

# Lecture 6: Colorimetry

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**CSC 292/572, Fall 2022**  
**Mobile Visual Computing**

# Logistics

Programming assignment 1 is up and is due Sept. 30 11:30 AM.

- Can work in groups of 2.

WA0 grades posted.

WA2/WA1 will be graded soon.

Abhishek's office hours: Wednesday 3pm - 4pm.

# The Roadmap

Theoretical Preliminaries

**Human Visual Systems**

Color in Nature, Arts, & Tech

(a.k.a., the birth, life, and death of light)

Digital Camera Imaging

Modeling and Rendering

Applications

Eye Anatomy

Spatial Vision

Color Vision

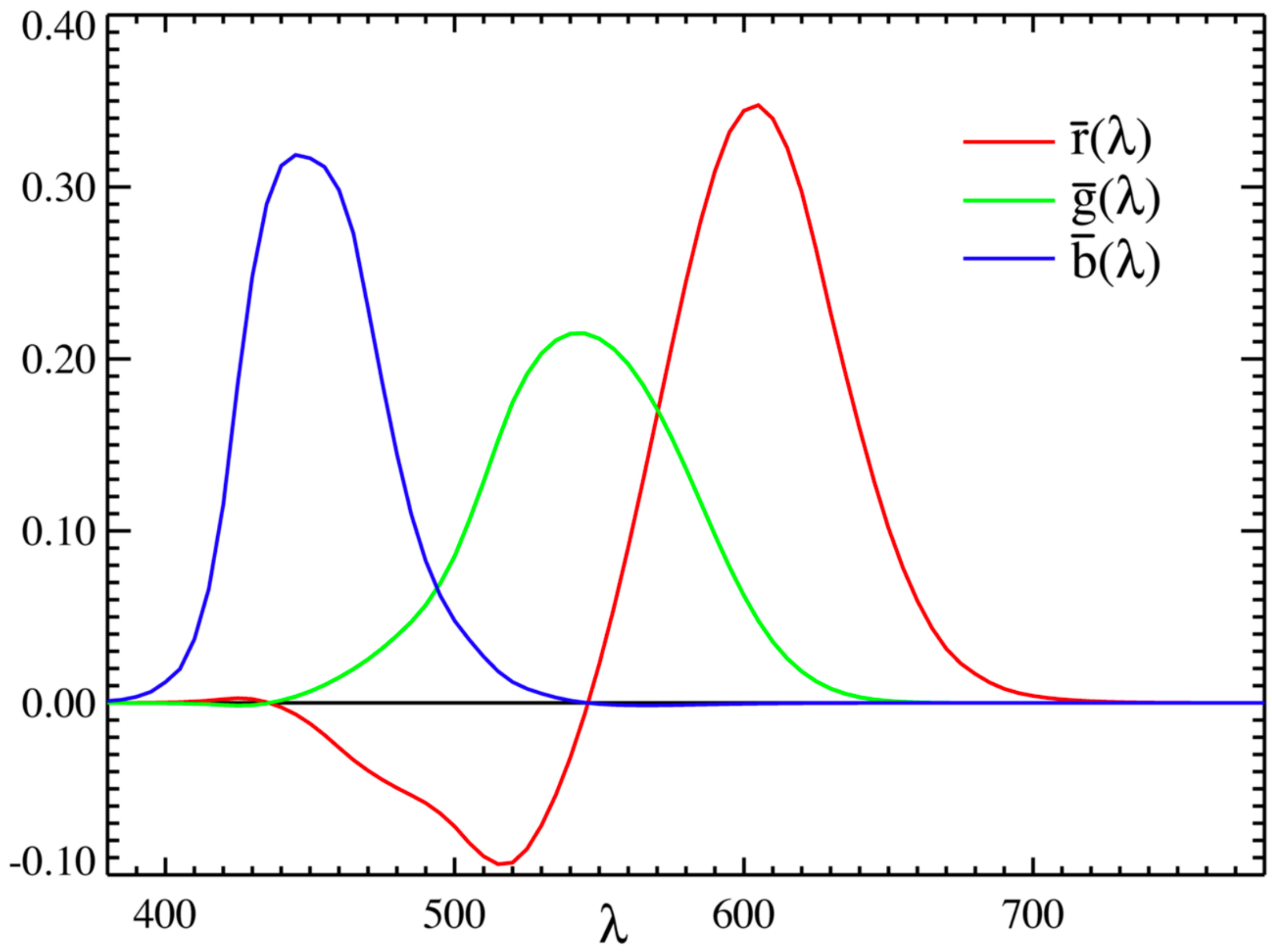
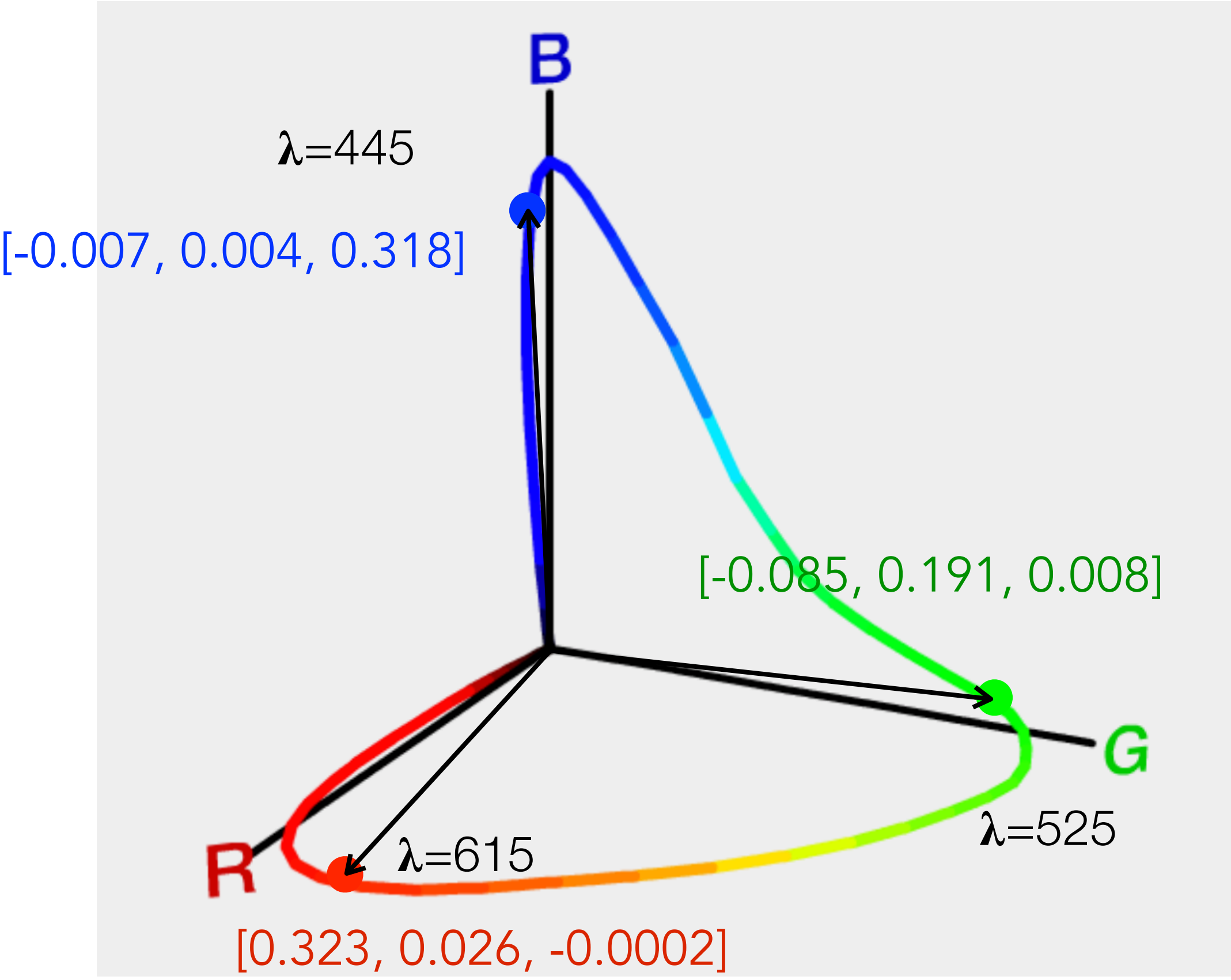
**Colorimetry**

Color science is a vastly interesting and broad discipline. We can only covering the basics. Dig (slightly) deeper using our interactive tutorials: <https://www.cs.rochester.edu/courses/572/colorvis/>.

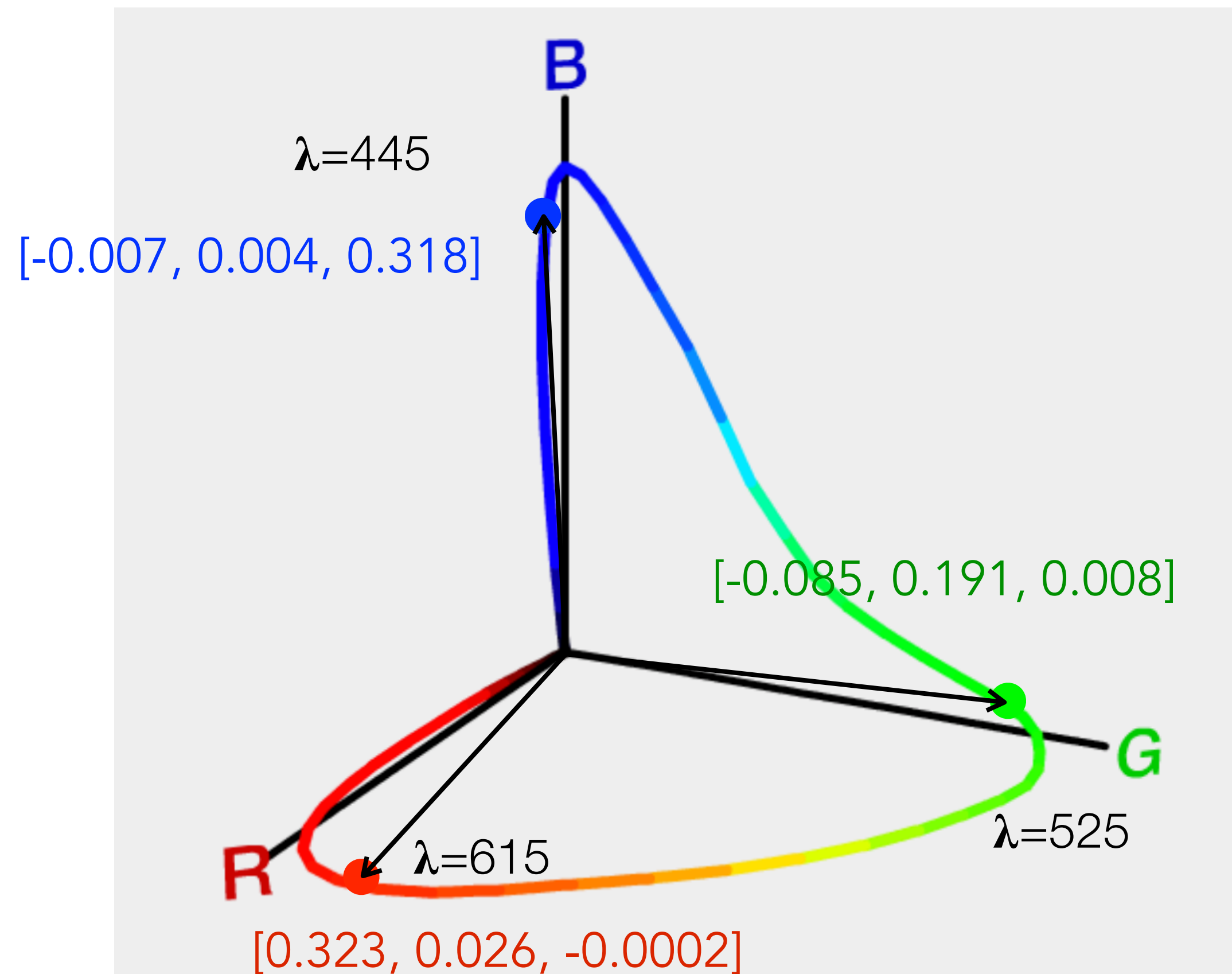
# XYZ Color Space



# Recall: CIE RGB Color Space



# A Slight Issue with CIE RGB Color Space



**“Issue”:** Some colors are expressed in negative RGB values, which is mathematically OK but not very intuitive.

**Goal:** design a color space so that all colors have positive values.

**How?** Transforming the coordinate system so that all colors are in the positive quadrant.

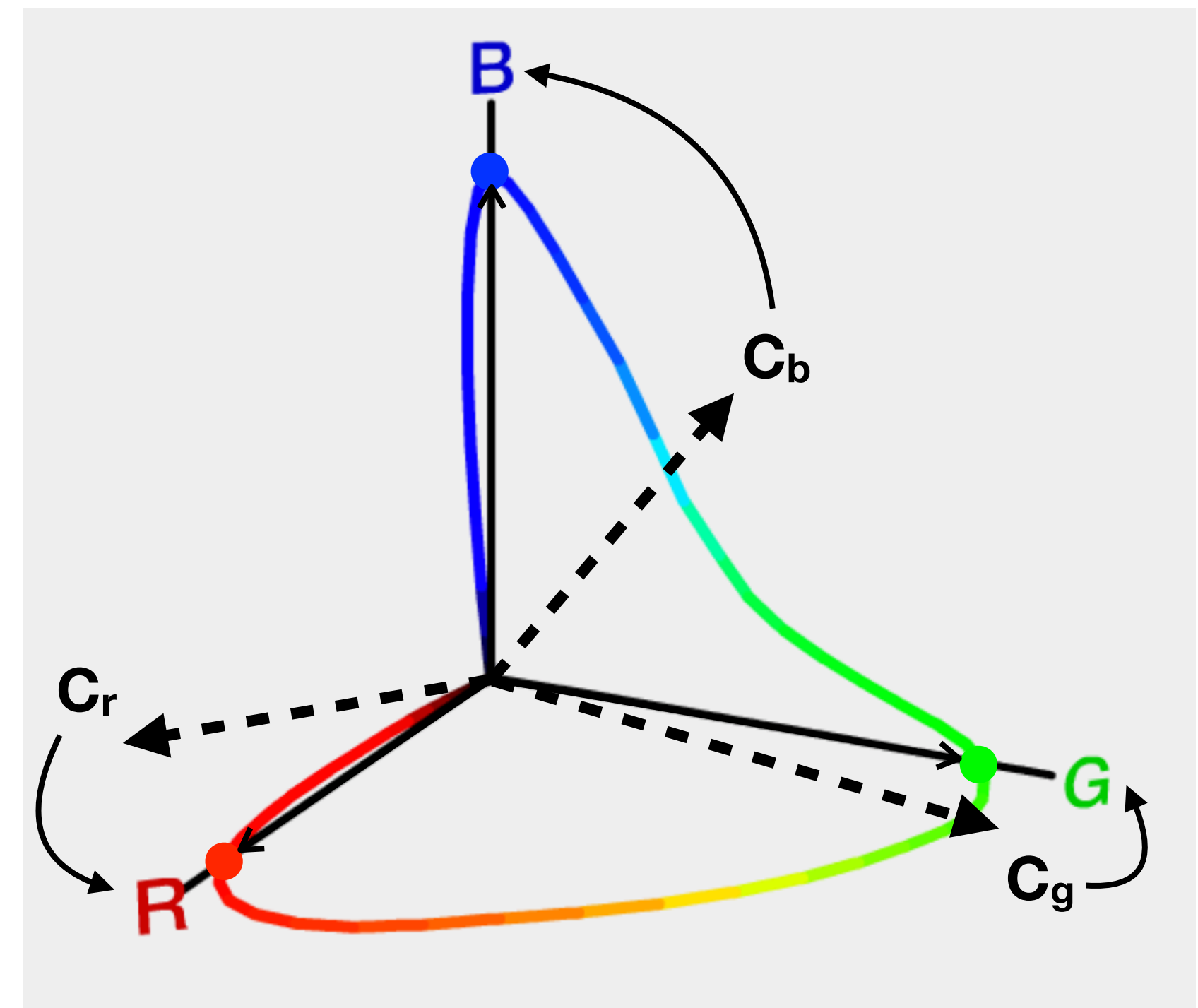
# CIE RGB to XYZ Color Space Transformation

CIE in 1931 constructed the XYZ color space for that purpose.

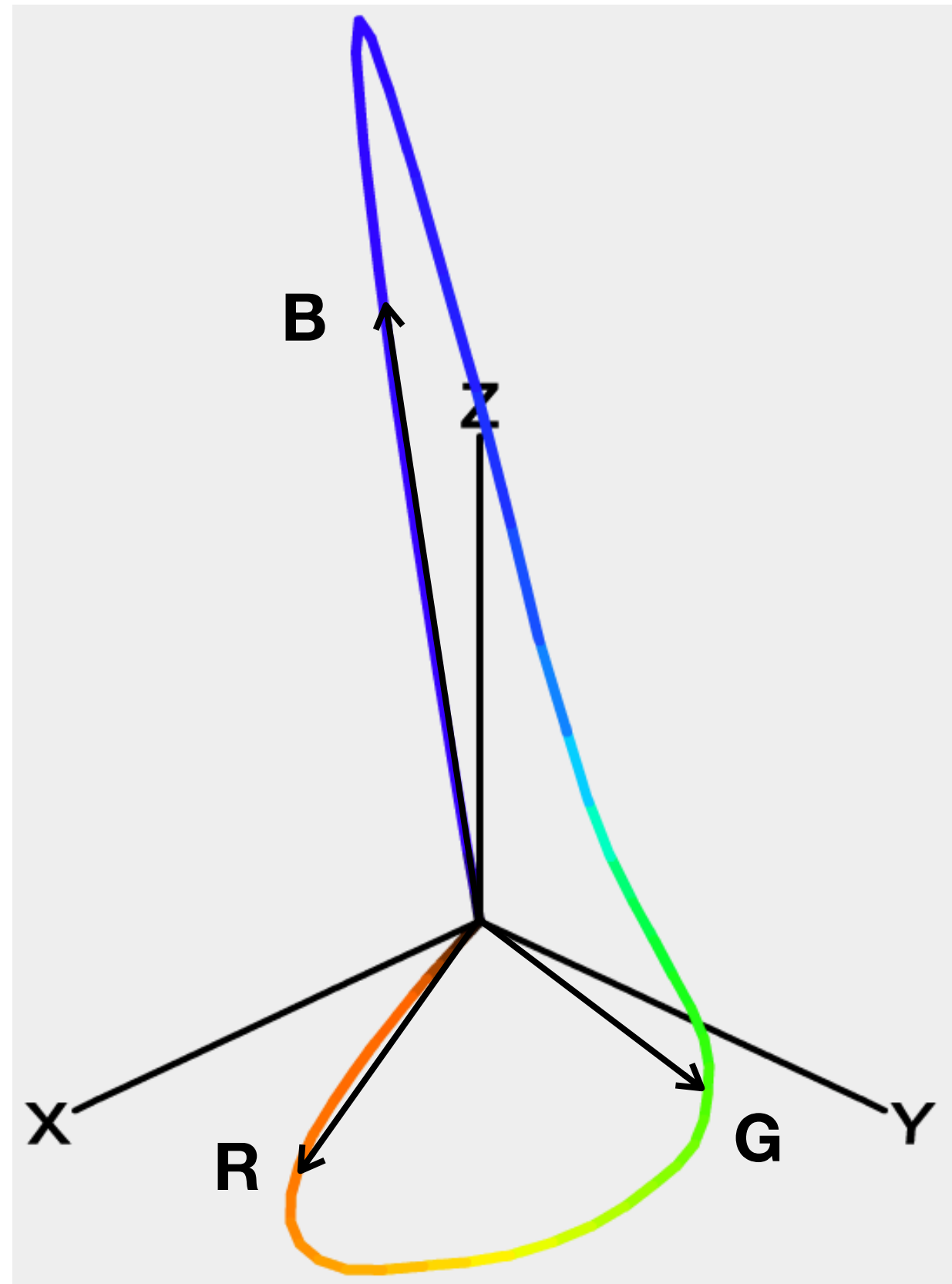
Every [R, G, B] value of a color is transformed to a [X, Y, Z] value.

See visualization here: <https://www.cs.rochester.edu/courses/572/colorvis/xyz.html>.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R_x & G_x & B_x \\ R_y & G_y & B_y \\ R_z & G_z & B_z \end{bmatrix} \times \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



# CIE RGB to XYZ Color Space Transformation



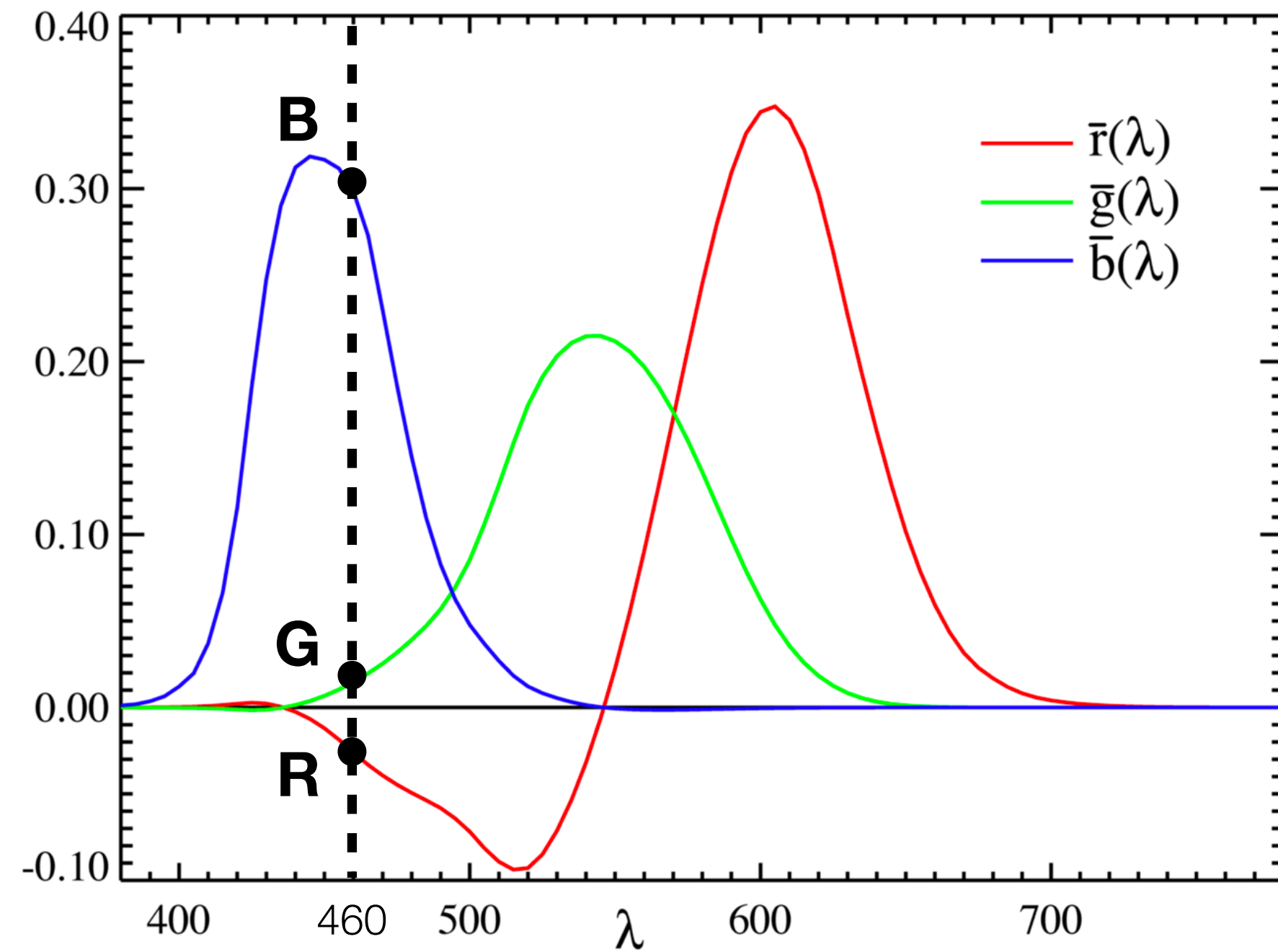
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4887180 & 0.3106803 & 0.2006017 \\ 0.1762044 & 0.8129847 & 0.0108109 \\ 0.0000000 & 0.0102048 & 0.9897952 \end{bmatrix} \times \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 2.3706743 & -0.9000405 & -0.4706338 \\ -0.5138850 & 1.4253036 & 0.0885814 \\ 0.0052982 & -0.0146949 & 1.0093968 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

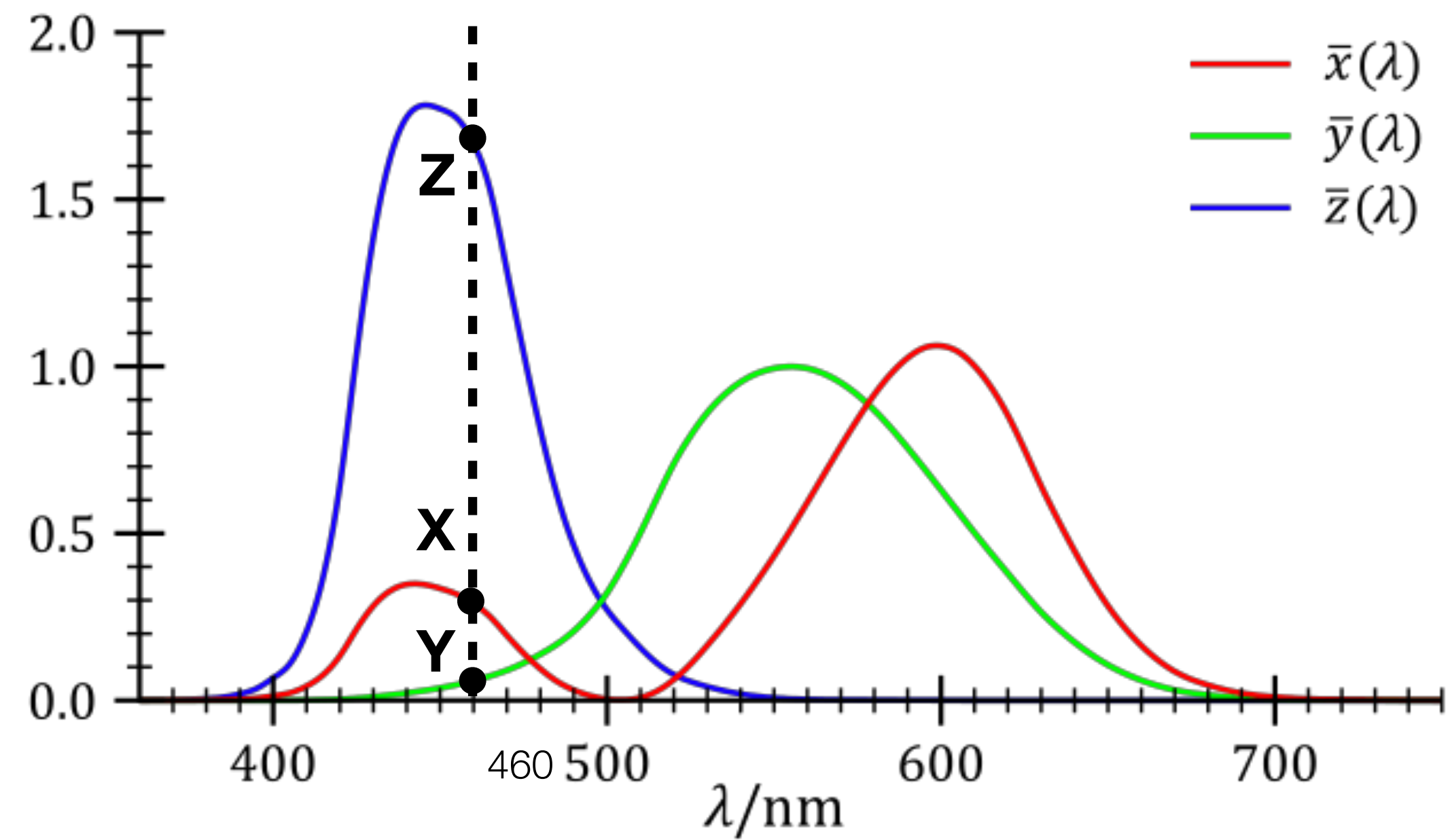
**XYZ to RGB transformation matrix.**

**CIE 1931 XYZ Space and the CIE 1931 RGB primaries in the XYZ space.**

# XYZ Color Matching Functions



CIE 1931 RGB color matching functions



CIE 1931 XYZ color matching functions

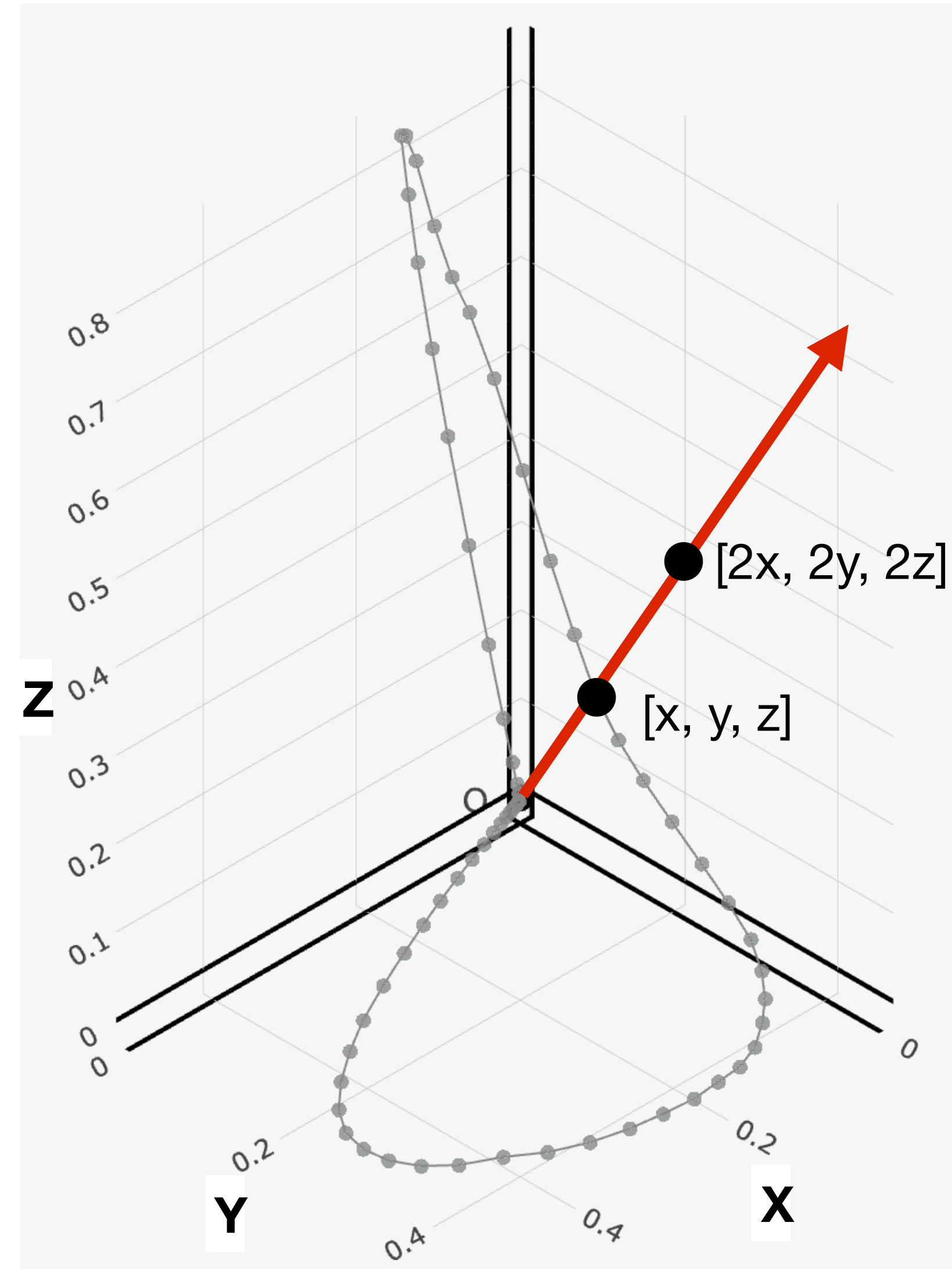
# Chromaticity and Gamut



# Scaling the Intensity of a Light

If we double the power of a light, intuitively its “color” doesn’t change. It just becomes brighter.

