more approachable narrative.
  ▶ My writing: How the CIE 1931 RGB Color Matching Functions Were Developed from the Initial Color Matching Experiments. Expanded math in a way that makes more sense to me.