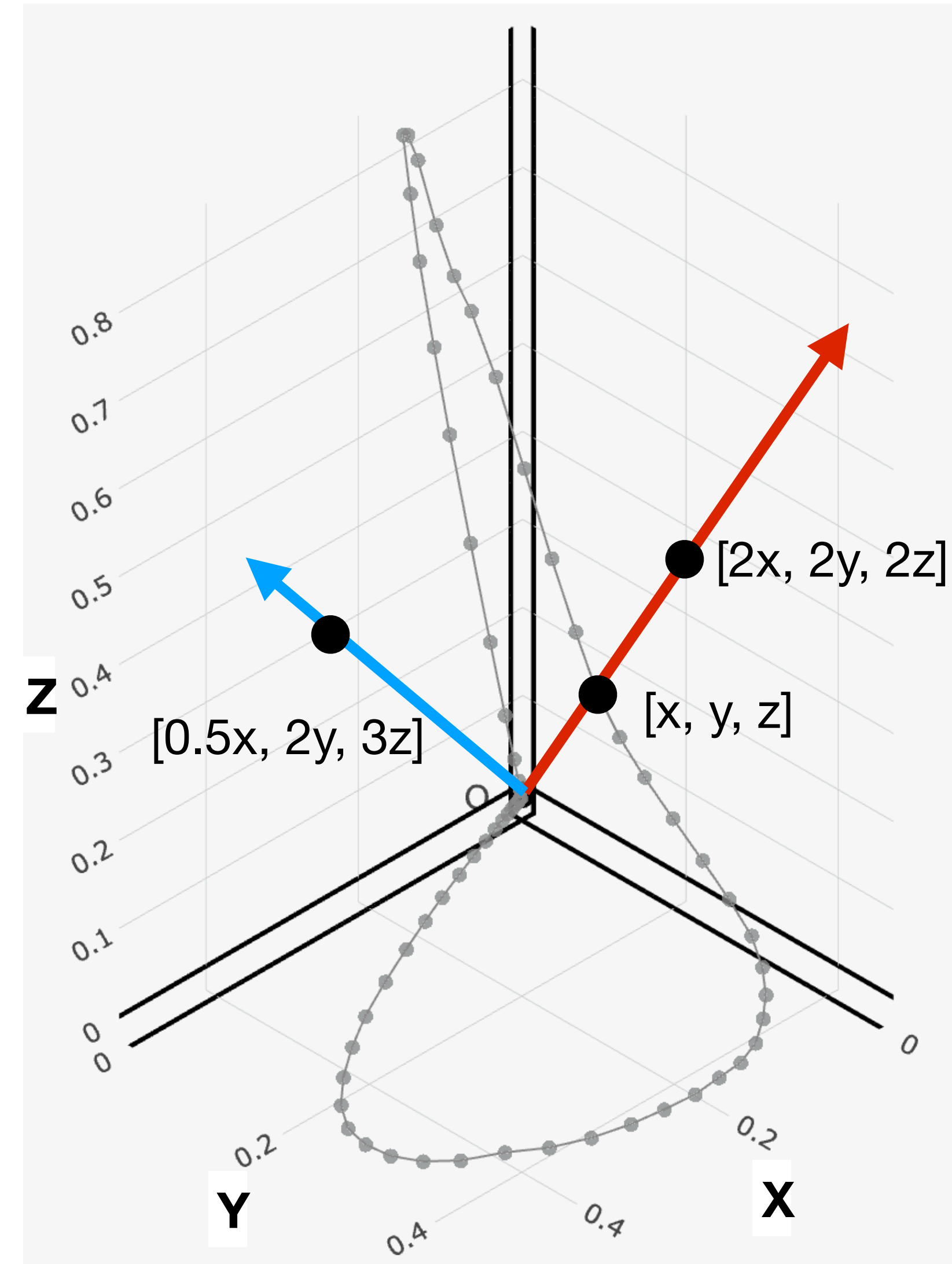
Doubling the power of a color  $[x, y, z]$  gets us another color  $[2x, 2y, 2z]$ . Both colors have the same **ratio** of the primary contributions. They lie on the same line that goes through the color space origin.



# Separating Intensity From "True" Color

Another color, say,  $[0.5x, 2y, 3z]$ , has different primary contributions, and is not simply scaled from the color  $[x, y, z]$ .

Goal: separate the objective quantity (intensity) and the **quality** of a color.





# Chromaticity

Normalize **XYZ** to **xyz**.

**xyz** are called the chromaticity, the “objective quality” of the color.

- Can extract chromaticity in any color space by the same calculations.

It's just a geometric transformation from  $[X, Y, Z]$  to  $[x, y, z]$ . What kind of transformation is this?

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

# Chromaticity

Two characteristics of this transformation.

1. Any color that has the same X:Y:Z ratio will be mapped to the same point.

- What points will have the same X:Y:Z ratio?

2. Given that  $x+y+z=1$ , all the transformed points are now on the same plane.

$$x = \frac{X}{X + Y + Z}$$

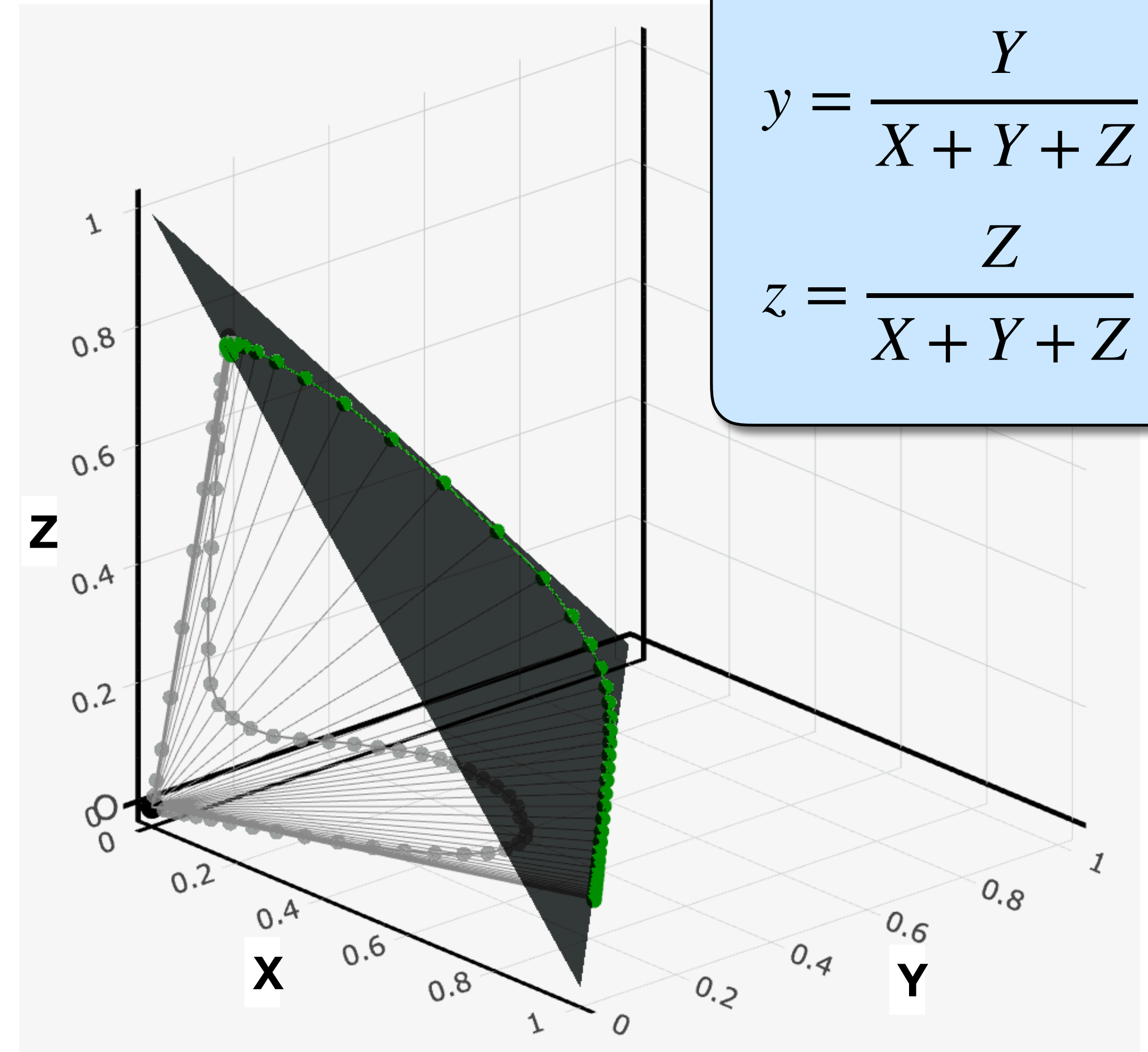
$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

# The Geometry of Chromaticity

We've effectively projected all the colors onto the same plane using what's called the **perspective projection**, which is exactly what a (pinhole) camera does.

See visualization here: <https://www.cs.rochester.edu/courses/572/colorvis/chromaticity.html>.

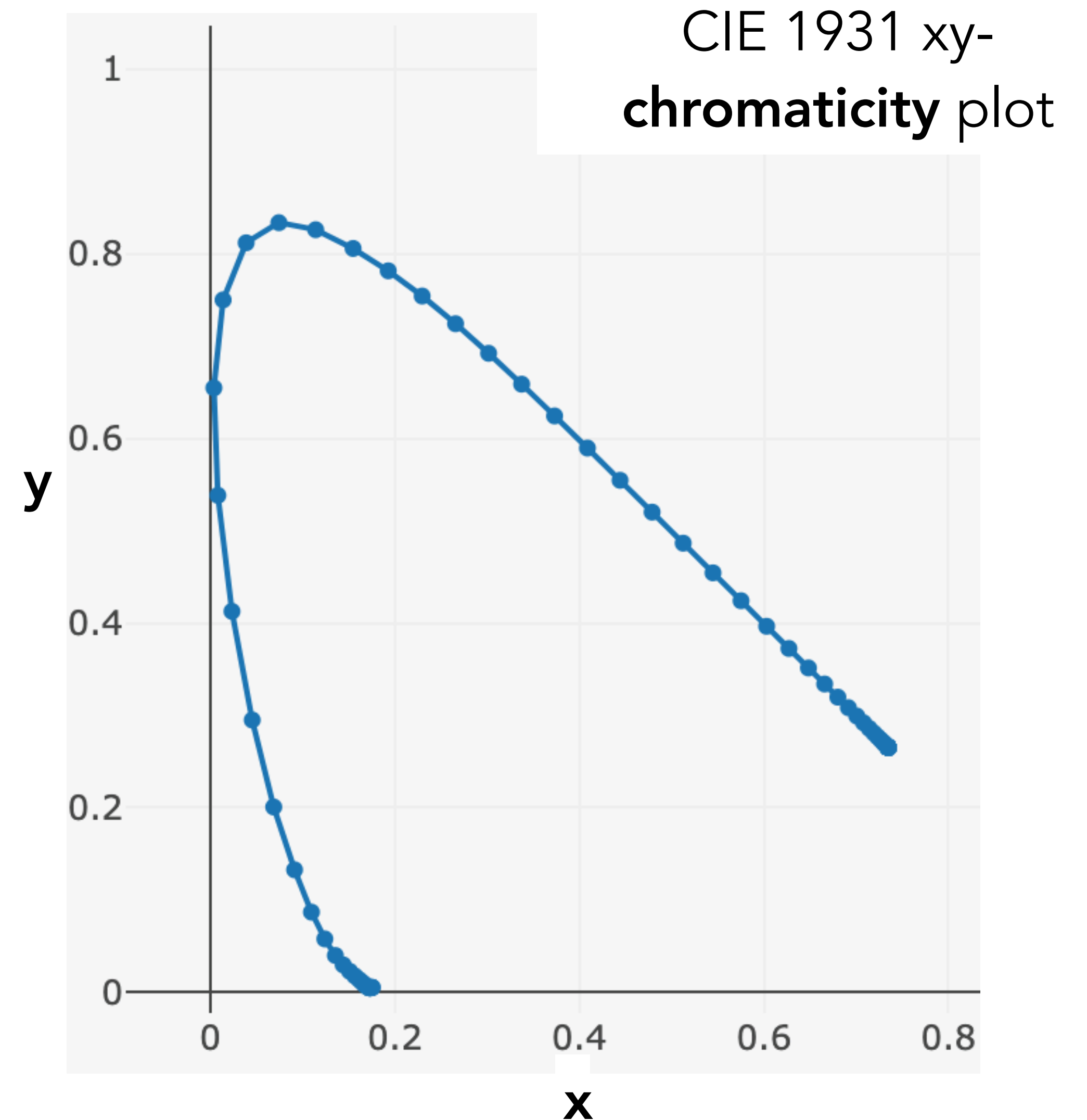


$$x = \frac{X}{X + Y + Z}$$
$$y = \frac{Y}{X + Y + Z}$$
$$z = \frac{Z}{X + Y + Z}$$

# xy-Chromaticity Plot

Since  $x+y+z=1$ , we need only two components to represent a color. That is, we can represent colors in a 2D space (except we lose the intensity information).

Plot the locus in a 2D xy plot.  $z$  is implicit (i.e.,  $z = 1 - x - y$ ).



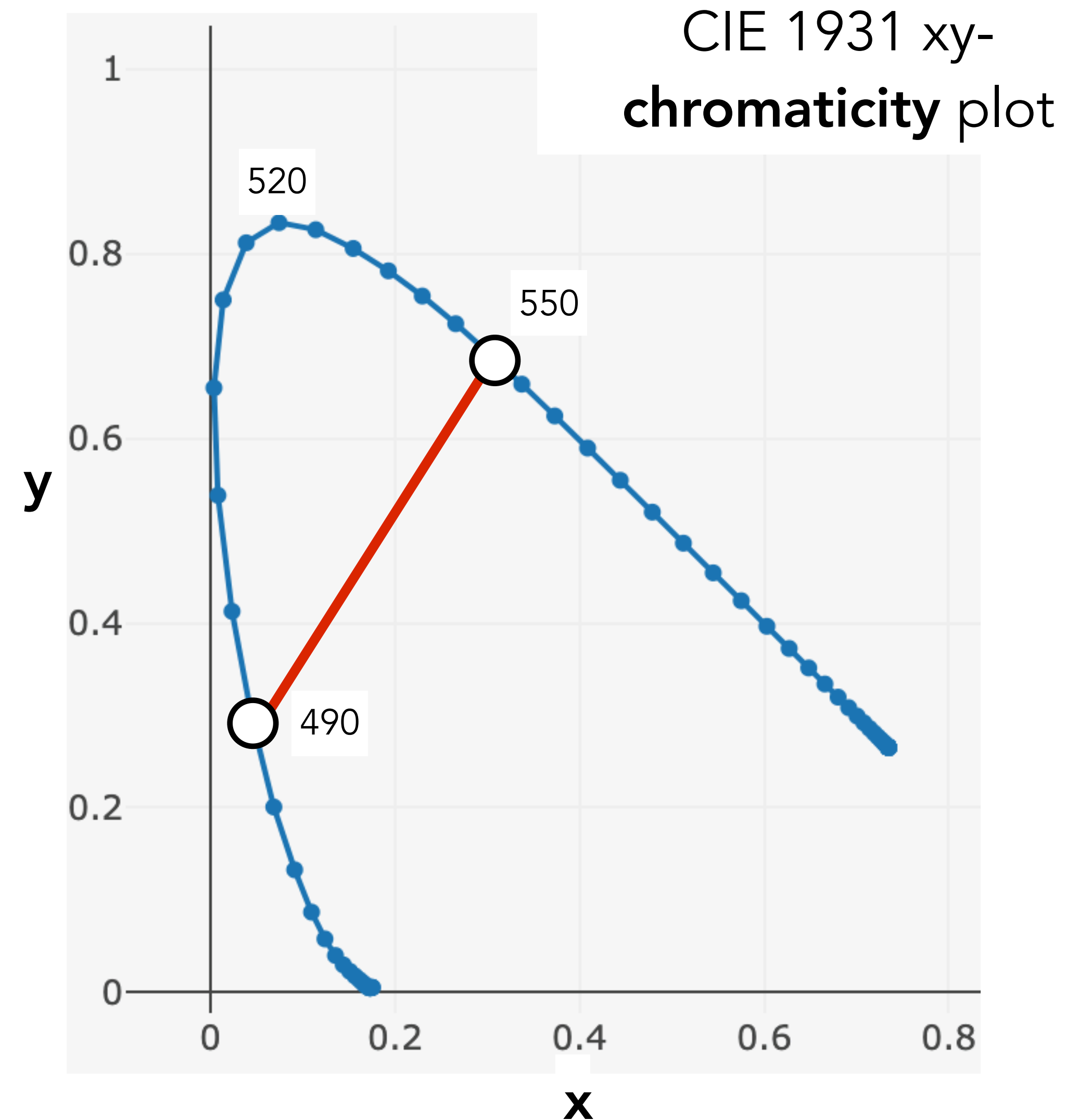
# Interpreting the $xy$ -Chromaticity Plot

The **curve** is the spectral locus.

Points **on locus**: spectral light.

Connect two points on the locus: all the colors that can be produced from mixing those two spectral lights.

- Because any point on a line segment is a **positive** linear combination of the two end points.

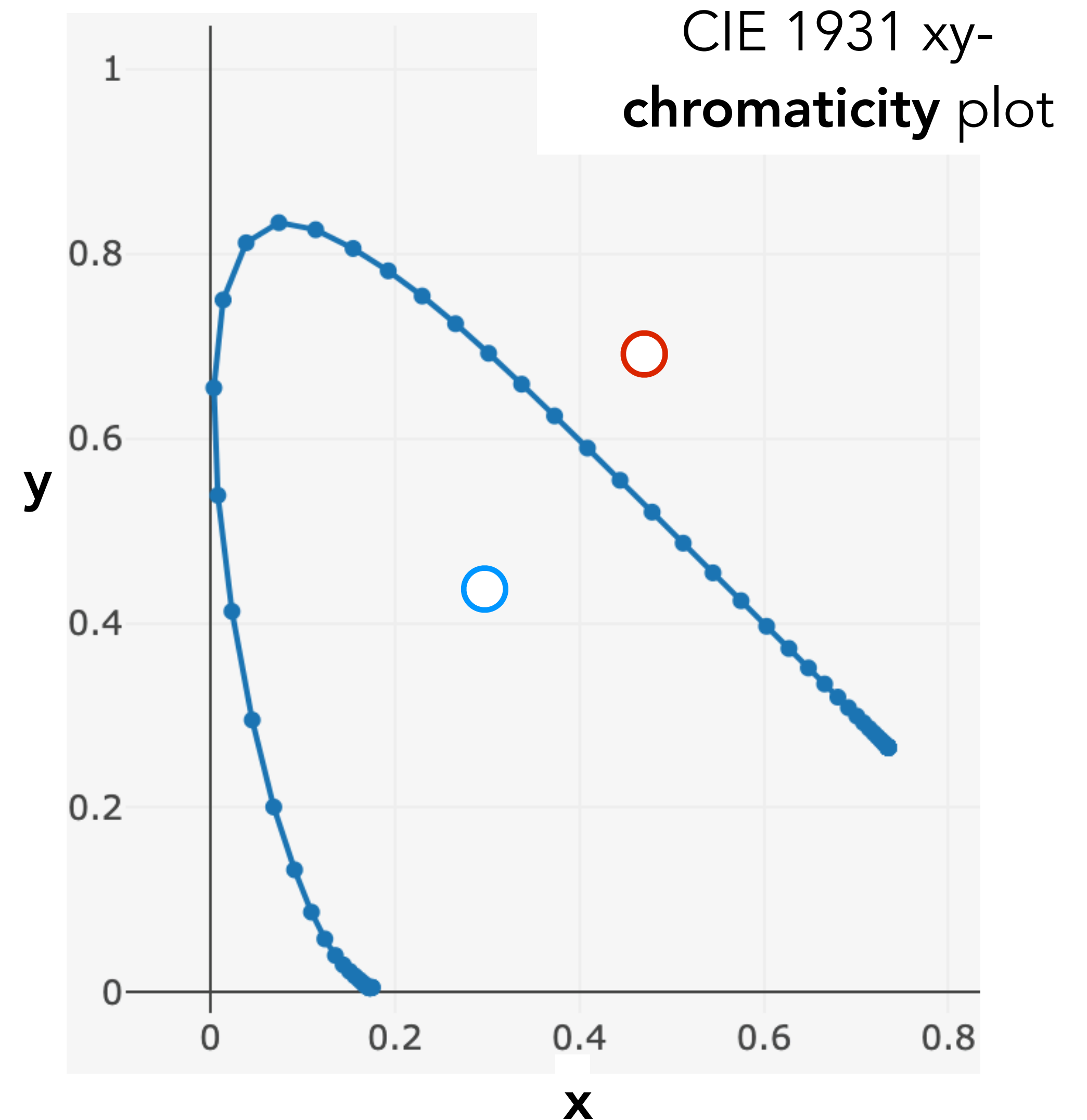


# Interpreting the xy-Chromaticity Plot

Points **inside locus**: colors of non-spectral lights. They can be produced from some combination of other lights.

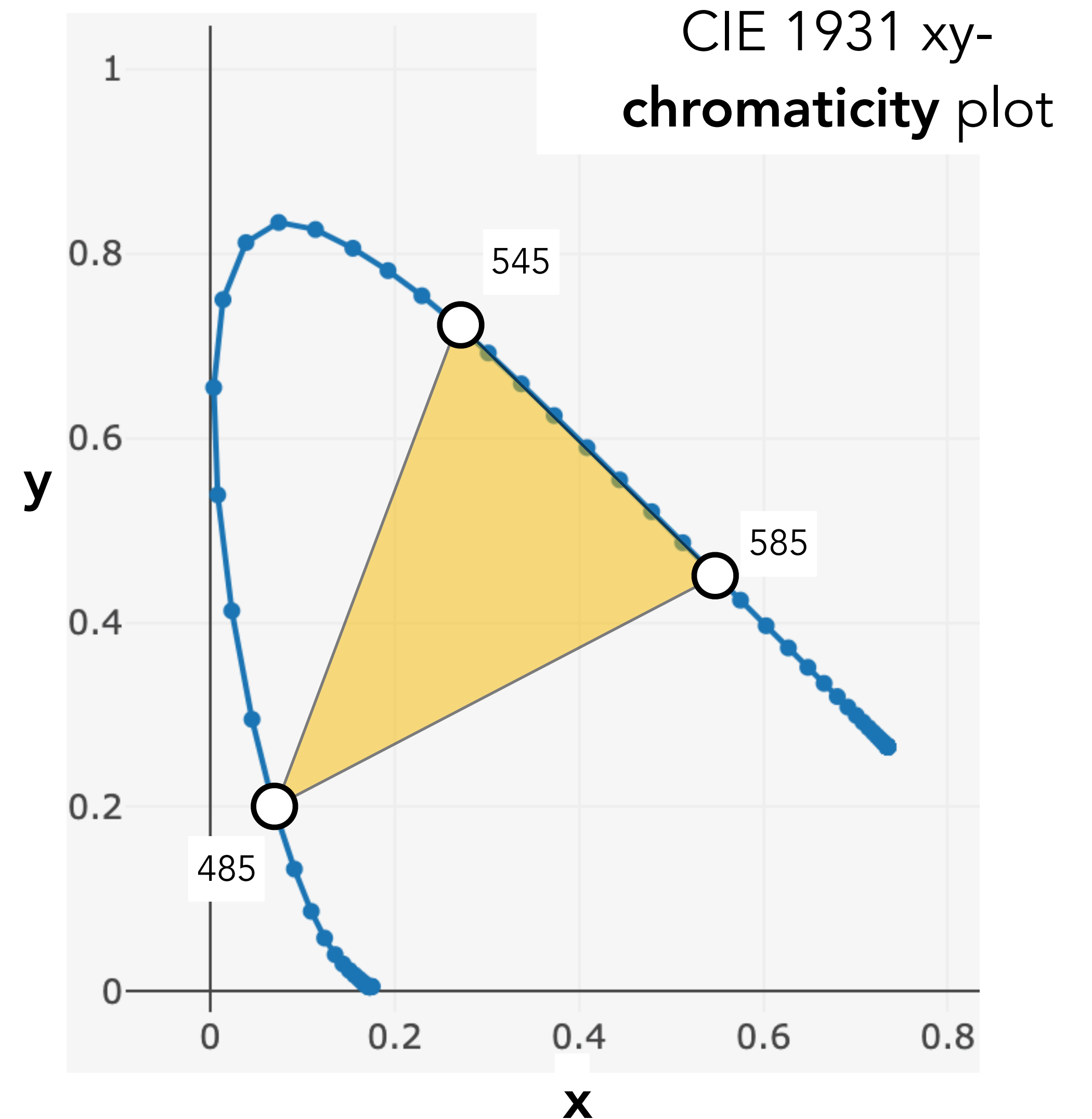
Points **outside locus** are imaginary colors; can't be produced from any combination of real lights.

The area enclosed by the spectral locus represents the **gamut of human visual system** — all the colors we can see.



# Representing a Color Space in $xy$ -Chromaticity Plot

Pick three points on or inside the locus as the primary lights.

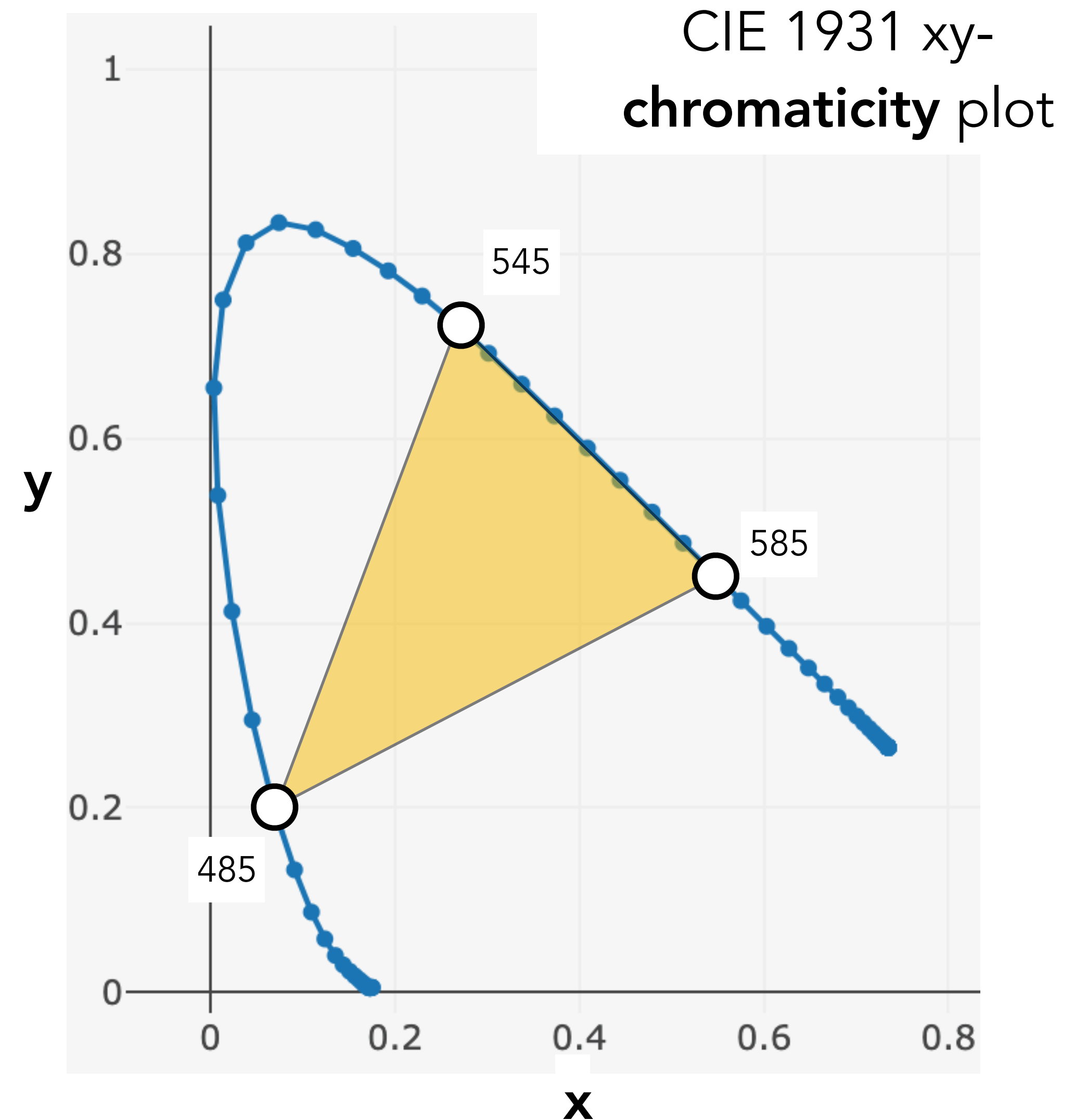




# Representing a Color Space in $xy$ -Chromaticity Plot

Pick three points on or inside the locus as the primary lights.

A point inside the triangle is a color that can be physically produced by mixing the chosen primaries lights.



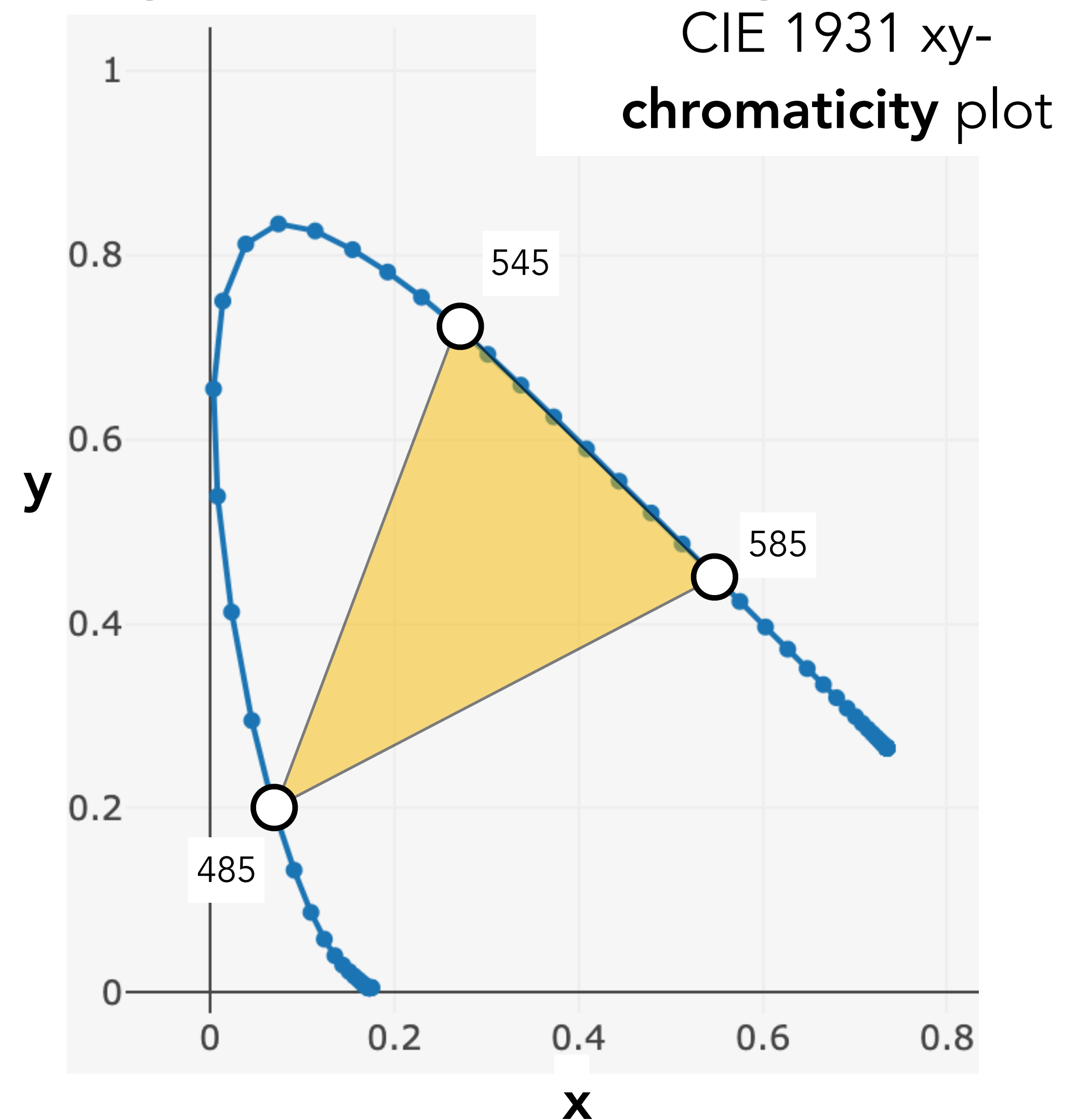


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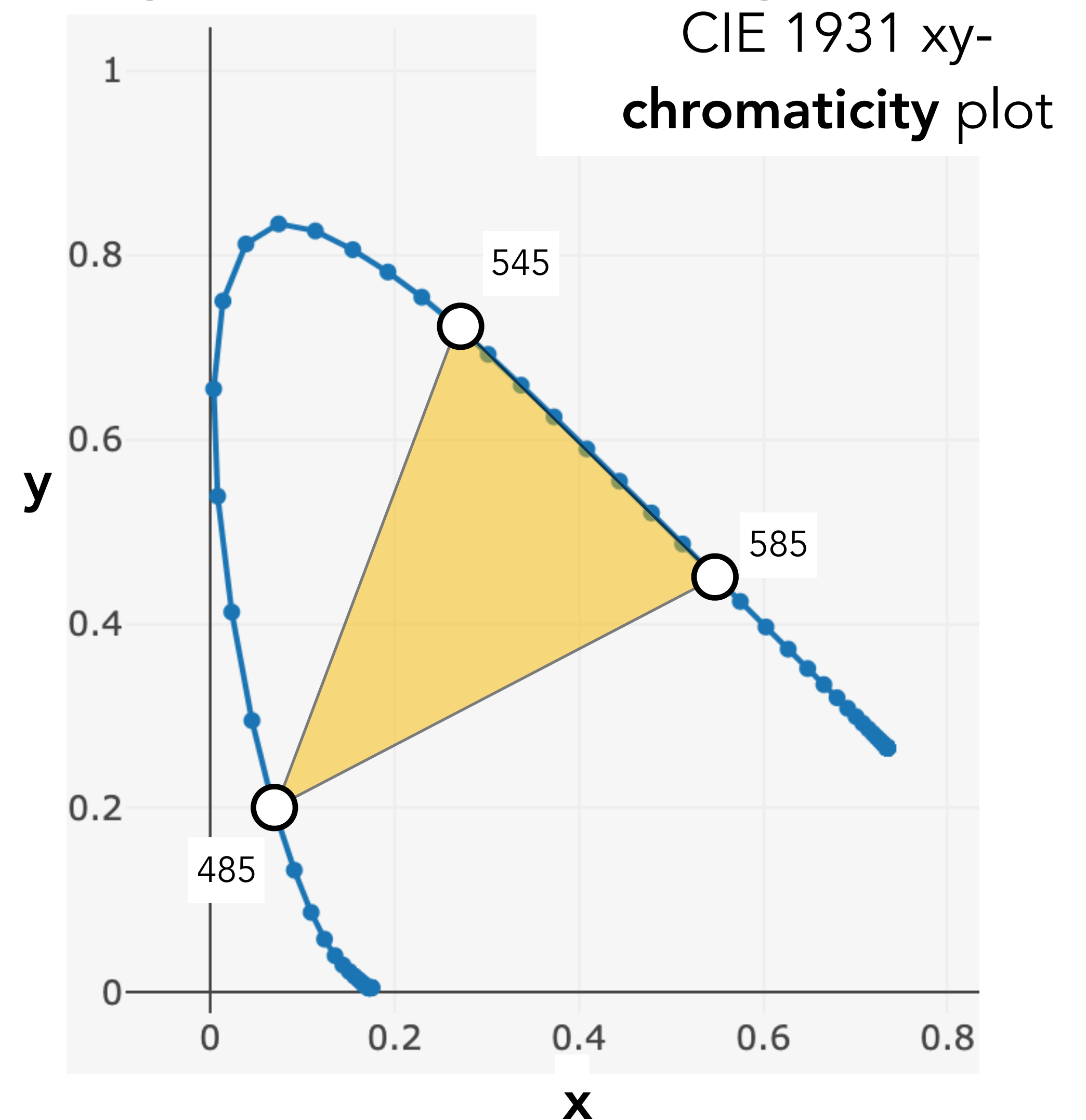
A point inside the triangle is a color that can be physically produced by mixing the chosen primaries lights.

- Because every point inside a triangle can be expressed as a **positive** linear combination of the vertices.



# Representing a Color Space in $xy$ -Chromaticity Plot

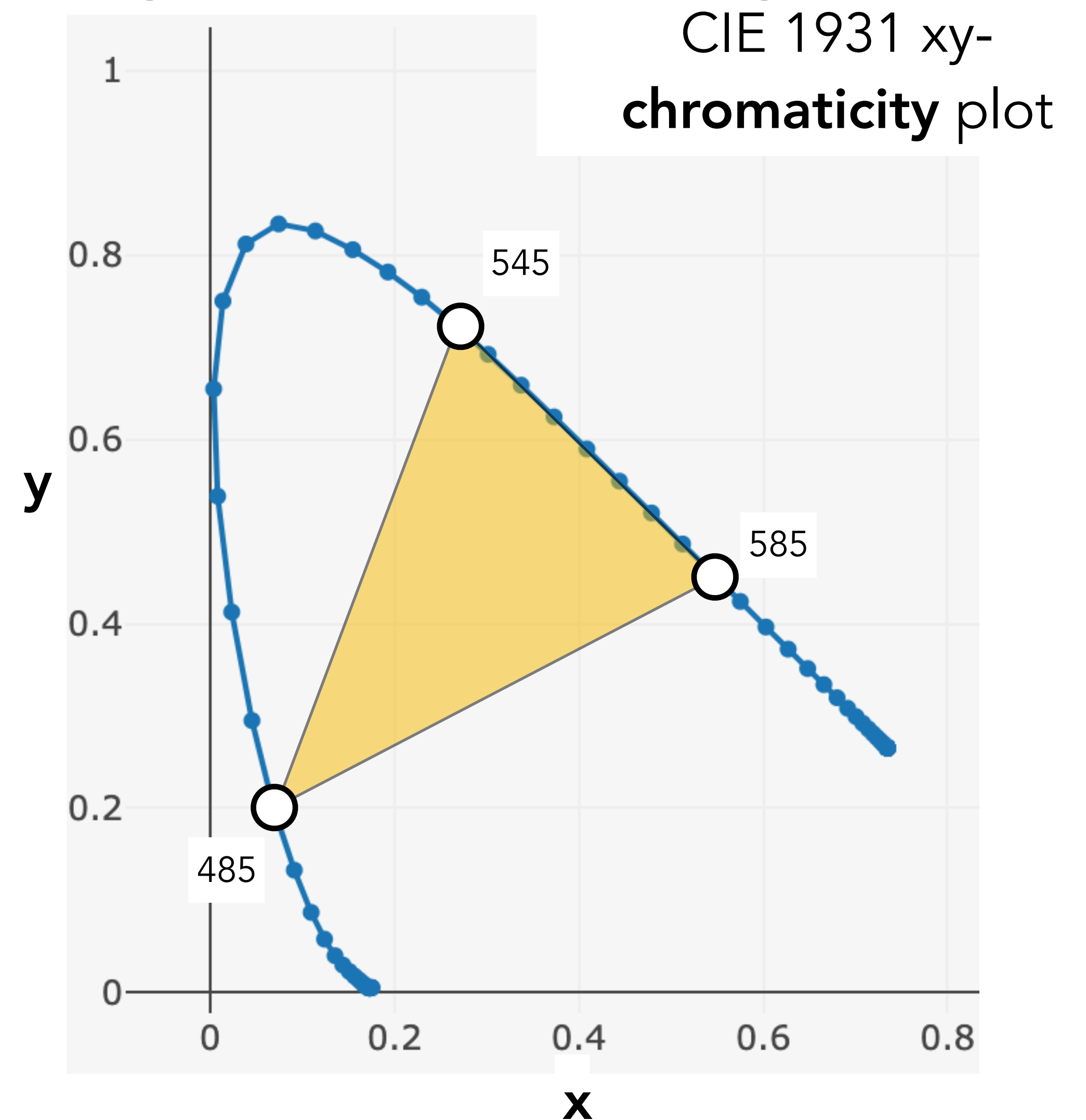
Another way to put it: the three chosen primaries form a color space, and the triangle represents the **color gamut** of the color space.



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- Rigorously speaking, the actual gamut of a color space is 3D. Will see it soon.

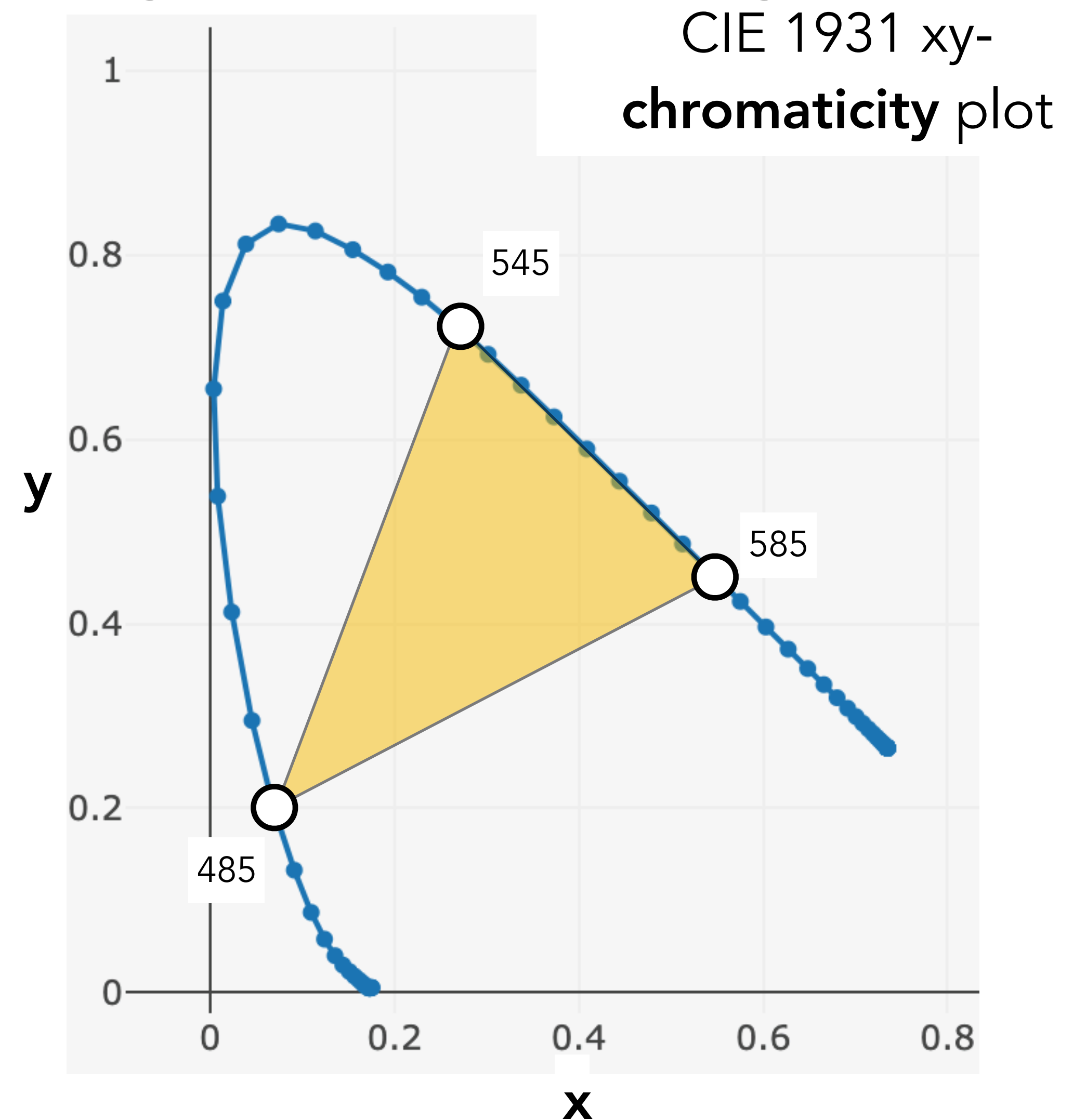


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Primary lights need not be spectral lights.

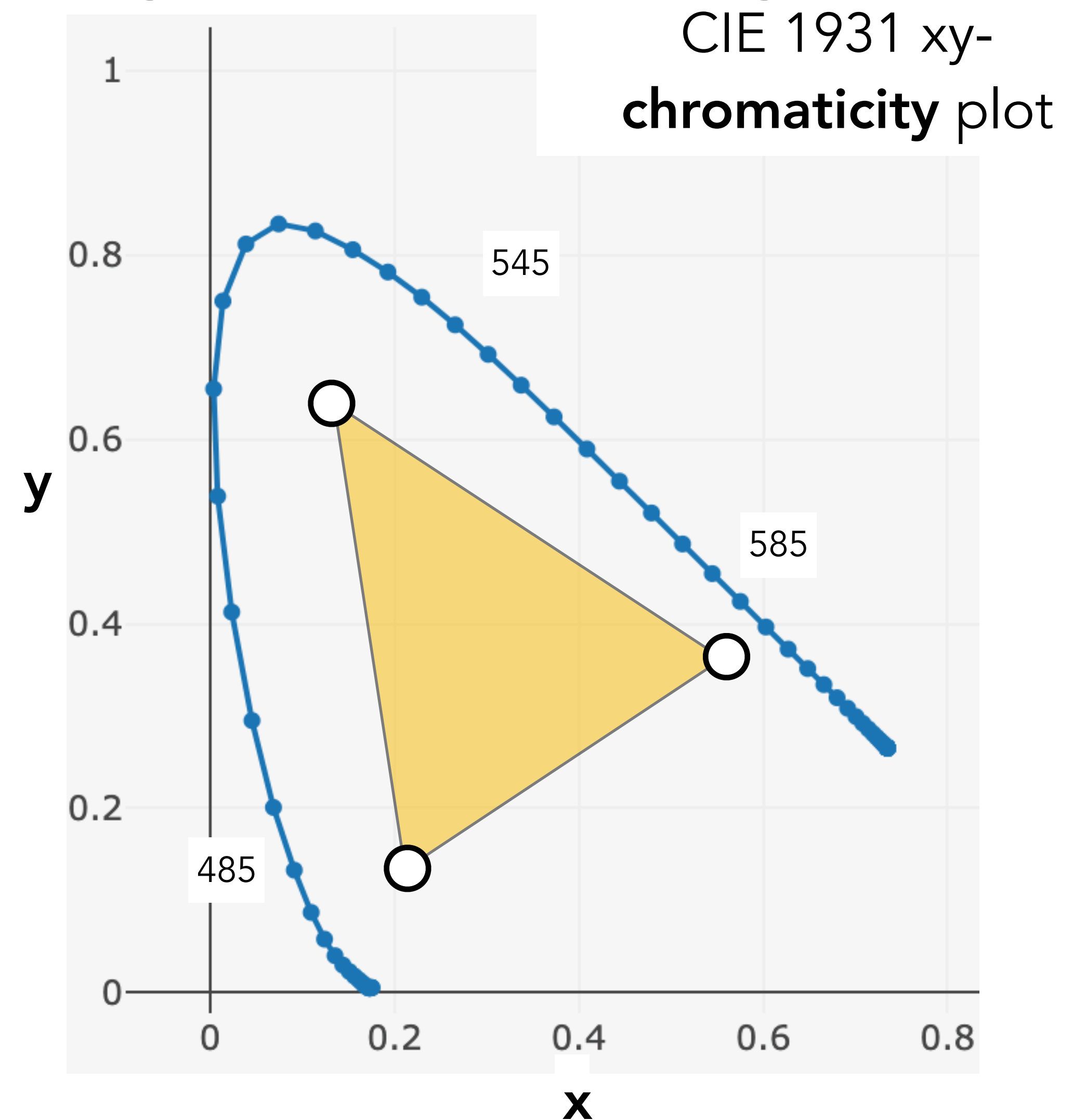


# Representing a Color Space in $xy$ -Chromaticity Plot

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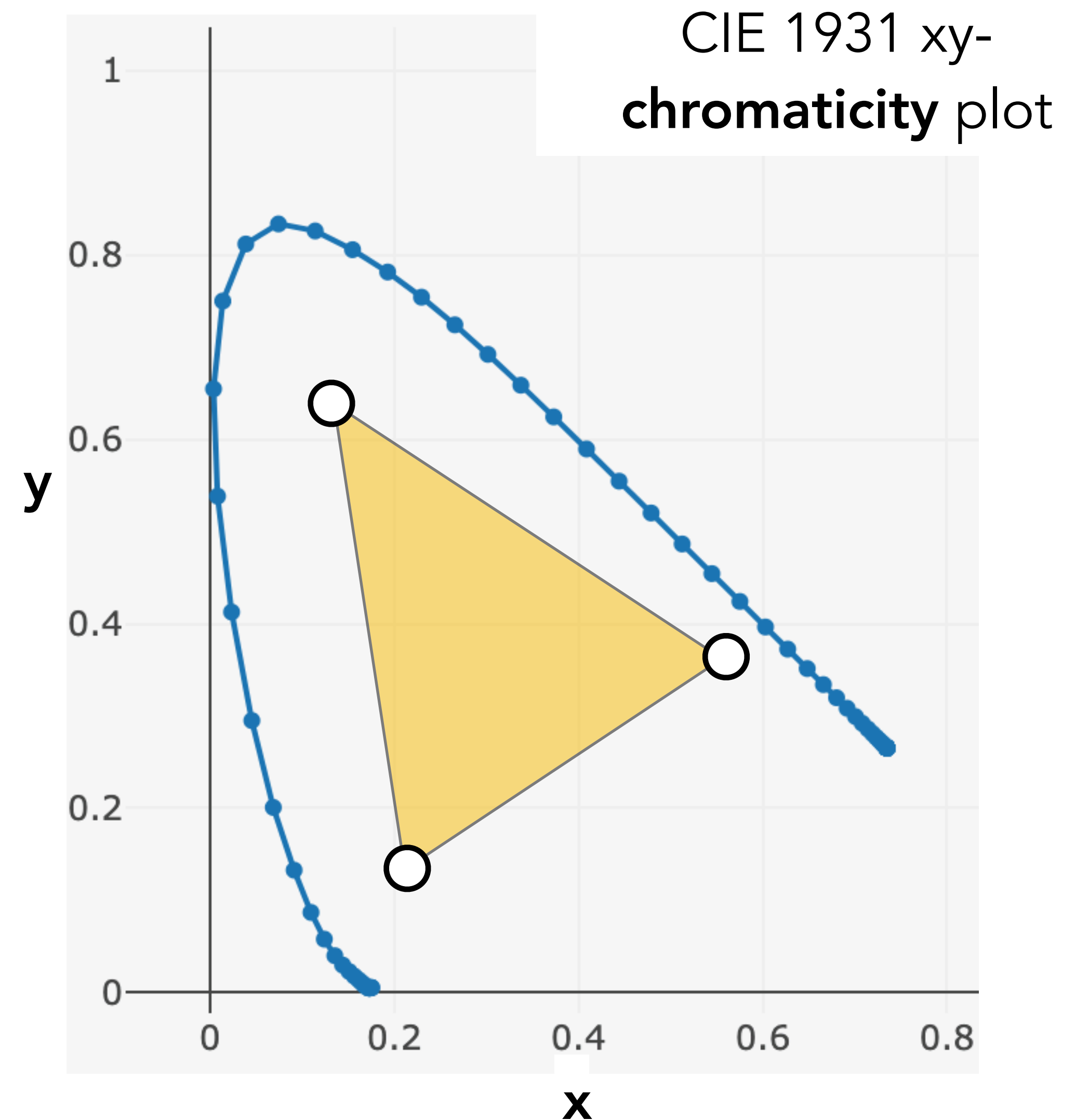
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# Gamut of a Color Space

Colors outside the triangle but inside the spectral locus are real colors, but they can't be physically produced by mixing the primary lights of this color space.

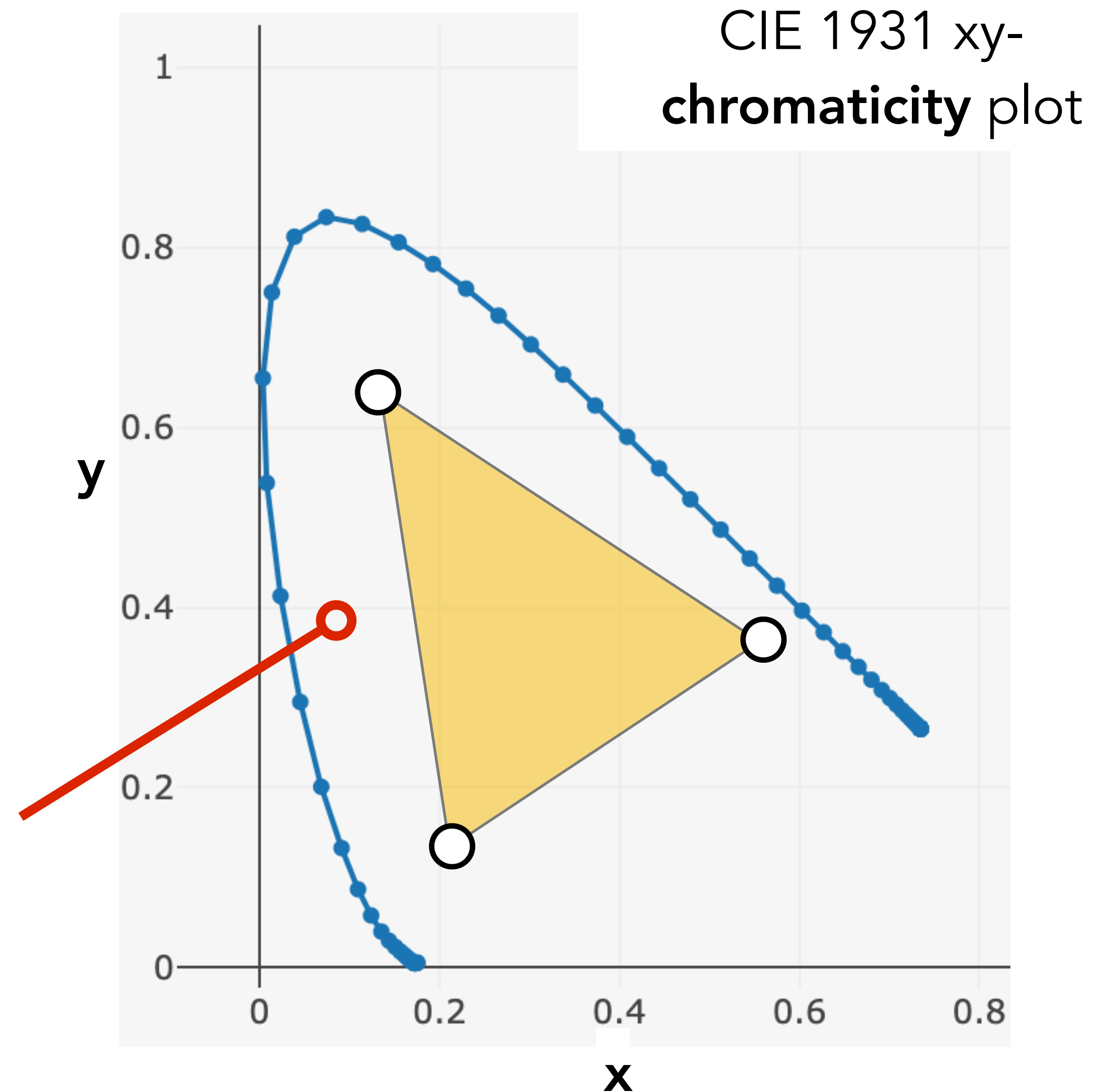




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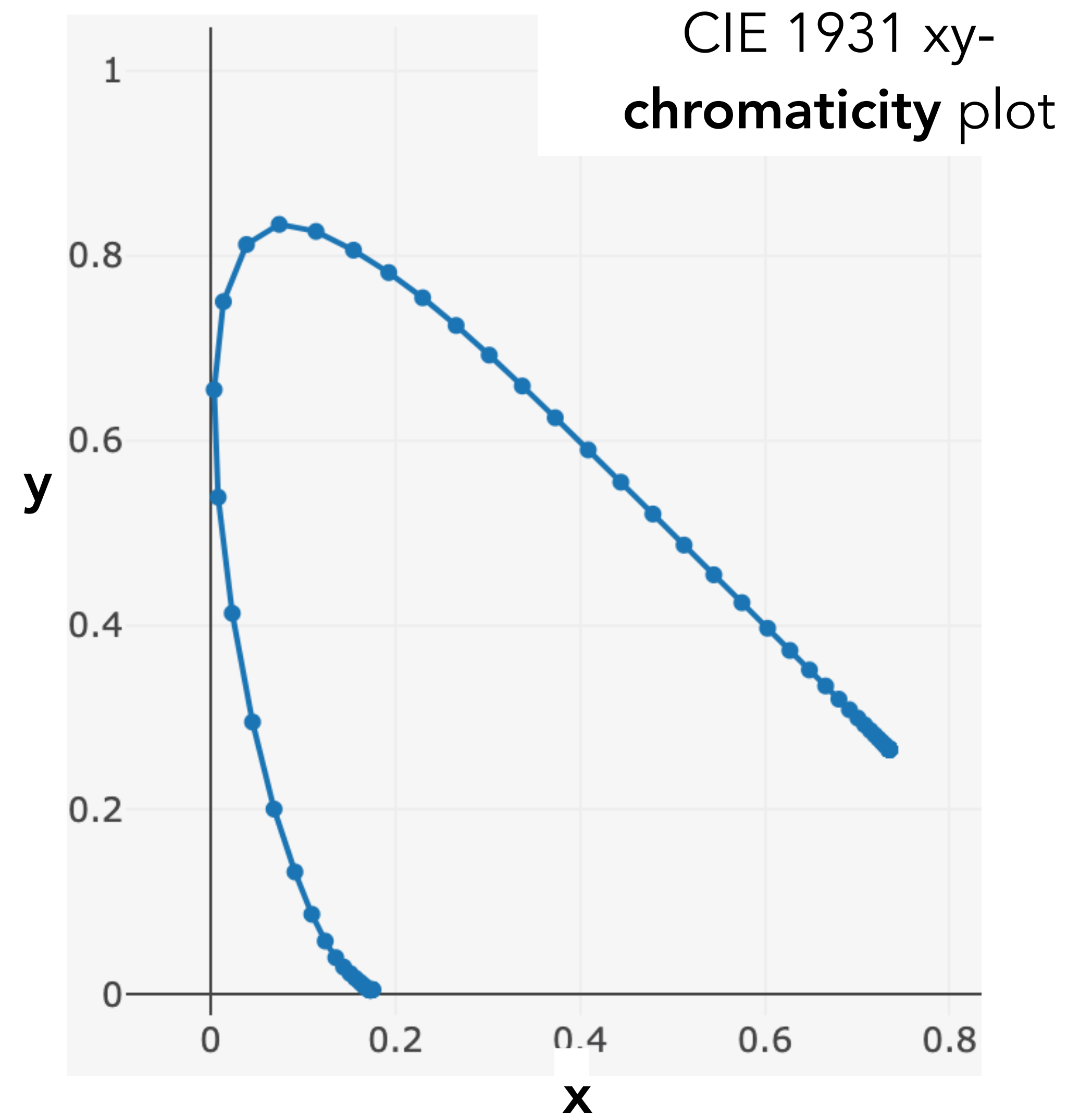
Color that can't be produced from this RGB color space



# A Fascinating Fact

No color space made of physically real lights can reproduce all visible colors.

- No matter how many primaries are used in the color space.
- Why? Can you give the intuition?



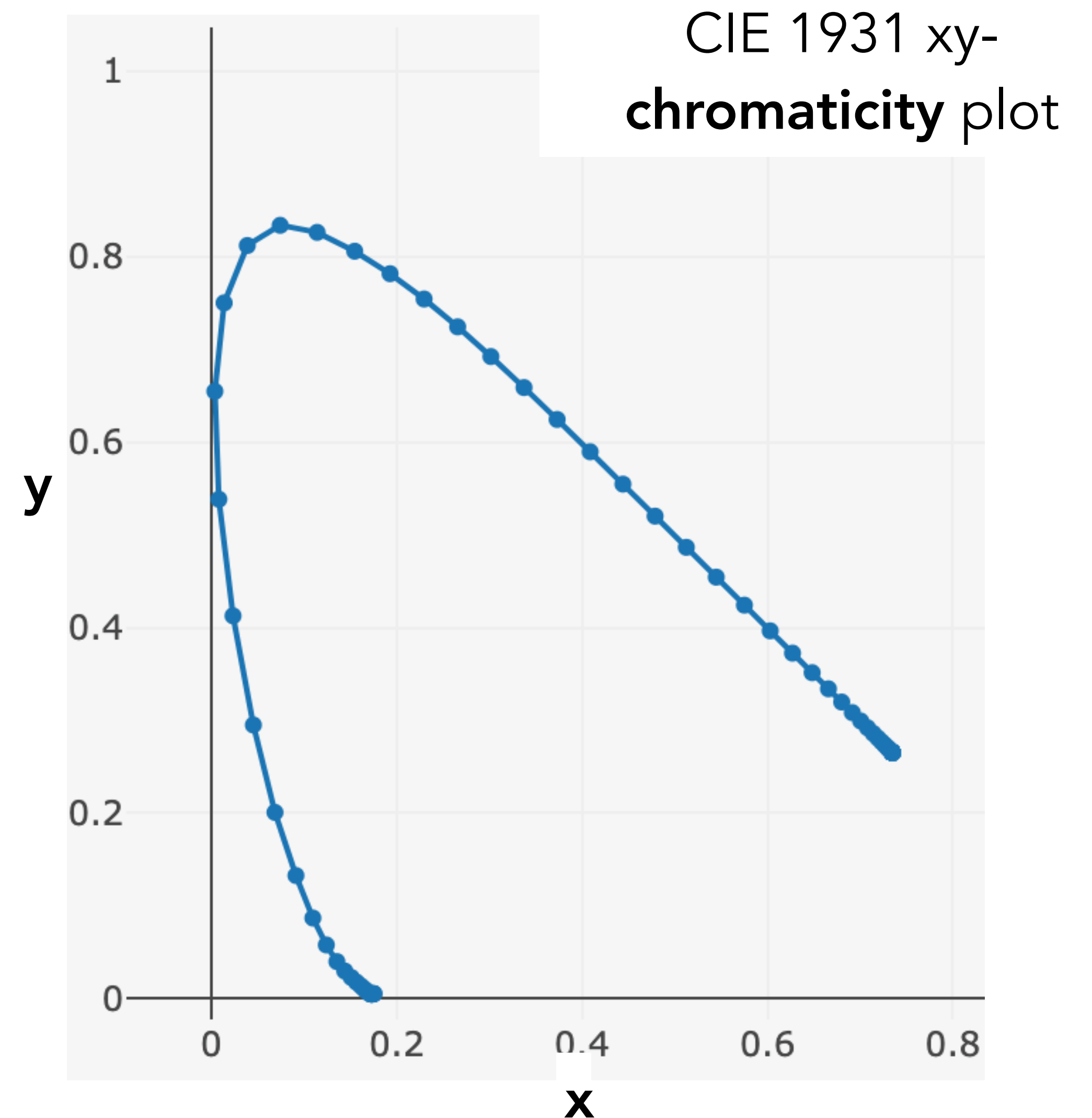


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The locus is **convex**. No polygon whose vertices are on or inside the spectral locus can cover the interior of the locus.



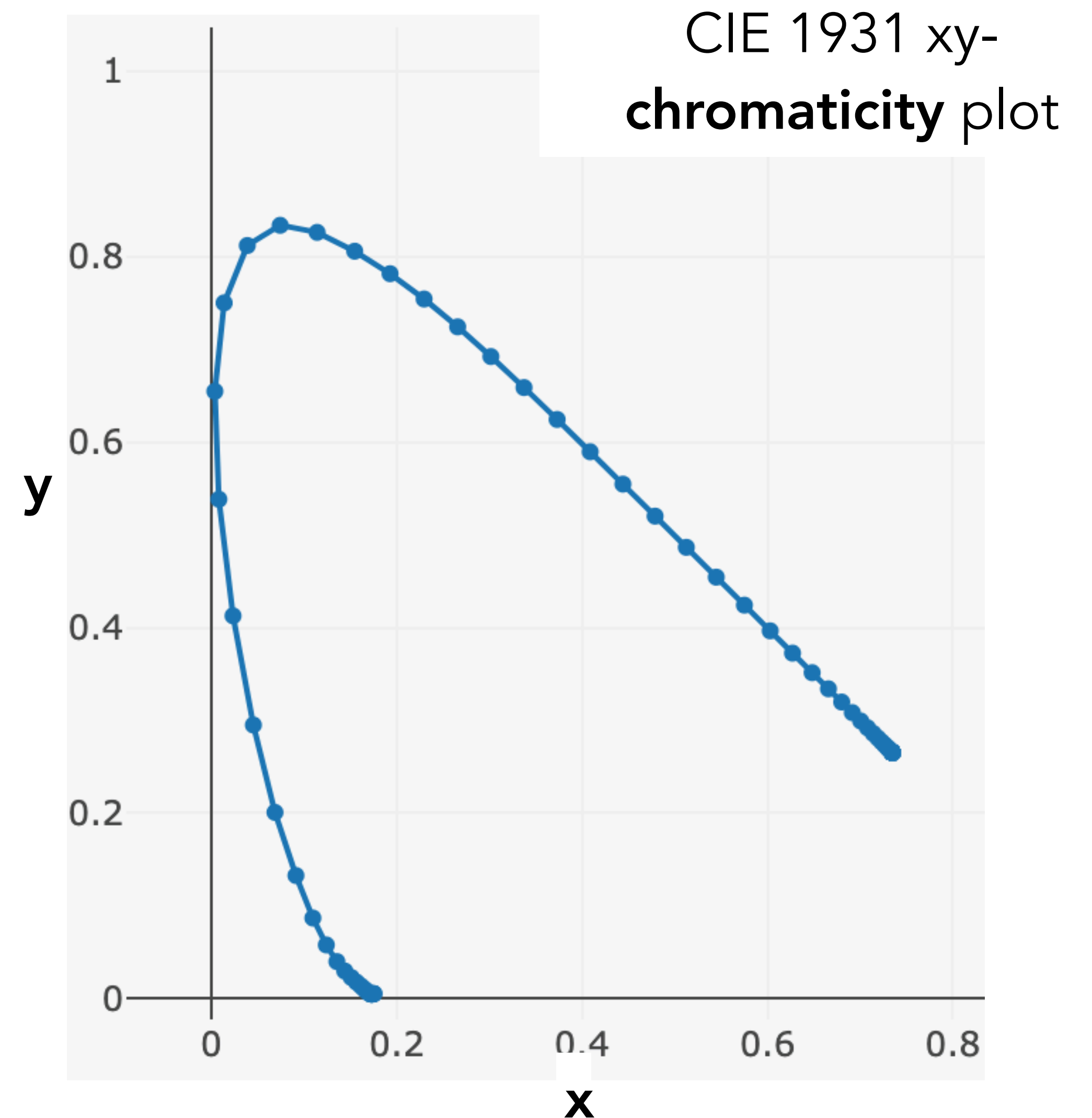
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No display can display all the colors.

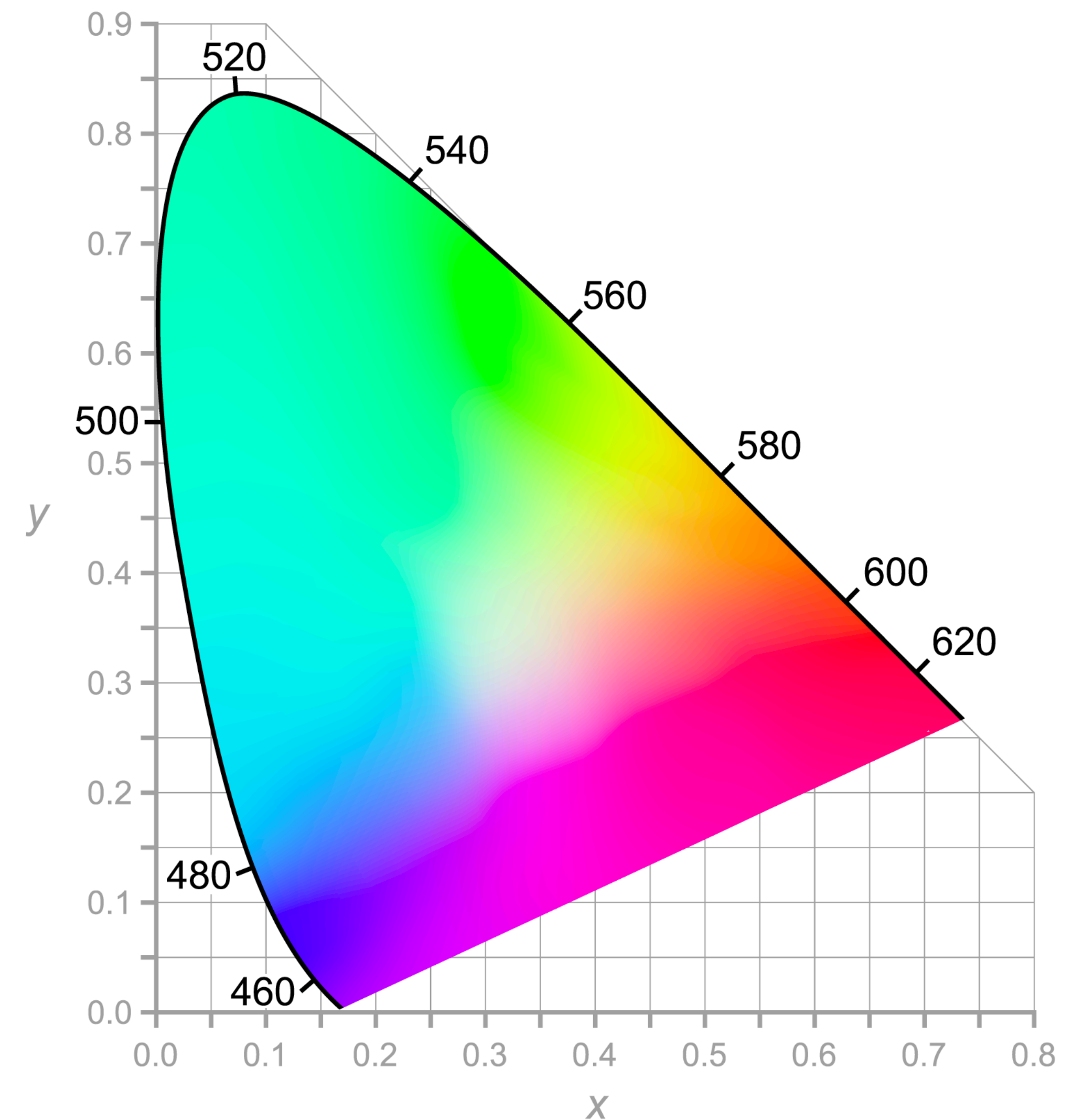


# Typical Visualization

It's a lie.

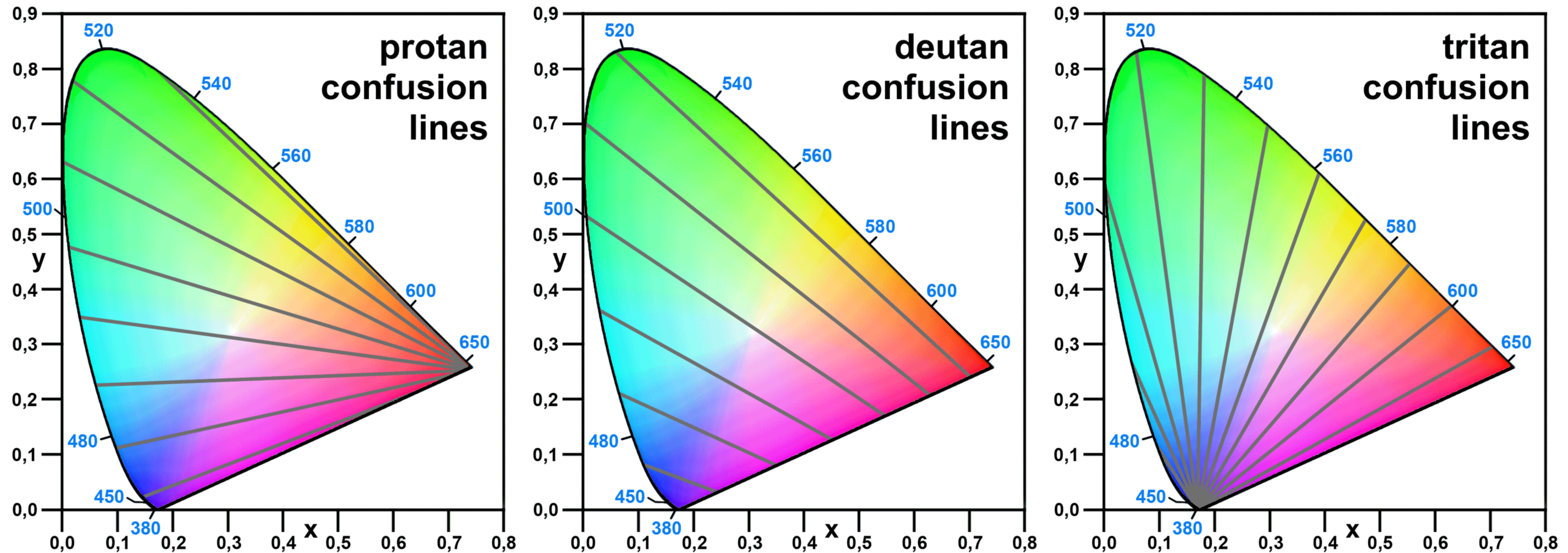
Technically we shouldn't color the entire gamut, since no display can display all those colors. Usually people find the closest displayable colors to show there.

Colors at the center are "white-ish"; they are mixed from relatively equal amount of other colors. More on white colors later.





# Lines of Confusion in Chromaticity Diagram



Lines of confusion are parallel to each other in the LMS/XYZ space, but converge to the same point in chromaticity diagram. Why?

# **Color Cube (a.k.a., Build Your Own Color Space)**

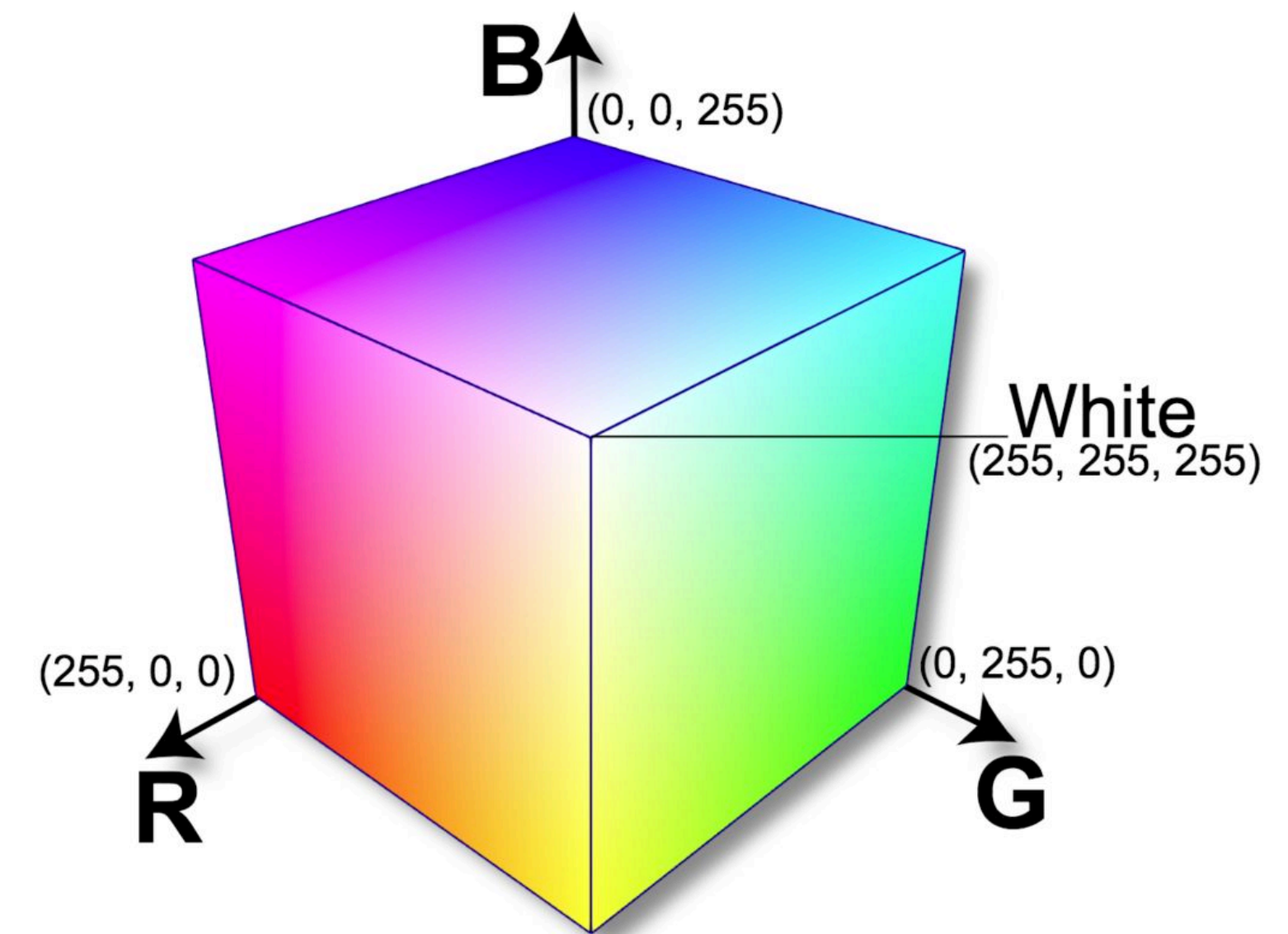
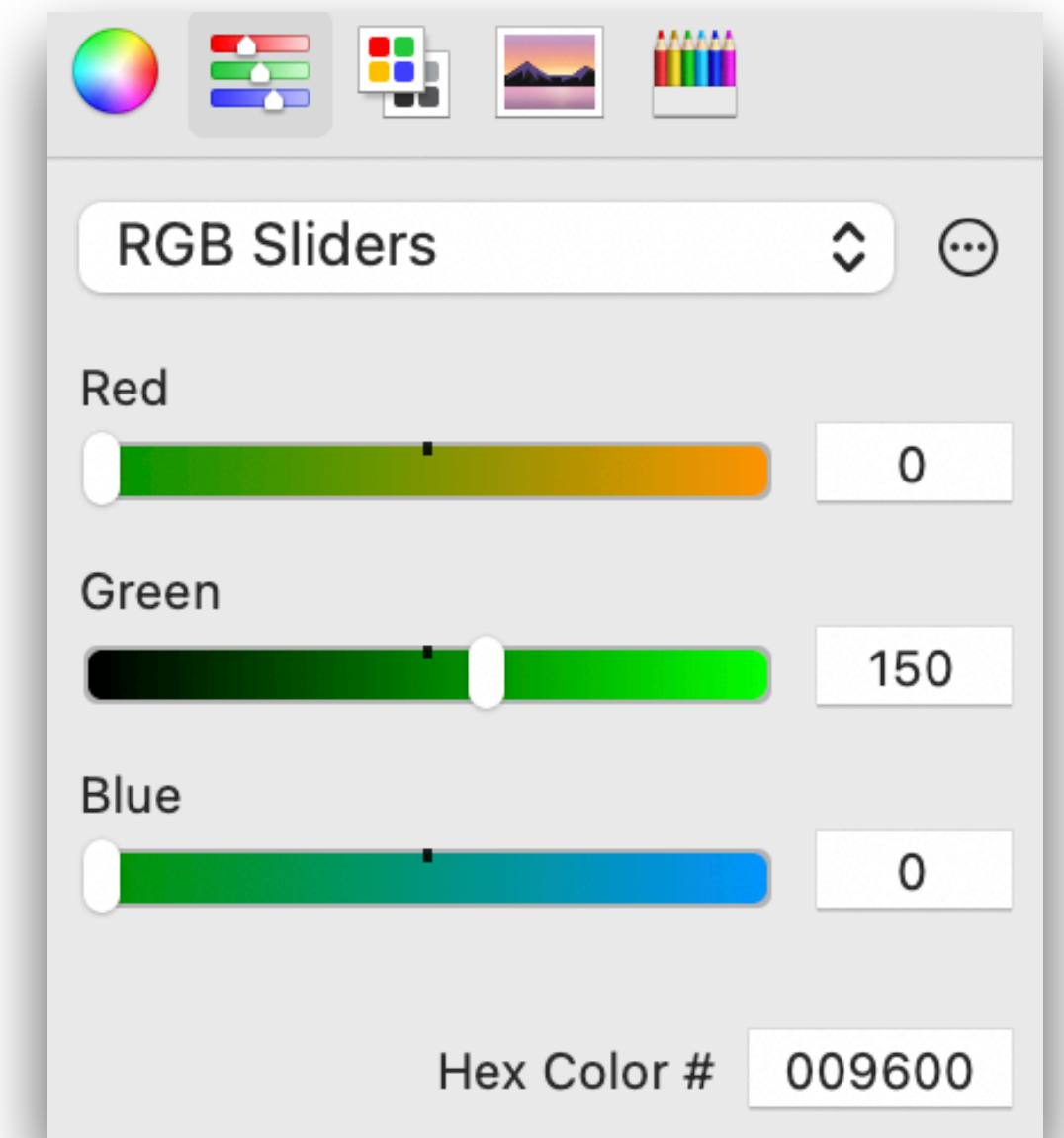


# What is a Color Cube?

In usual graphing applications we specify color by three integers, each between 0 and 255, or as a six-digit HEX value. For instance:

- Black [0, 0, 0] / 0x000000
- White [255, 255, 255] / 0xFFFFFFFF
- Red [255, 0, 0] / 0xFF0000
- Some sort of gray [146, 146, 146] / 0x929292

Intuitively, it allows us to specify a color as a mixture of three primary colors. The numerical value represents how much of a primary color we want.



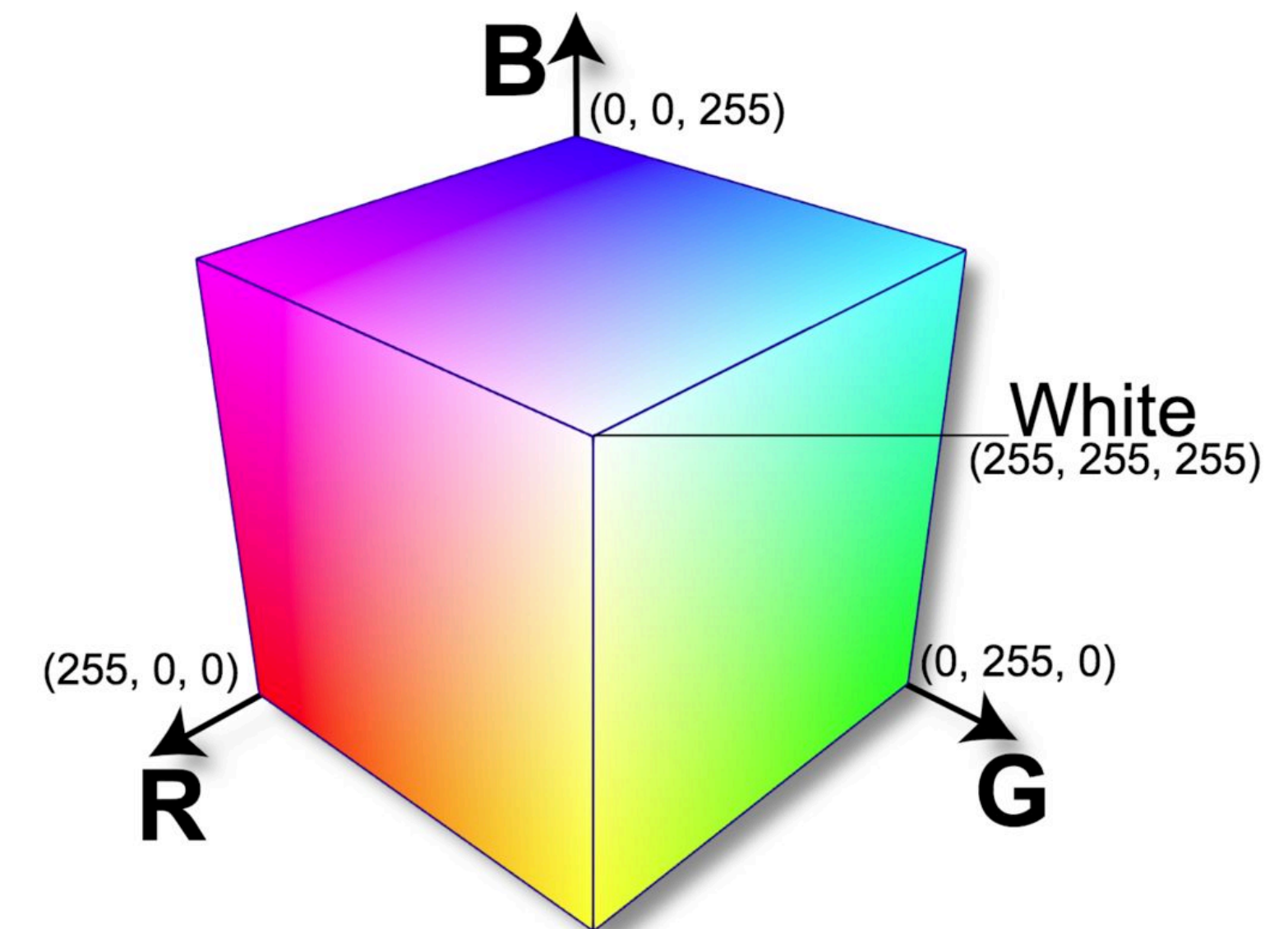
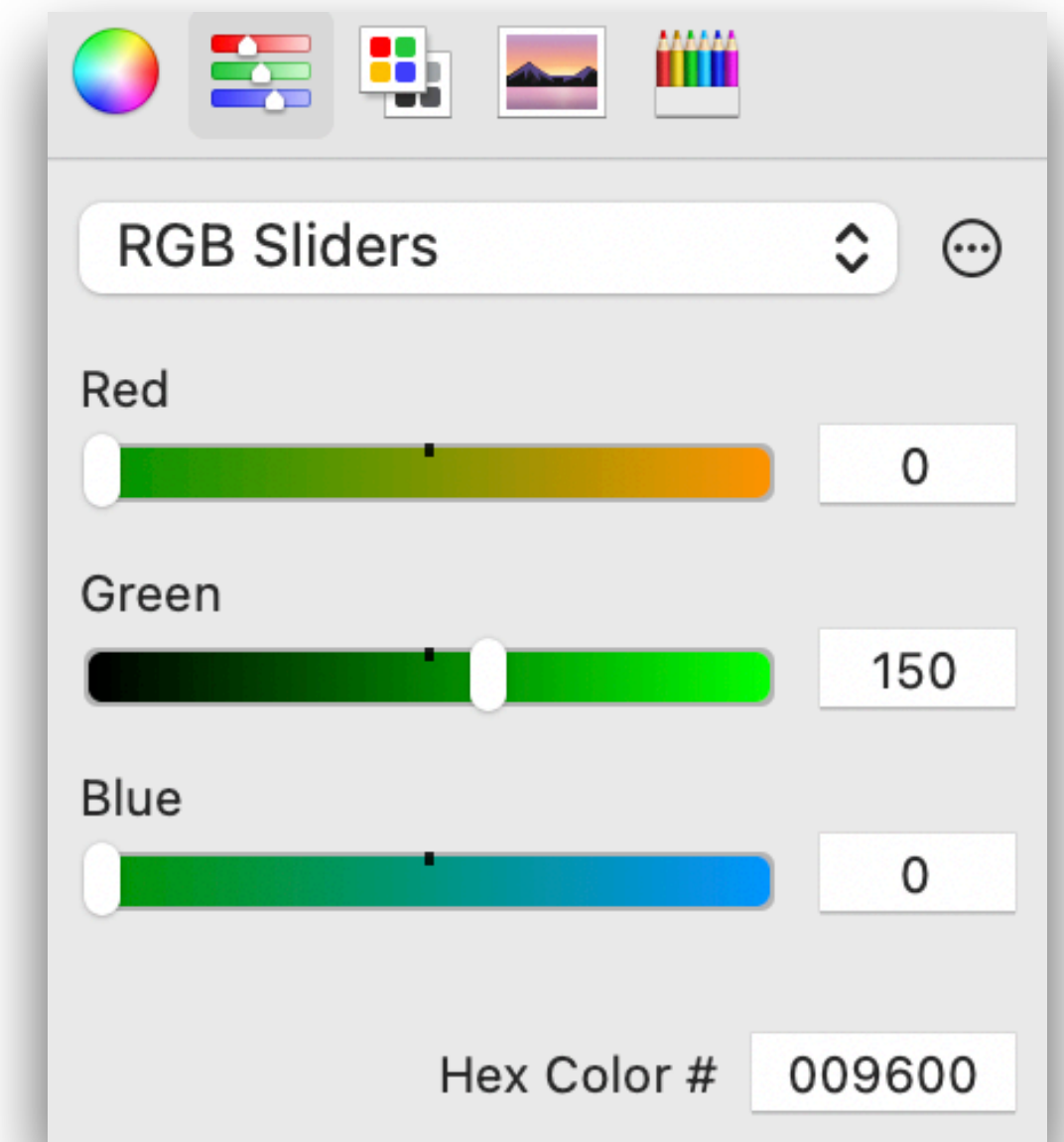
# How to Build a Color Cube?

We need to define the color space first.

- Surprisingly, we need not only the three primaries, but also the white point.

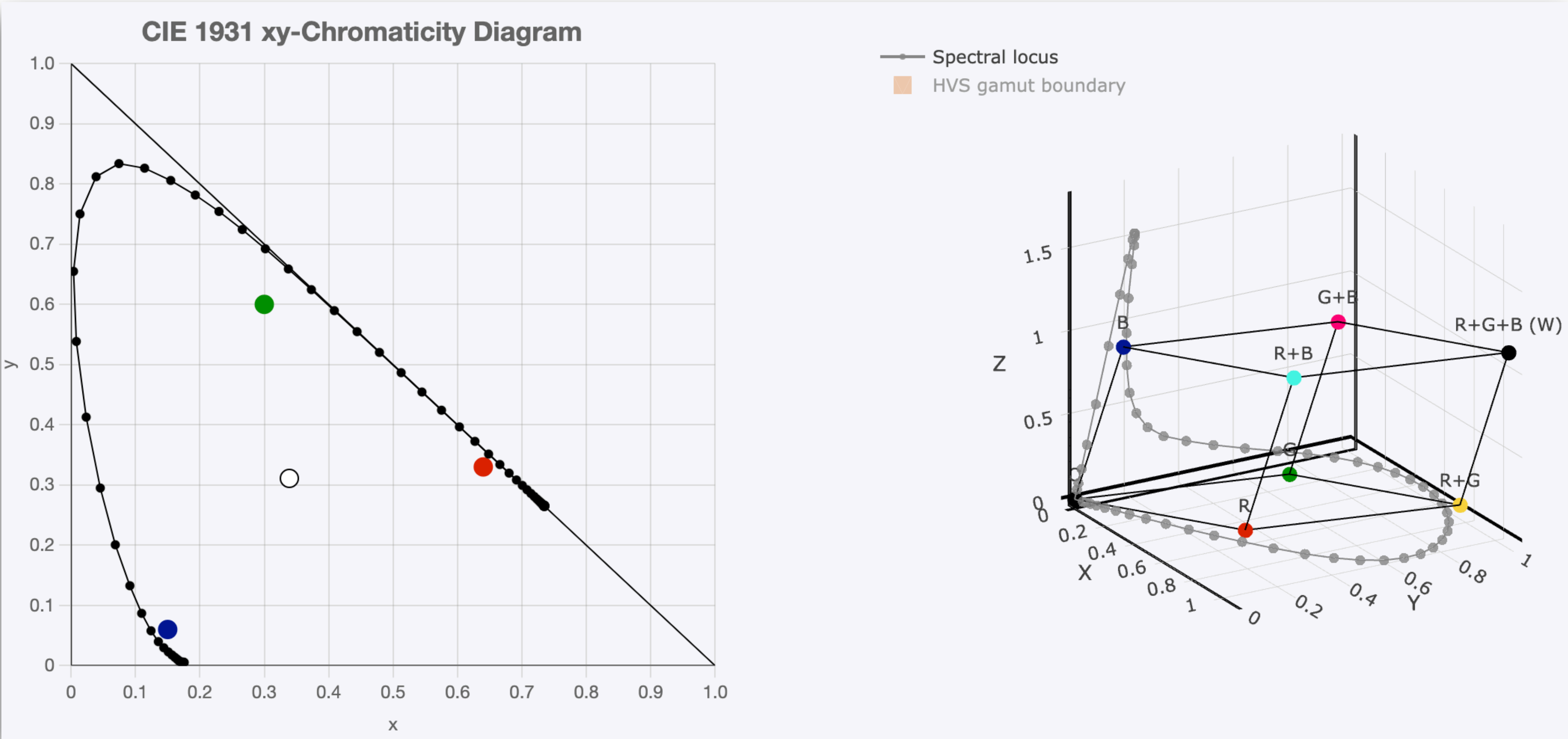
We will then figure out how to **quantize** the color space: there are infinitely many colors in a color space, but we can afford to represent only  $2^{24}$  colors.

- Assuming 8 bits per primary (color channel).
- There are encodings that use more bits per channel.



# Intuition

<https://www.cs.rochester.edu/courses/572/colorvis/colorcube.html>





# What is White?

**Intuitively:** equal amount of maximum primary lights in a color space is white.

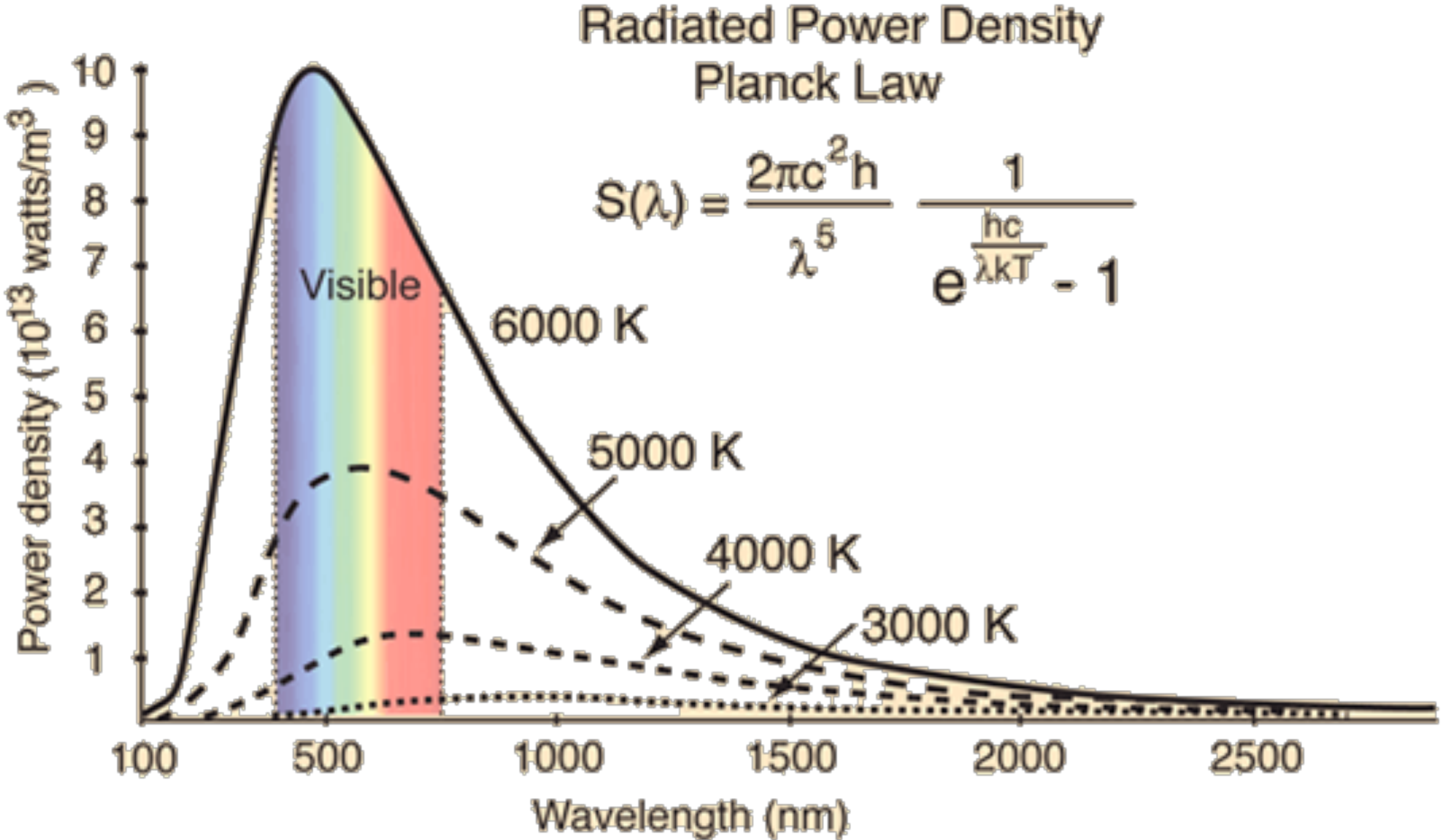
- So the notion of "white" is specific to a color space.

**Scientifically:** white refers to colors from **black-body radiation** under different temperatures.

Many colors closer to colors of black-body radiation can be chosen as the white point of a color space.



# Black Body Radiation

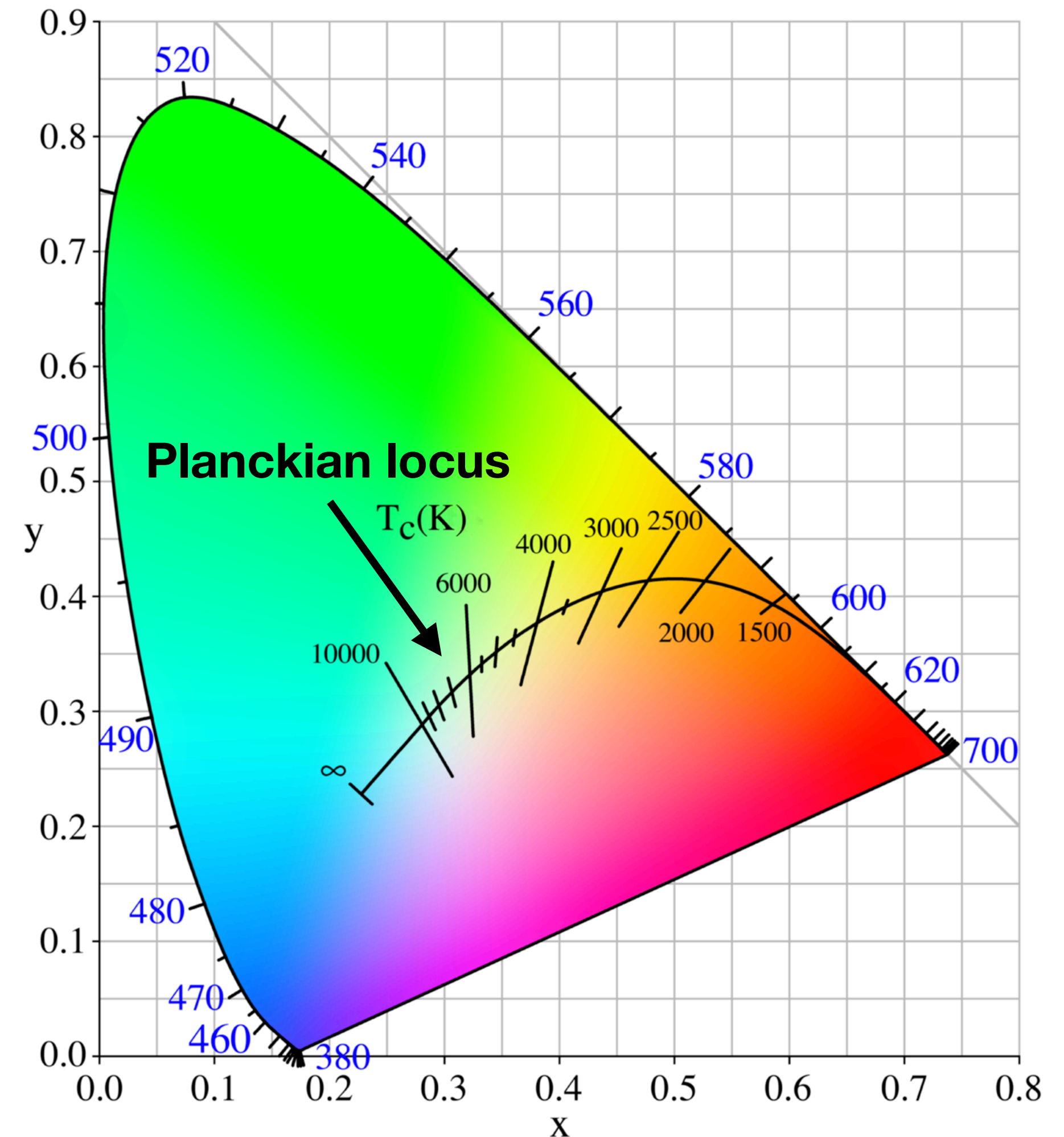
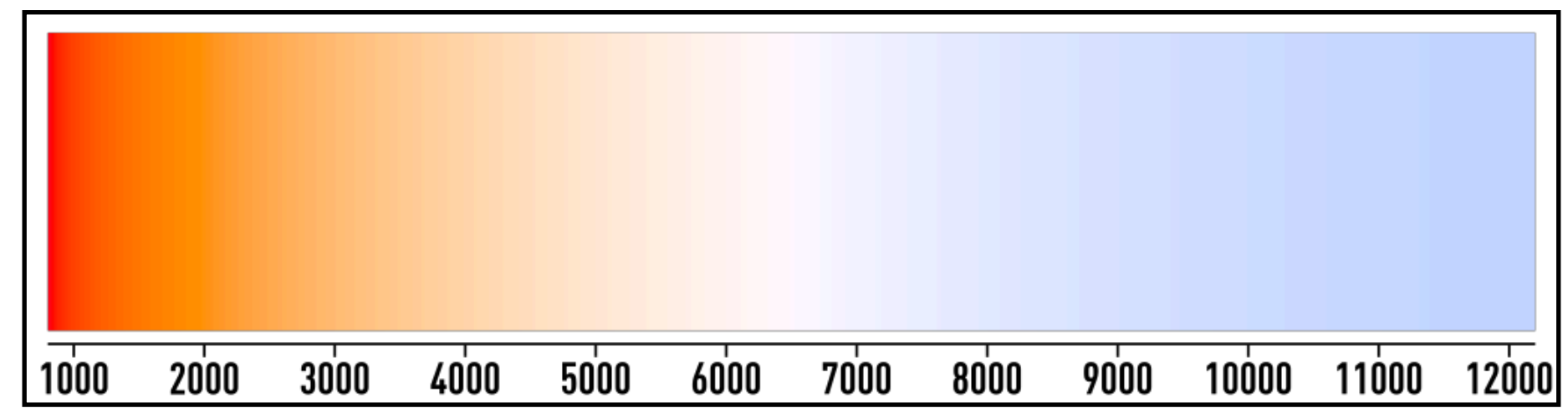




# Color Temperature

The colors of black body radiation under different temperatures form a curve when plotted in the xy-chromaticity diagram. It's called the Planckian locus.

A color on the locus has a color temperature.



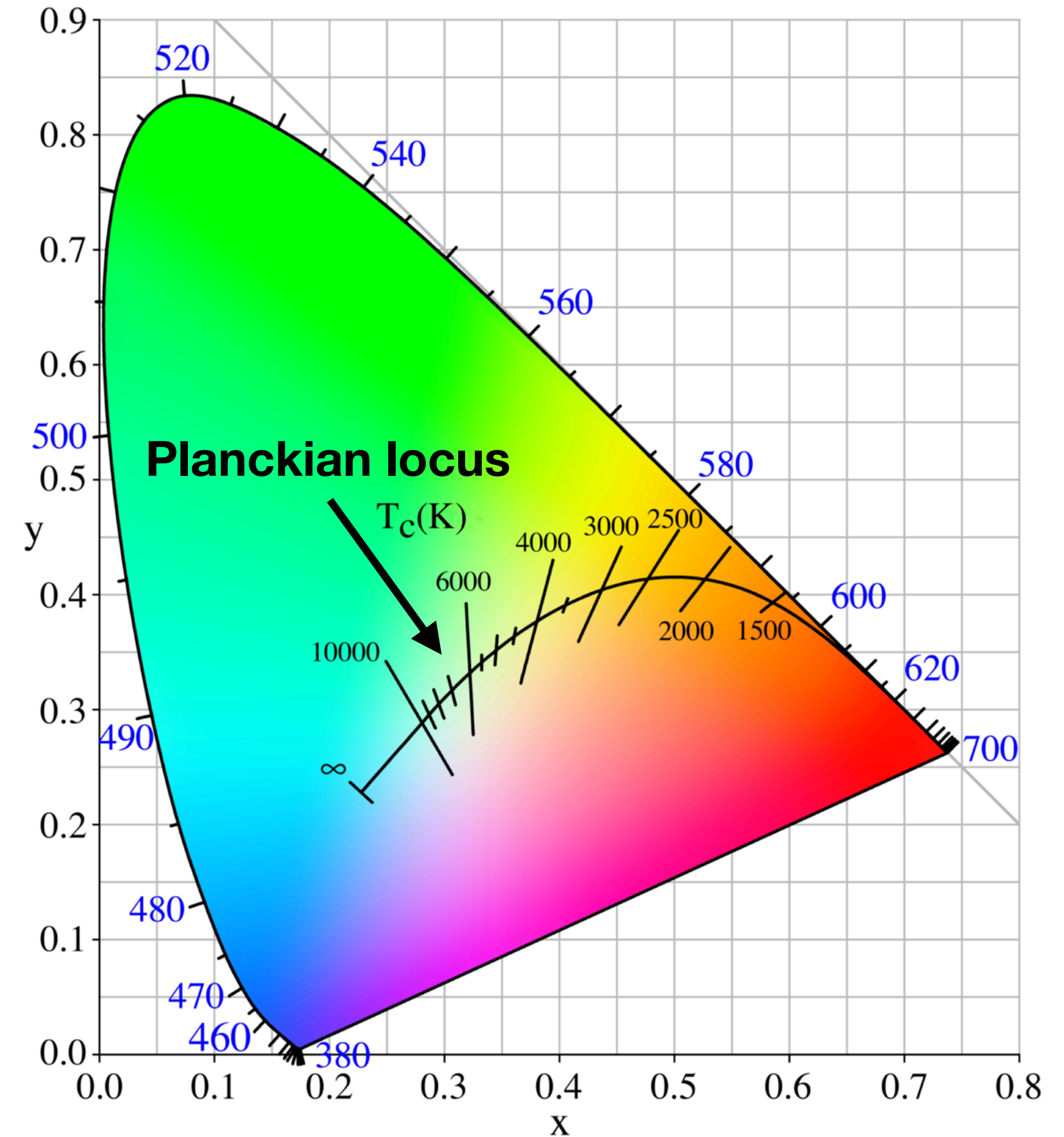
# Correlated Color Temperature (CCT)



← CCT →  
"Warm" "Cool"

Colors of actual lights closer to a color of black body radiation can be assigned a *correlated* color temperature.

The two are (mis)used interchangeably.





# CHOOSING THE RIGHT COLOR



## Soft White, Warm White

The standard color of incandescent bulbs.



## Neutral White, Cool White

Good for kitchens and work spaces.



## Daylight

Good for reading.



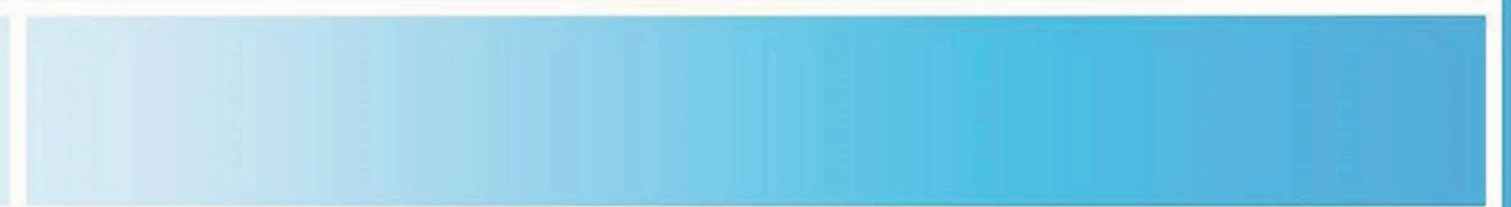
2200K

3000K



3500K

4100K

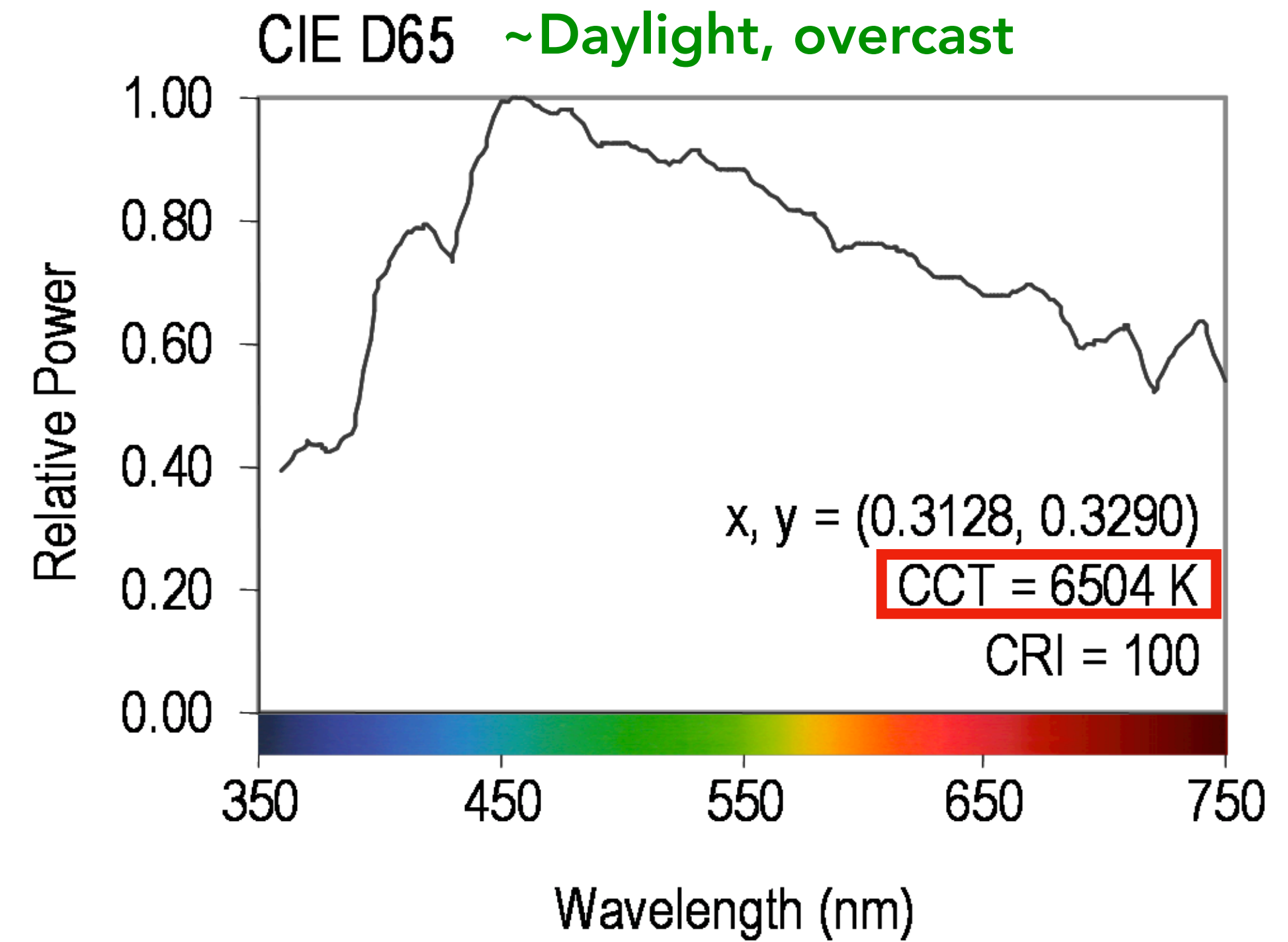
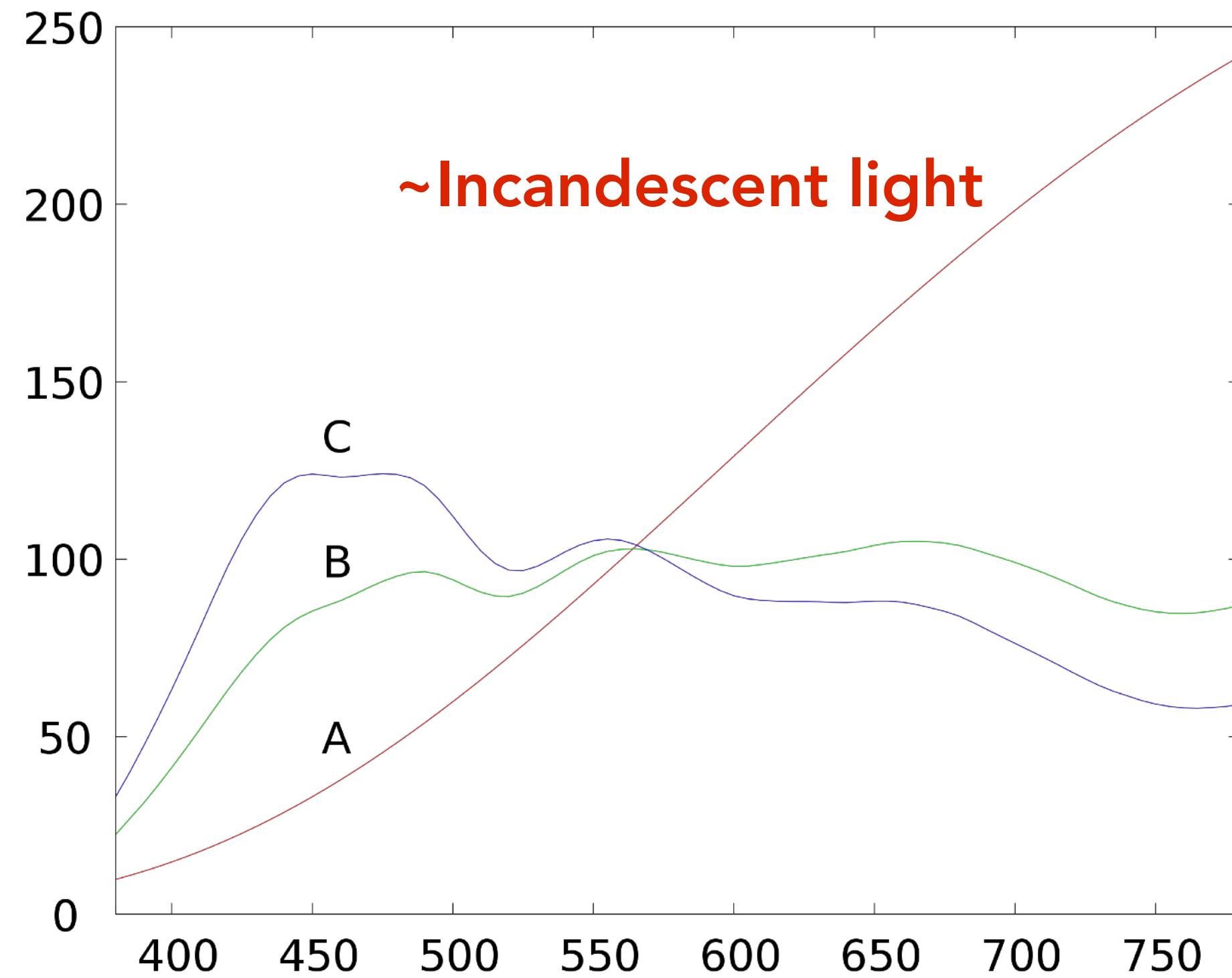


5000K

6500K



# CIE Standard Illuminants



# Color Spaces in Chromaticity

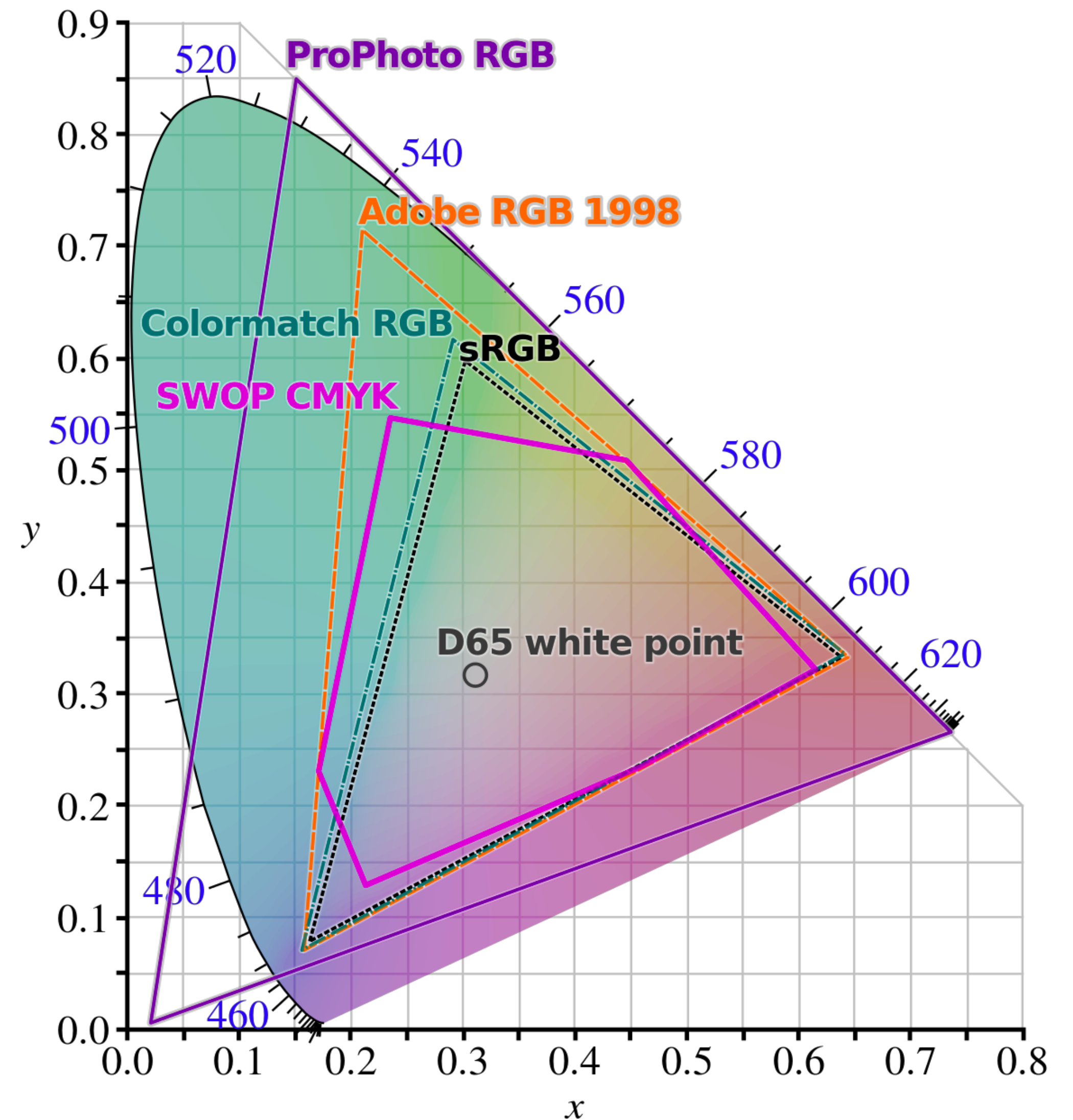
D65 is the most commonly used white.

sRGB is the most common color space, but its gamut is very small.

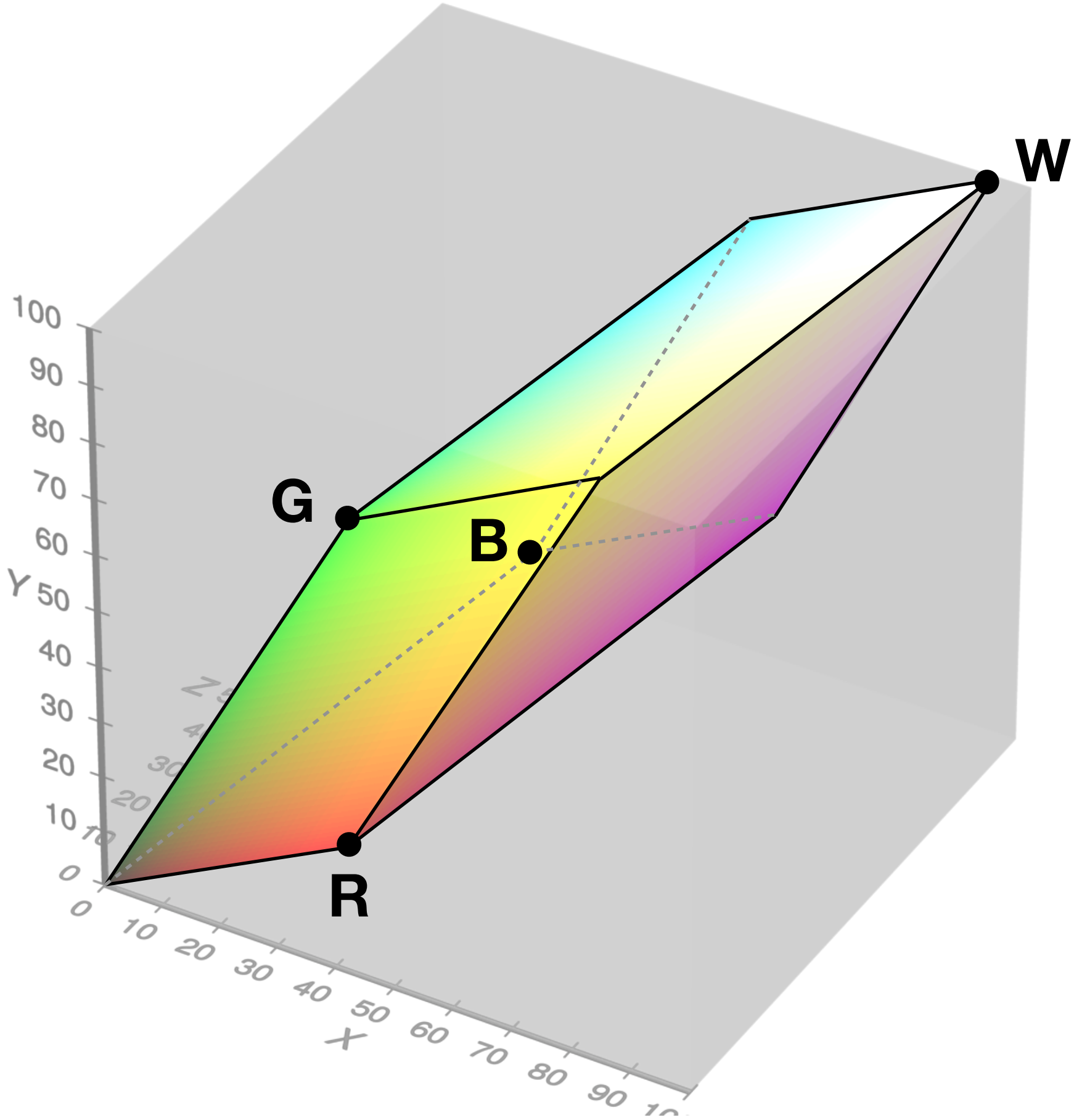
CMYK is used in printers. More later.

ProPhoto RGB is a conceptual color space; its primaries are not real colors.

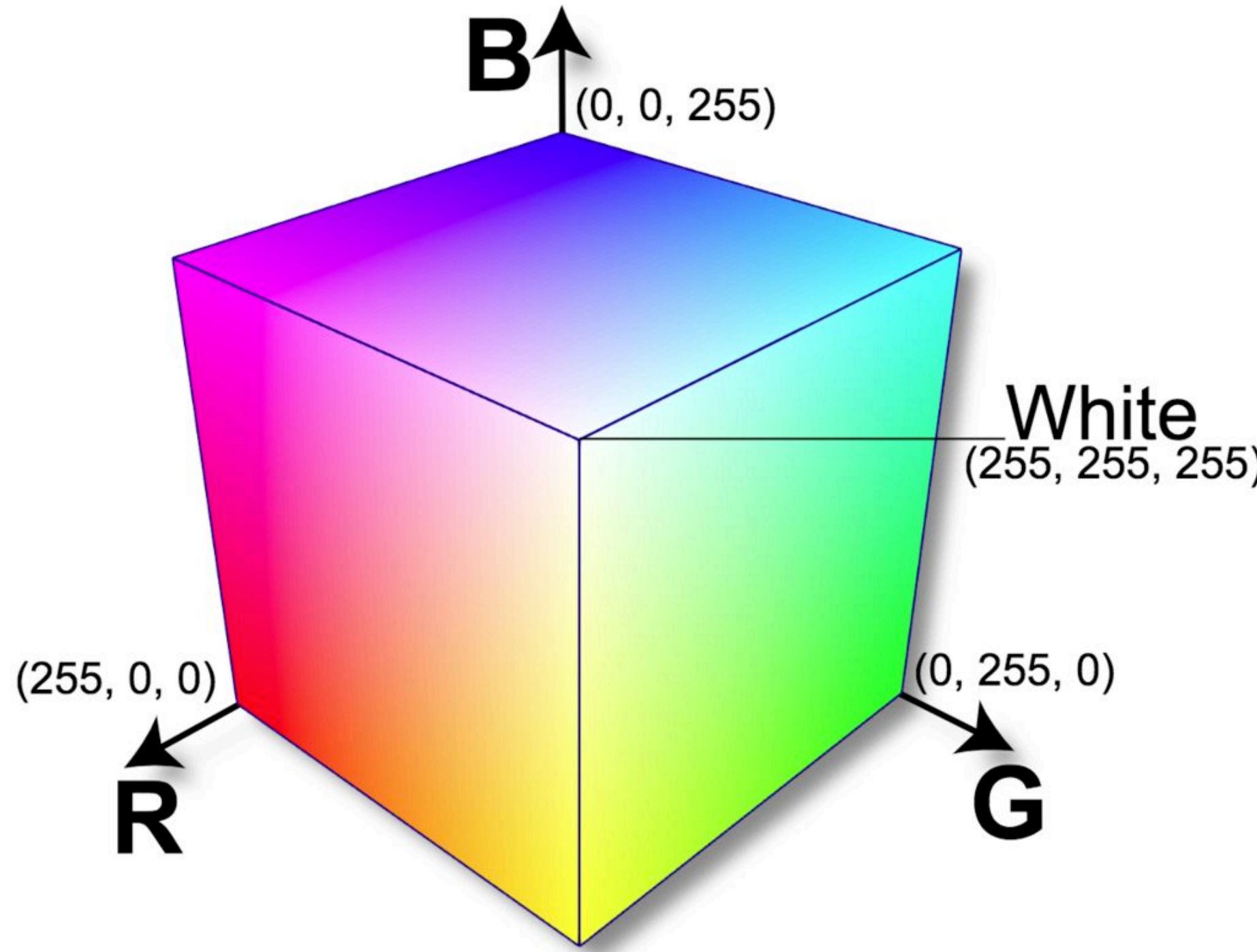
**All RGB color spaces are just one linear transformation away from each other.**



# Map the Gamut to a Cube



Another geometric transformation

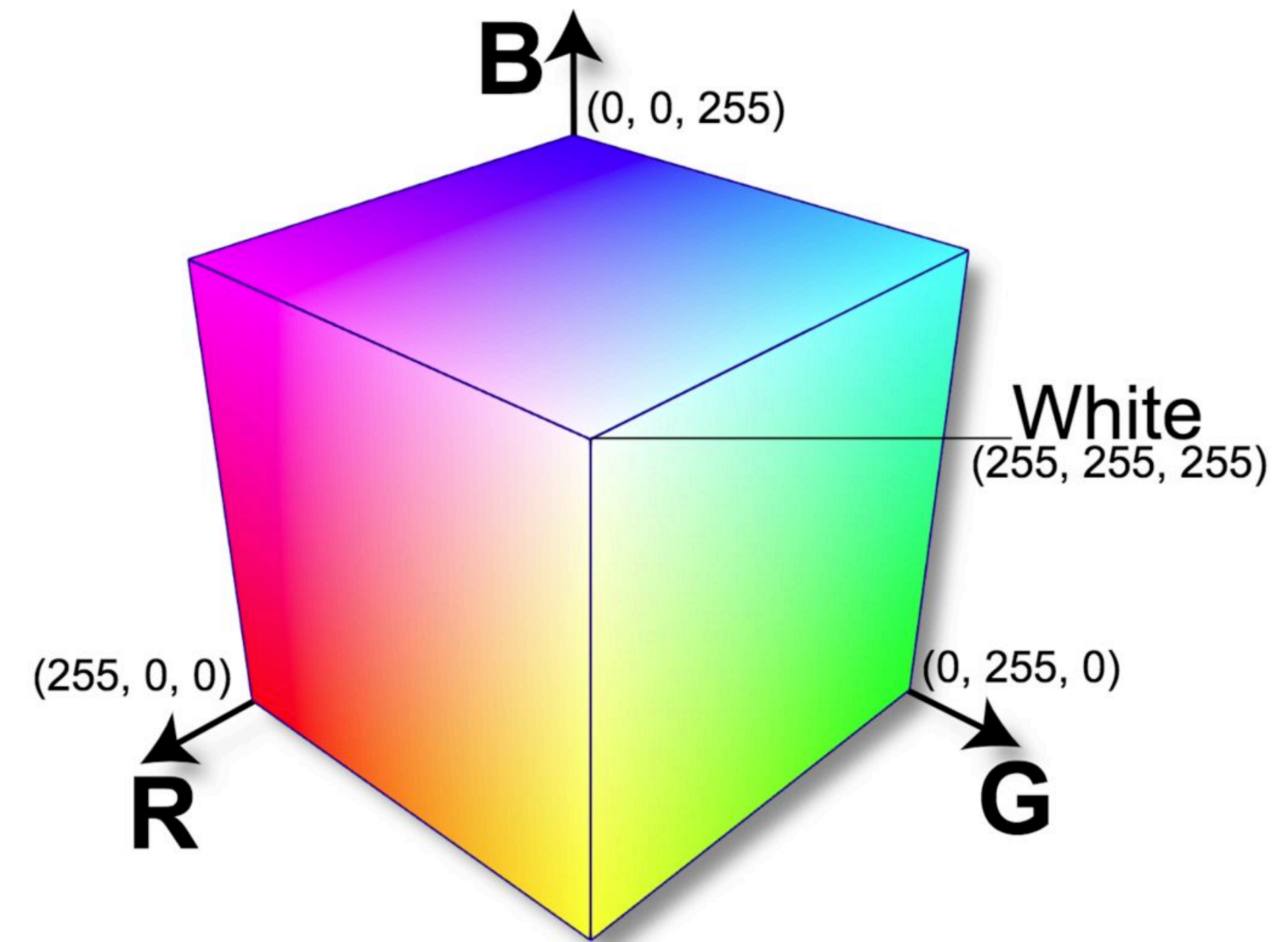




# Quantize the Color Cube

The color space so far is continuous, but we have to discretize/**quantize** the space since we have limited bits. Popular sRGB color space uses 8 bits per primary. That is, we can represent at most  $2^{24}$  colors.

Even quantization? Uneven? What's the criterion?



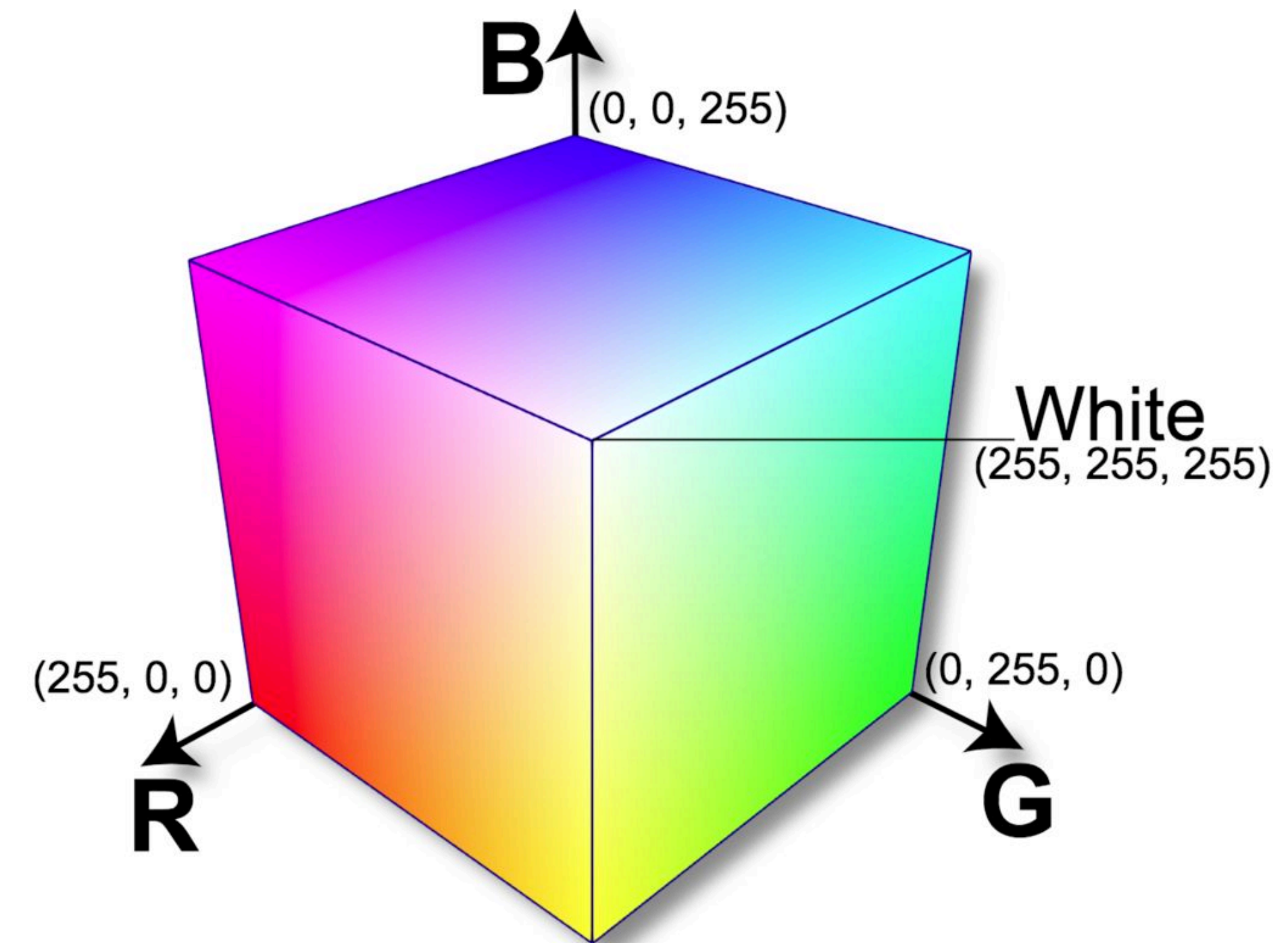
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Even quantization? Uneven? What's the criterion?

Ideally, we want to evenly divide the brightness.

- Increasing **R** from 0 to 1 should add the same amount of perceived redness as increasing **R** from 240 to 241.





# Quantize the Color Cube

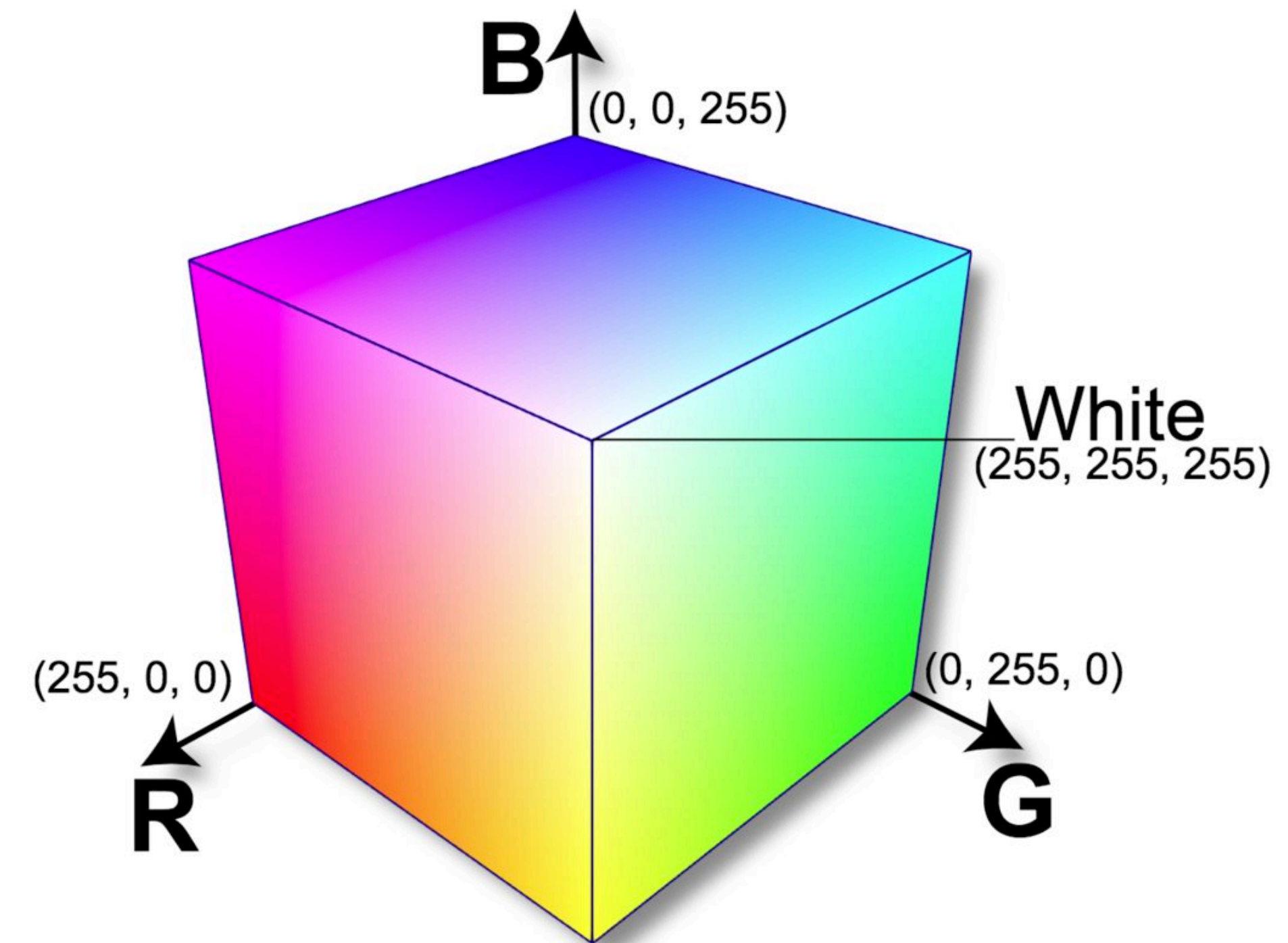
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Ideally, we want to evenly divide the brightness.

- Increasing **R** from 0 to 1 should add the same amount of perceived redness as increasing **R** from 240 to 241.

Does our perceived brightness increase uniformly with light power?



# Brightness

# Thought Experiment

Compare the following two scenarios:

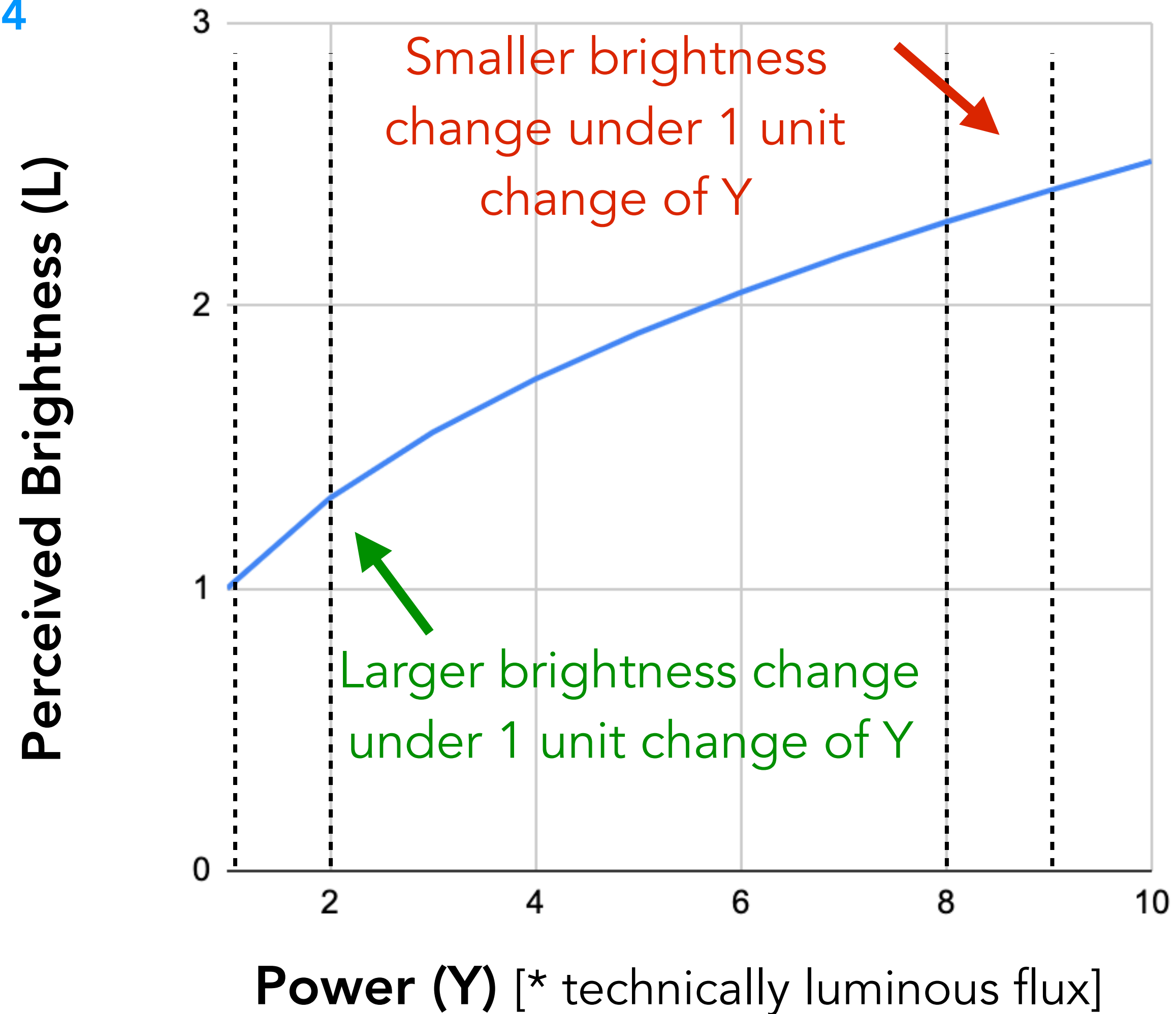
- Walking into a completely dark room and turn one light on
- Walking into a room with 10 lights already on and turn on one more light

In which scenario would you feel a stronger brightness increase?

Brightness vs. power relationship isn't linear.

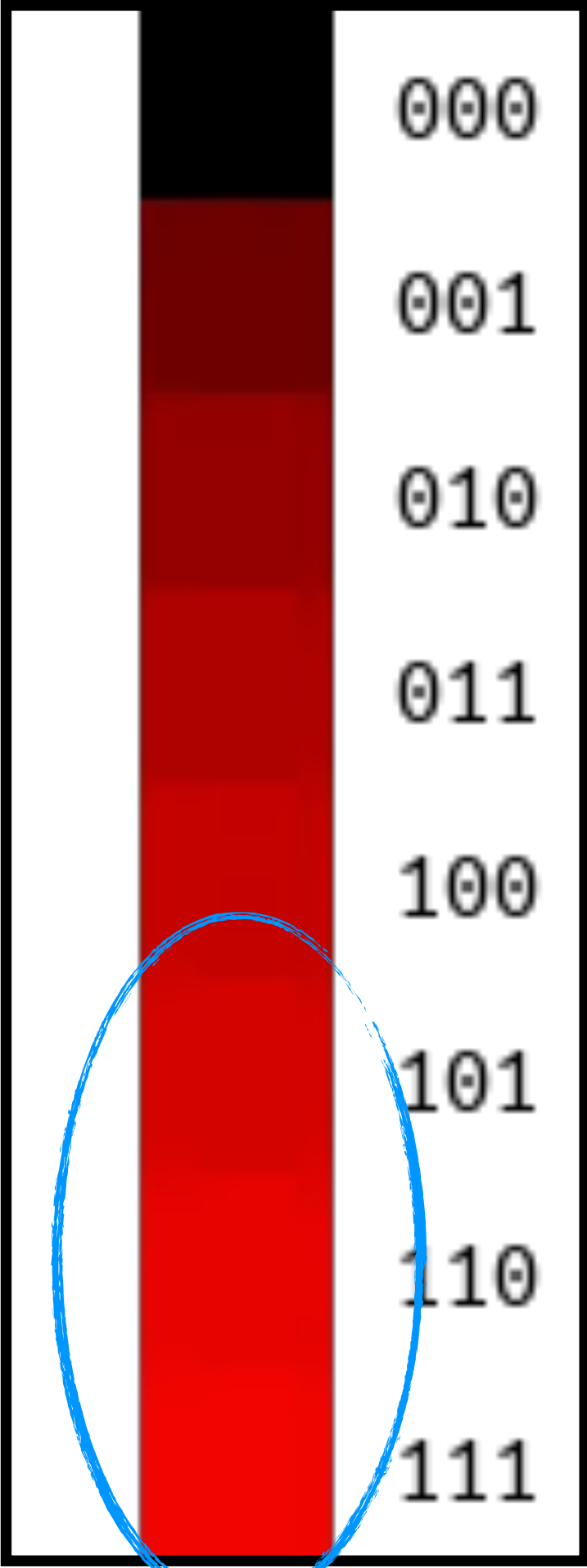
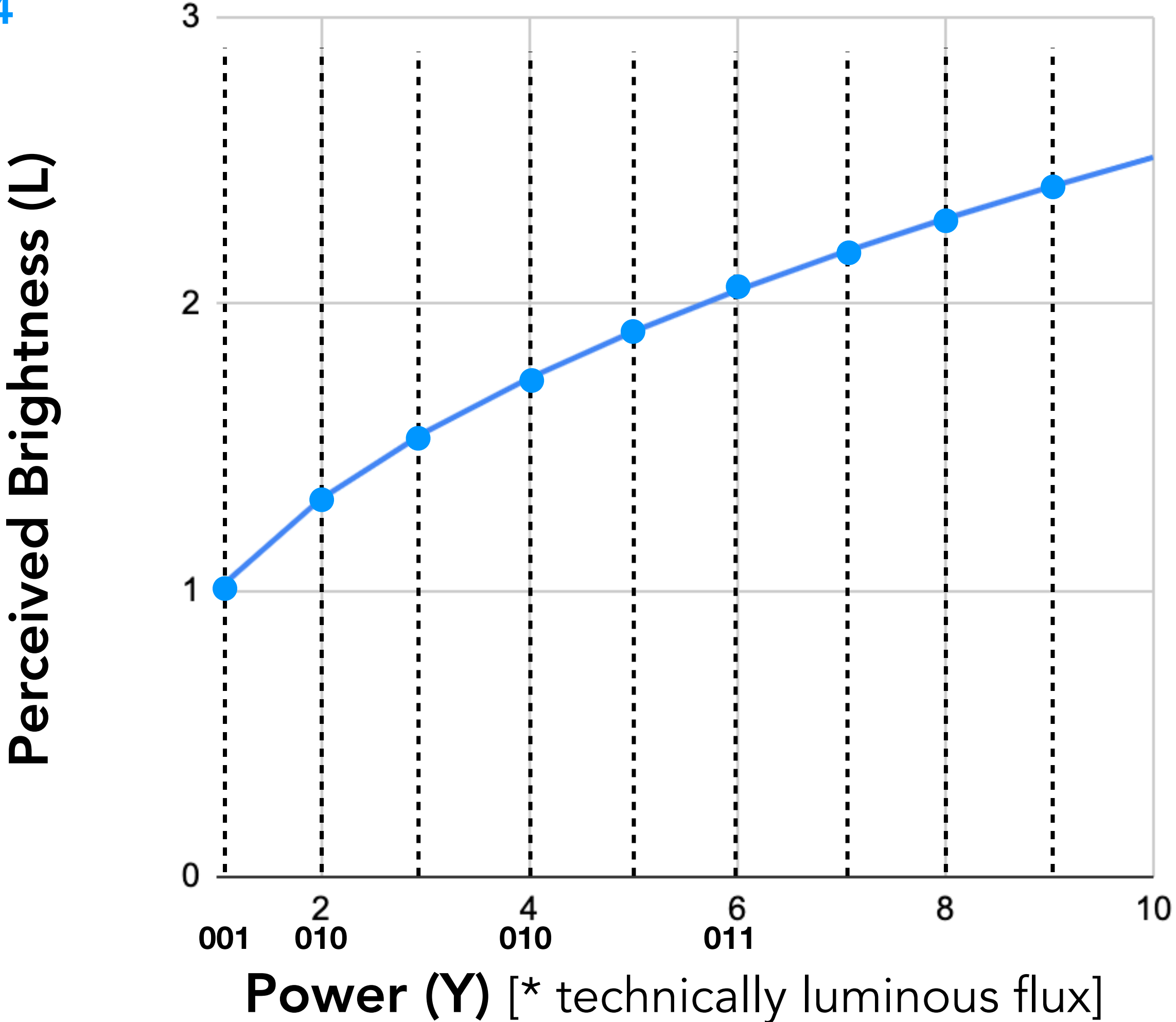
# Power vs. Perceived Brightness

$$L = Y^{0.4}$$



# Encoding Power Evenly

$$L = Y^{0.4}$$



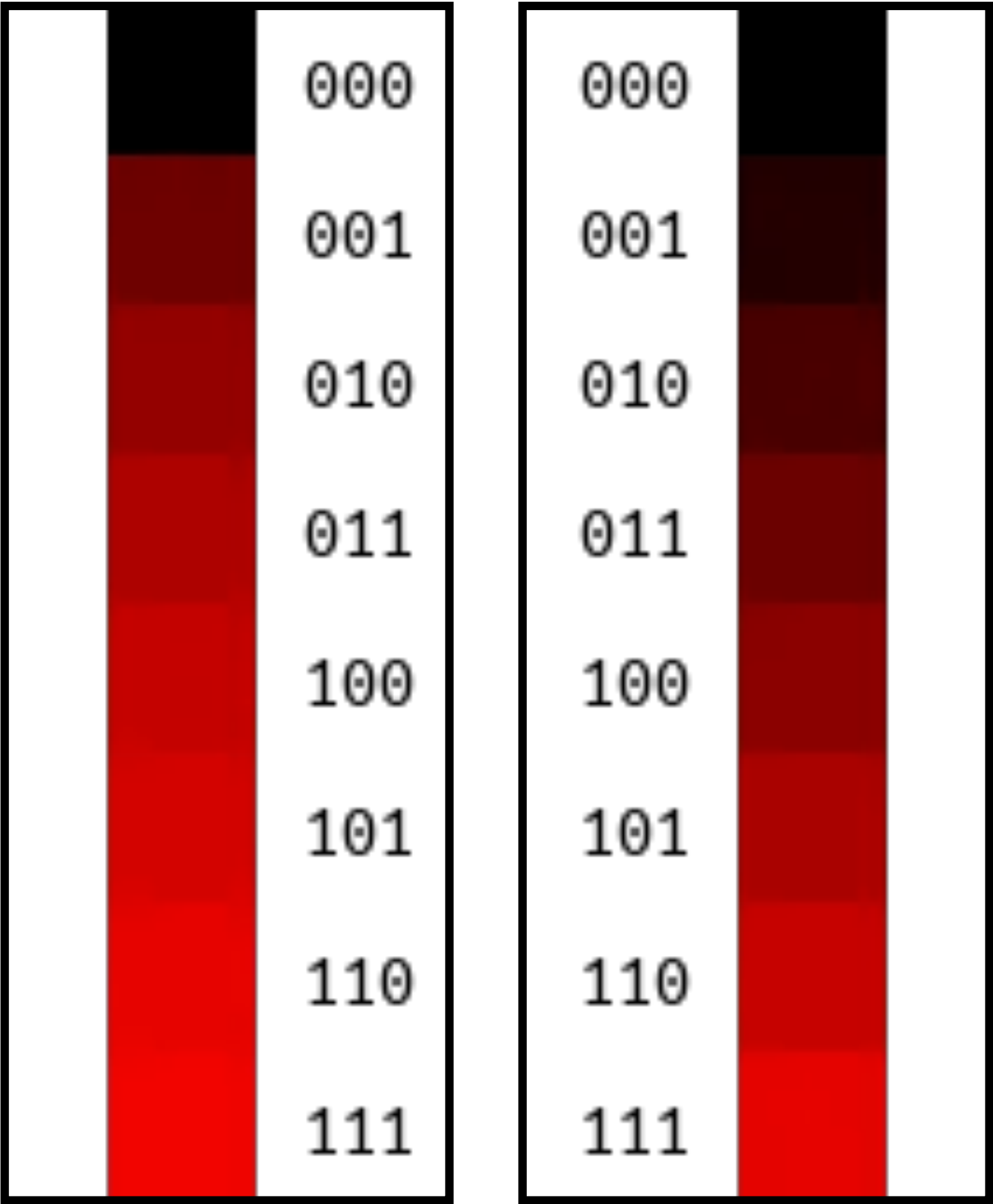
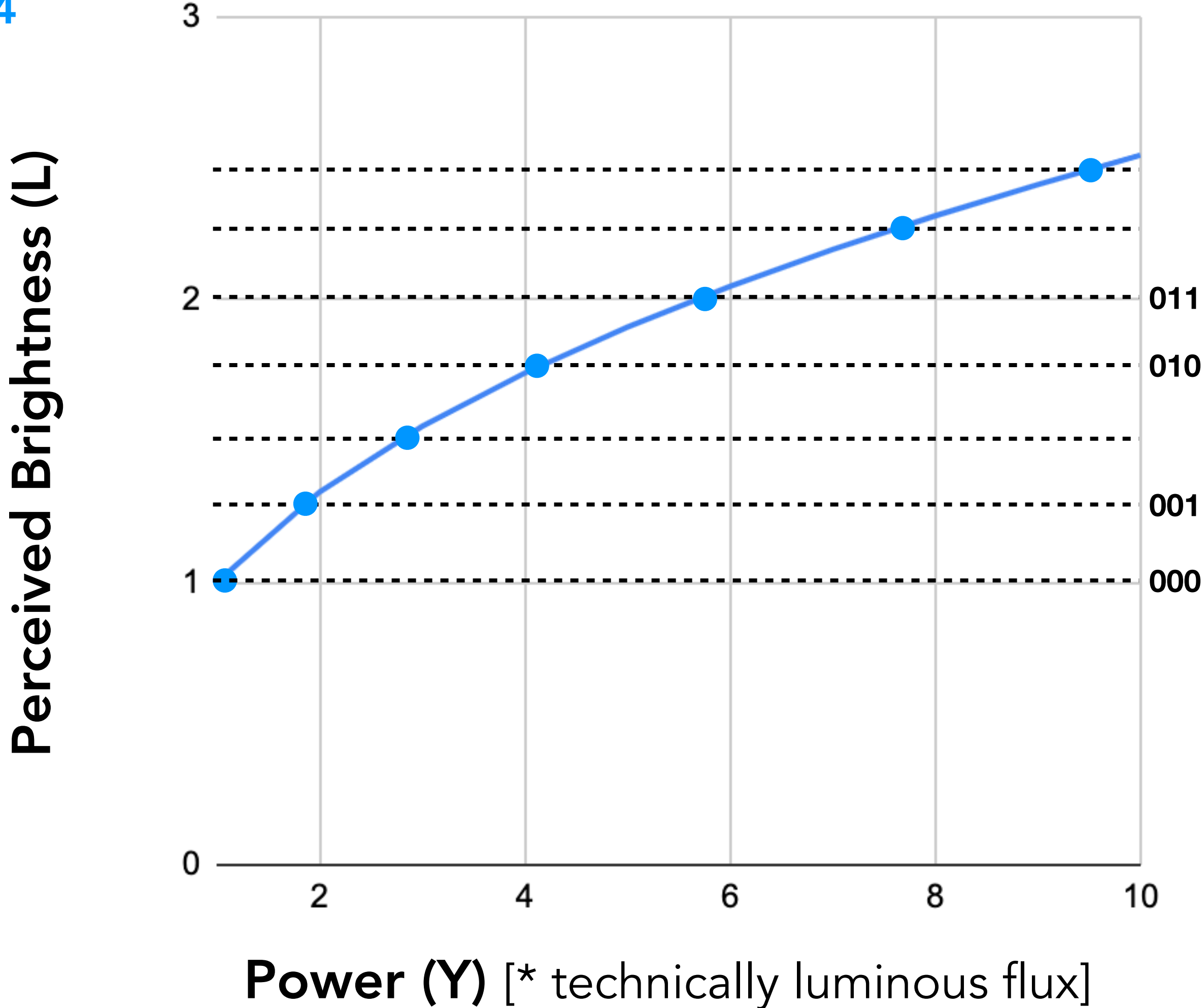
Undistinguishable in the high luminance region



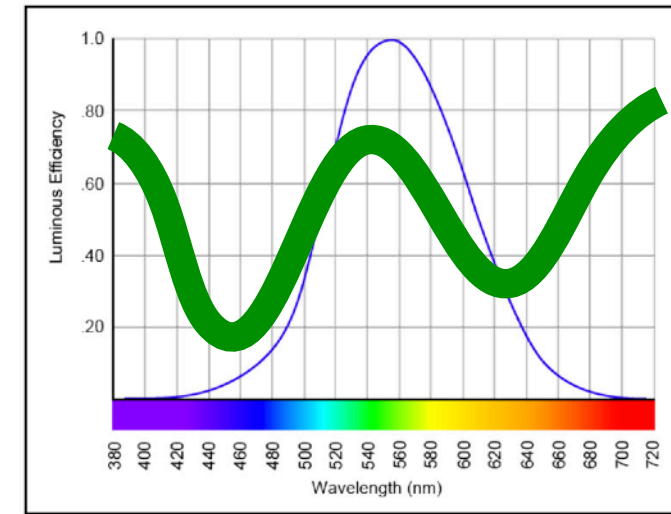
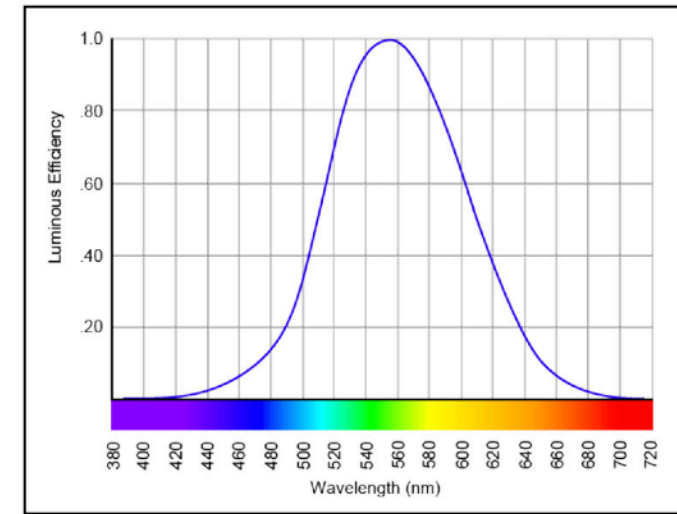
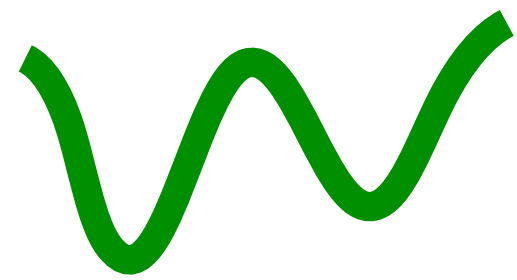
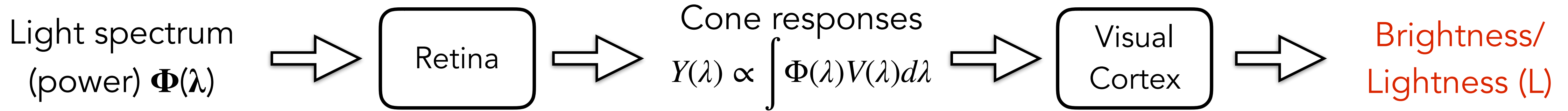
# Encoding Brightness Evenly

Perceptually uniform

$L = Y^{0.4}$



# Brightness/Lightness of a Light



Luminous efficiency function

$$L = Y^{0.4}$$

The logarithmic relationship between subjective sensation and objective physical stimulus intensity is called the “Weber–Fechner law”.

- Pain, hearing, taste, etc.

# Gamma Color Encoding

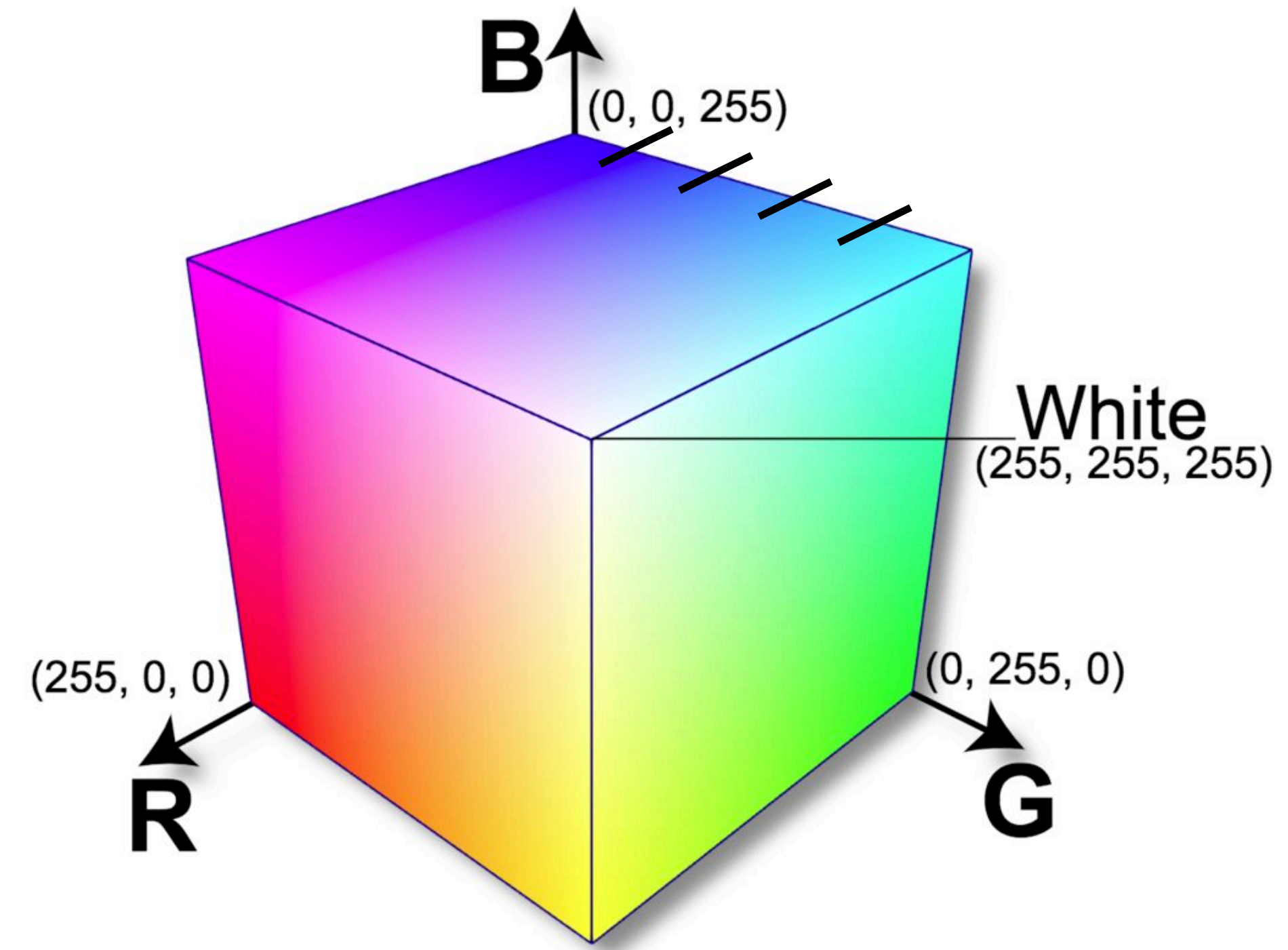
Assuming after linear transformation,  
R/G/B are in the [0, 1] range

$$R_{8bit} = \lfloor R^{1/2.2} \times 255 \rfloor$$

$$G_{8bit} = \lfloor G^{1/2.2} \times 255 \rfloor$$

$$B_{8bit} = \lfloor B^{1/2.2} \times 255 \rfloor$$

Gamma used by sRGB



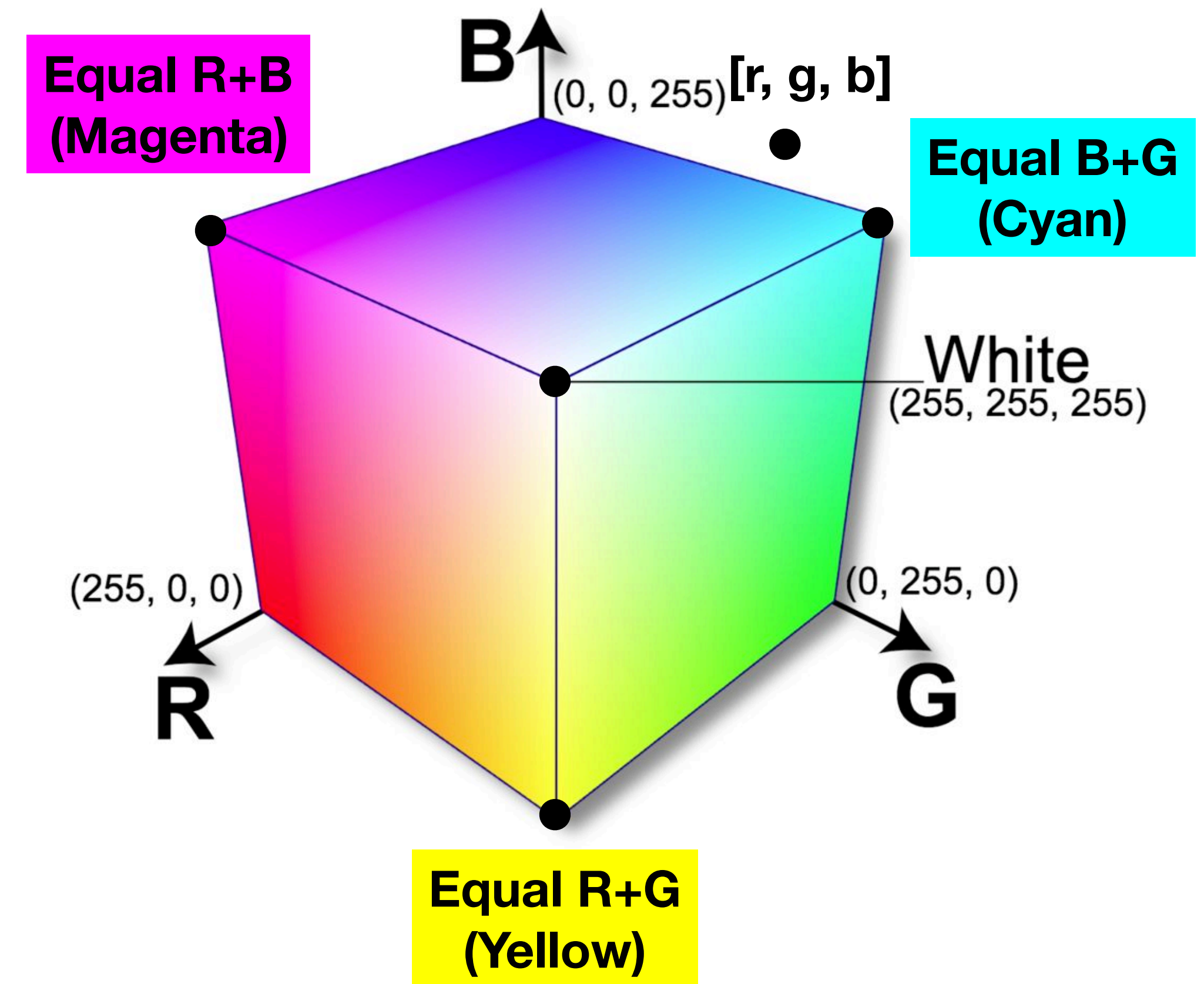
\* sRGB actually uses a piece-wise gamma

# Color Cube

Colors on vertices have equal mix of 2 or 3 primaries with maximum quantity.

Colors outside of the cube are either:

- physically unrealizable or
- realizable but can't be produced from the chosen primaries.



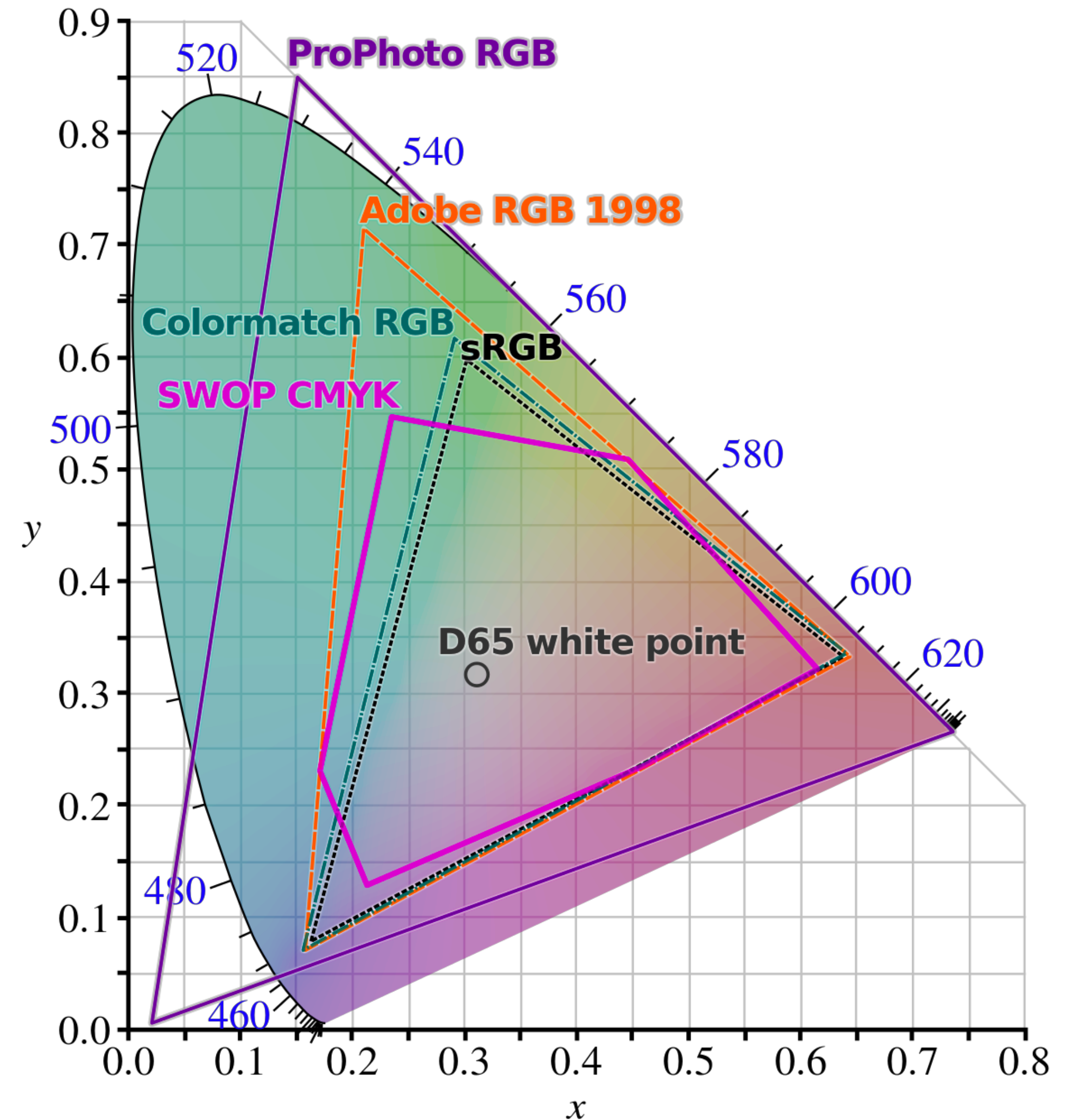
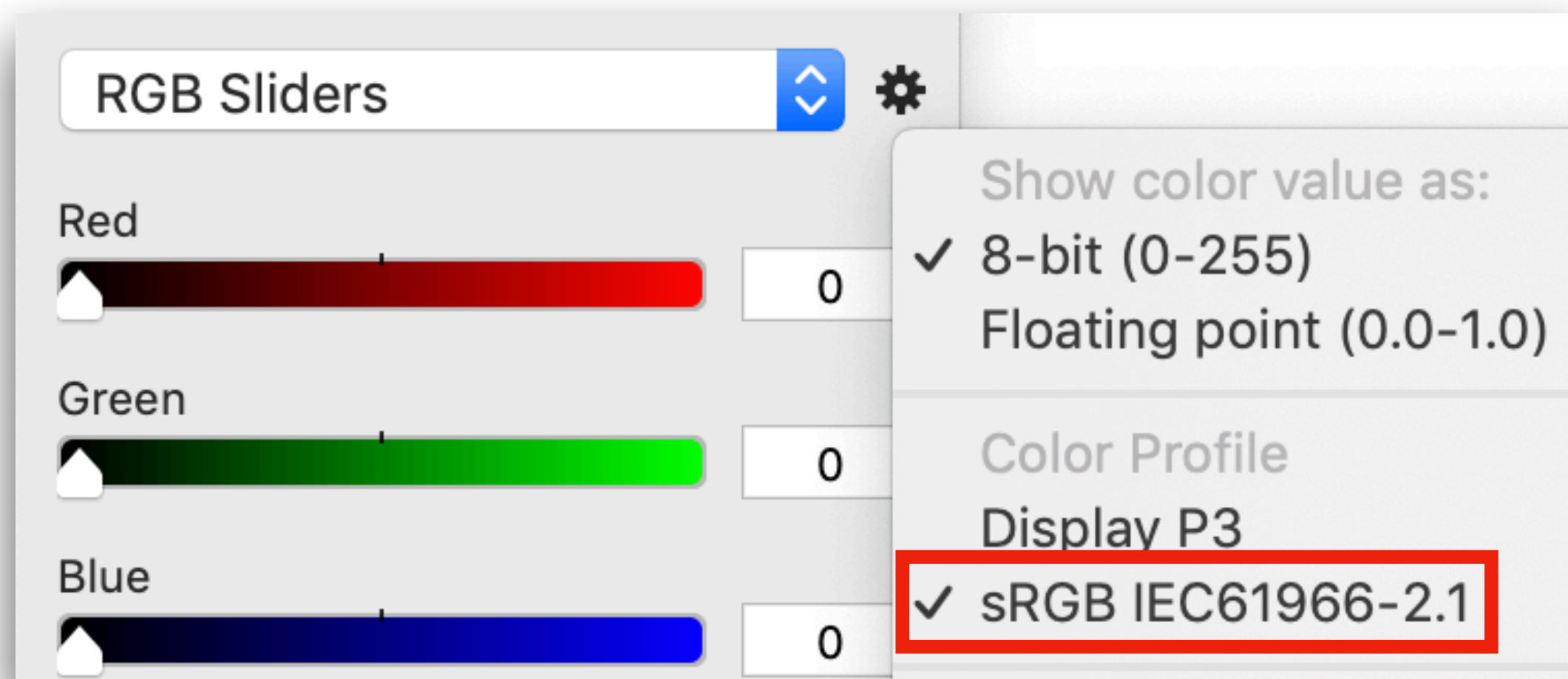
# A Tour of Color Spaces



# RGB-based Color Spaces

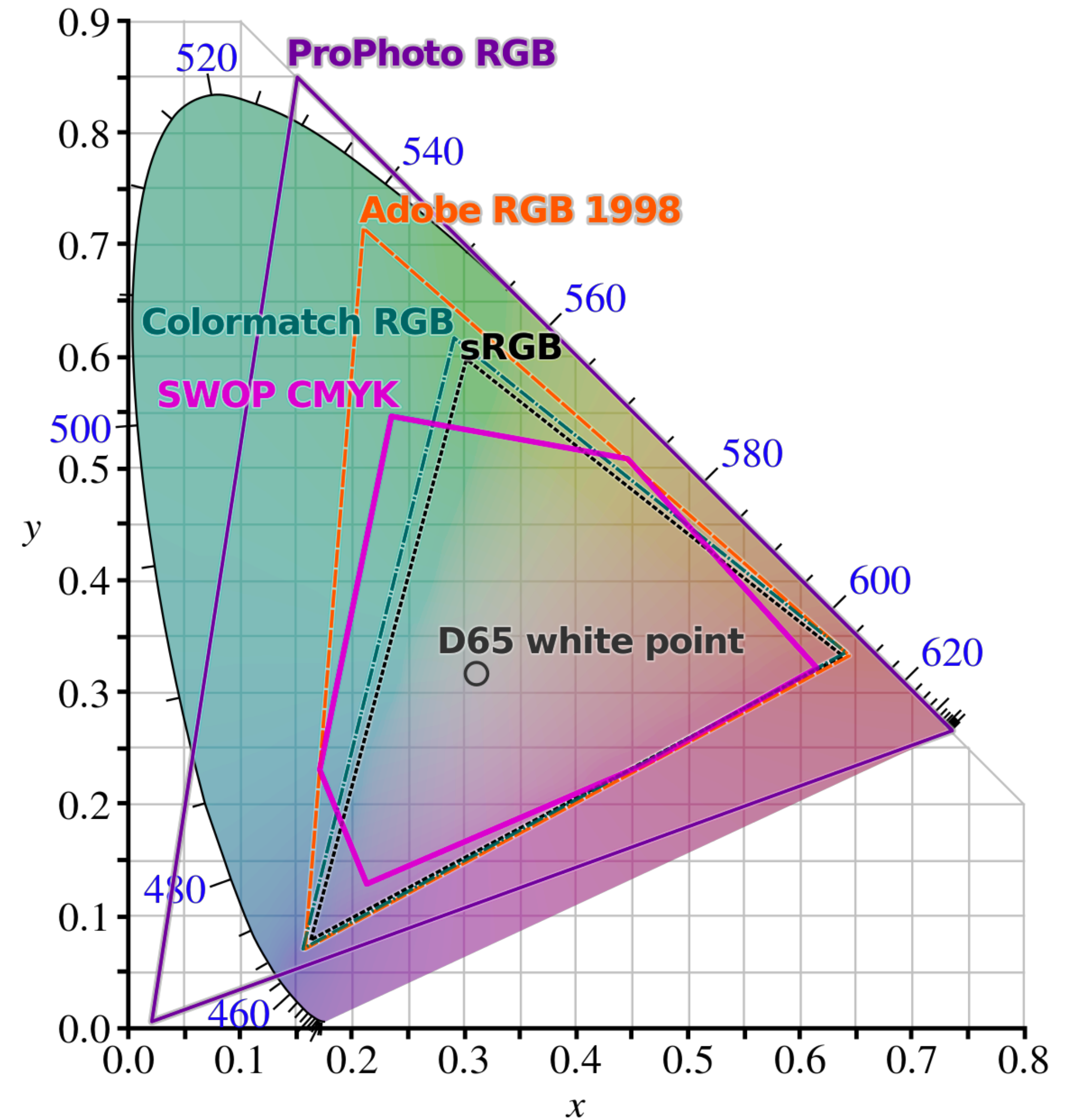
sRGB is what we commonly refer to as the RGB space in the real world.

Adobe RGB has a wider gamut.



# ProPhoto RGB

ProPhoto RGB has a wide gamut, but ~13% of is imaginary. Two of the three primaries are not physically realizable.

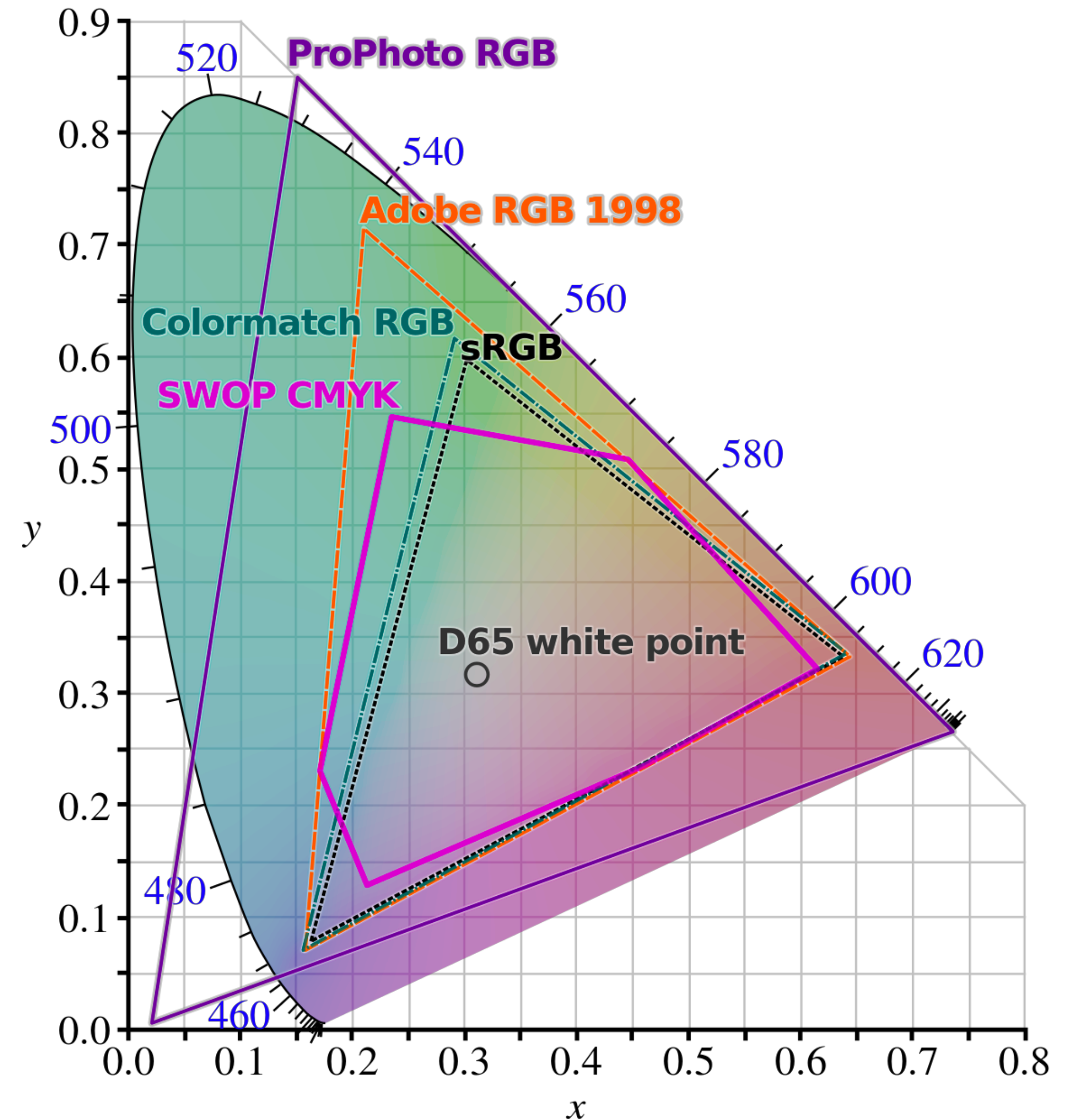




# ProPhoto RGB

ProPhoto RGB has a wide gamut, but ~13% of is imaginary. Two of the three primaries are not physically realizable.

Q: Can you build a display that shows all physically real colors in ProPhoto RGB?

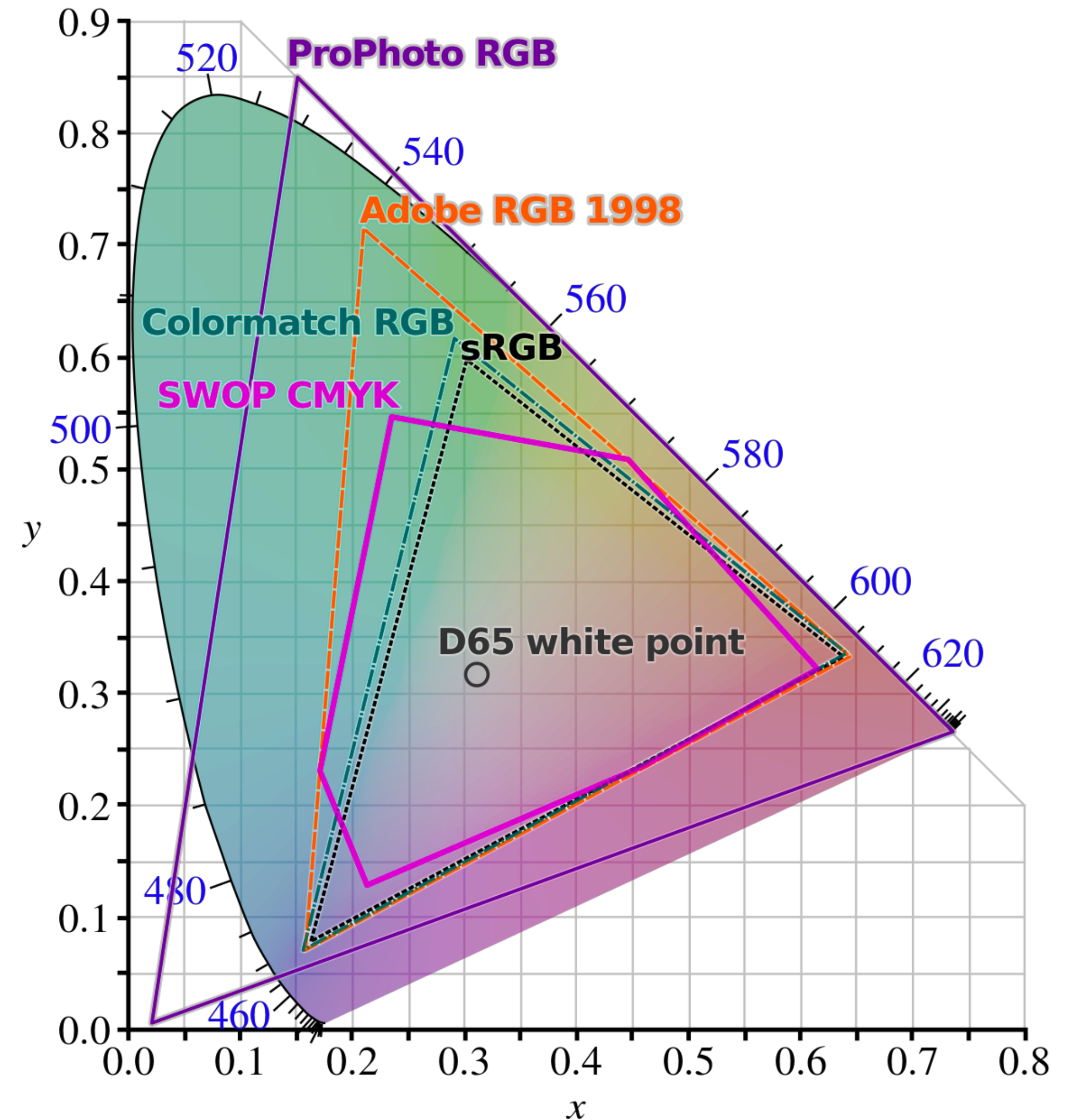


# ProPhoto RGB

ProPhoto RGB has a wide gamut, but ~13% of is imaginary. Two of the three primaries are not physically realizable.

Q: Can you build a display that shows all physically real colors in ProPhoto RGB?

- Use >3 primaries in the display





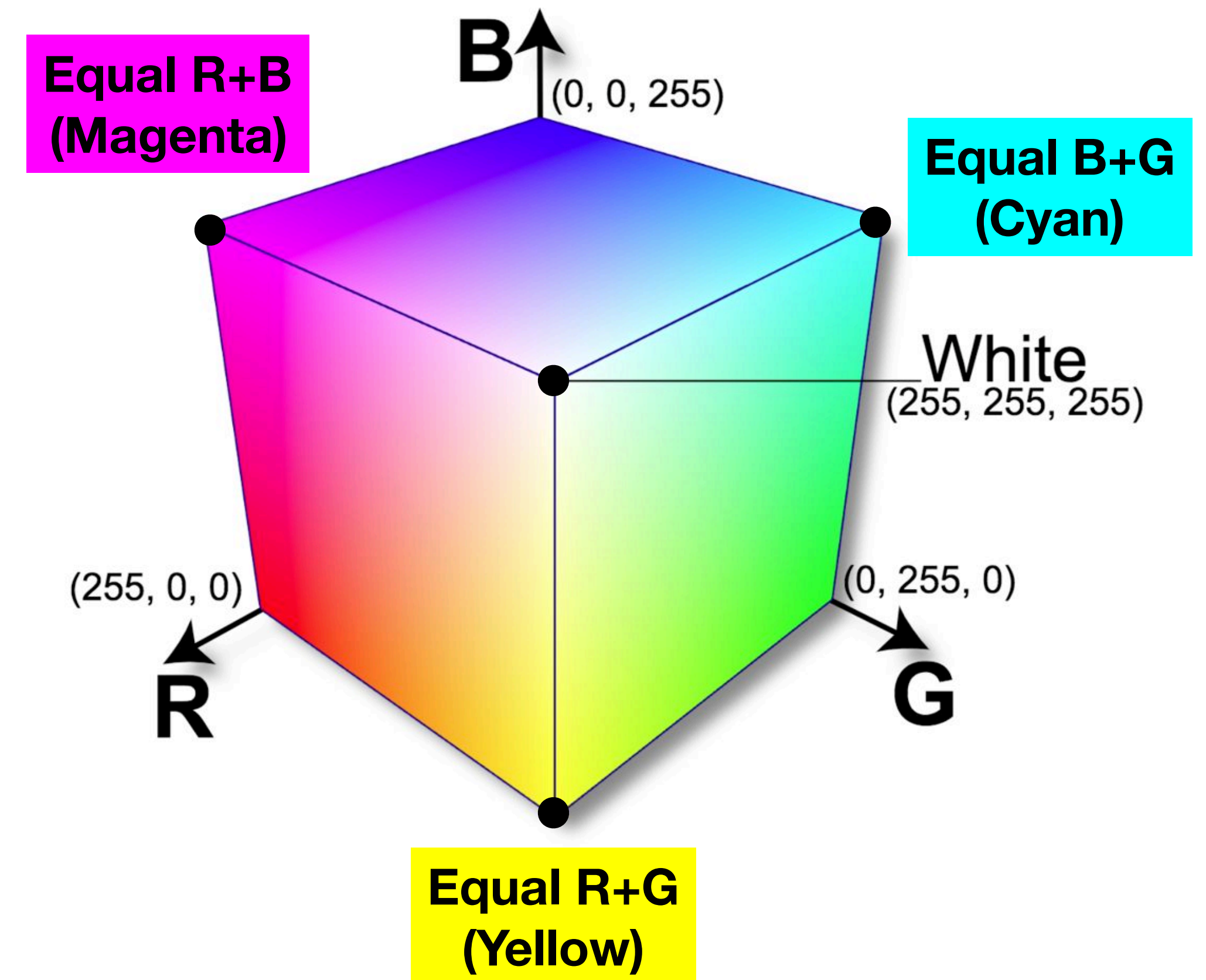
# Additive vs. Subtractive Color Space

RGB color spaces are additive: adding multiple colors to form a new color.

- Good for generated lights (e.g., display)

Cyan material looks cyan because it absorbs red light.

- Cyan = White - Red = Blue + Green
- Yellow = White - Blue = Red + Green
- Magenta = White - Green = Red + Blue

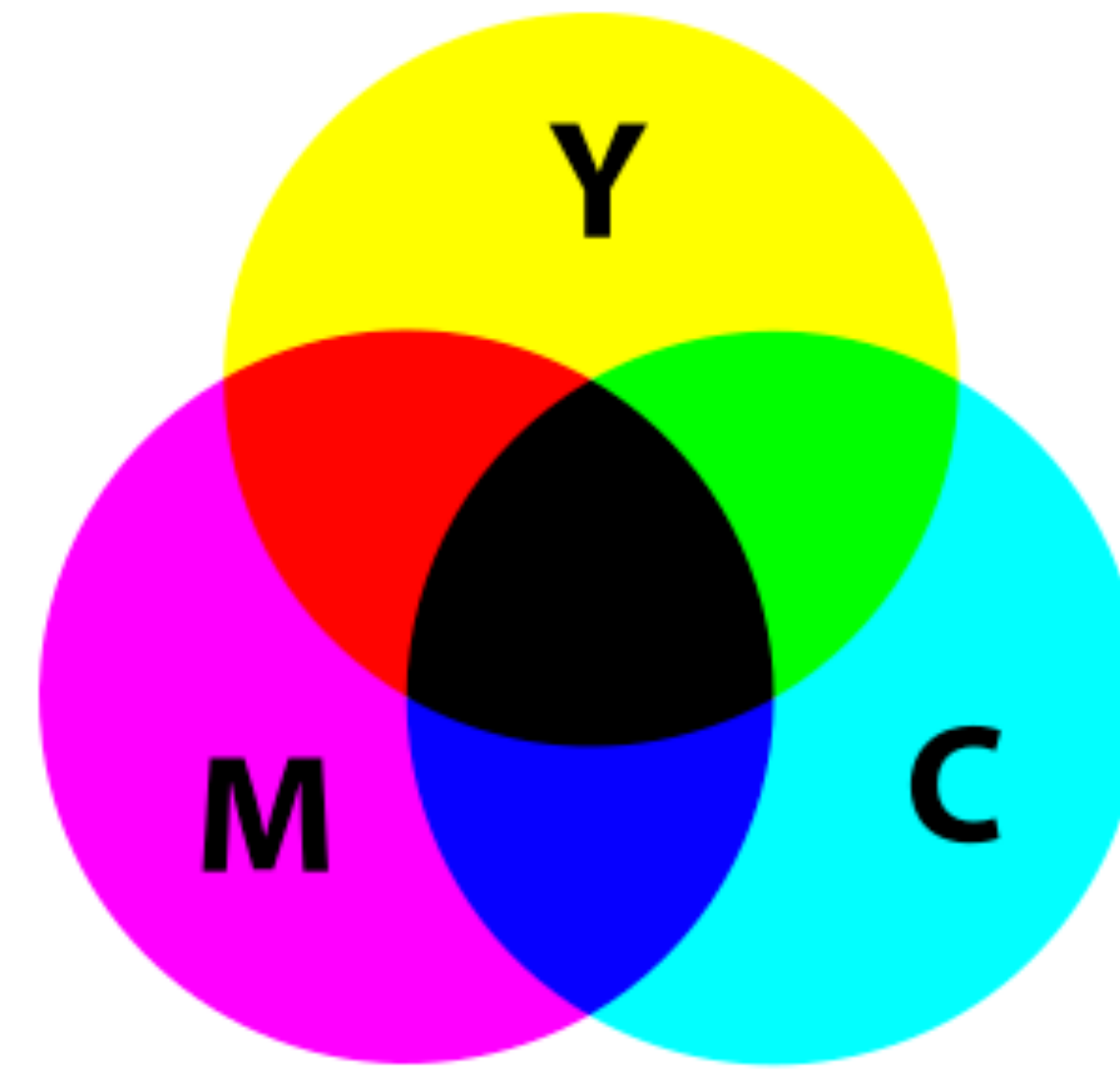


# Subtractive Color Space: CMYK

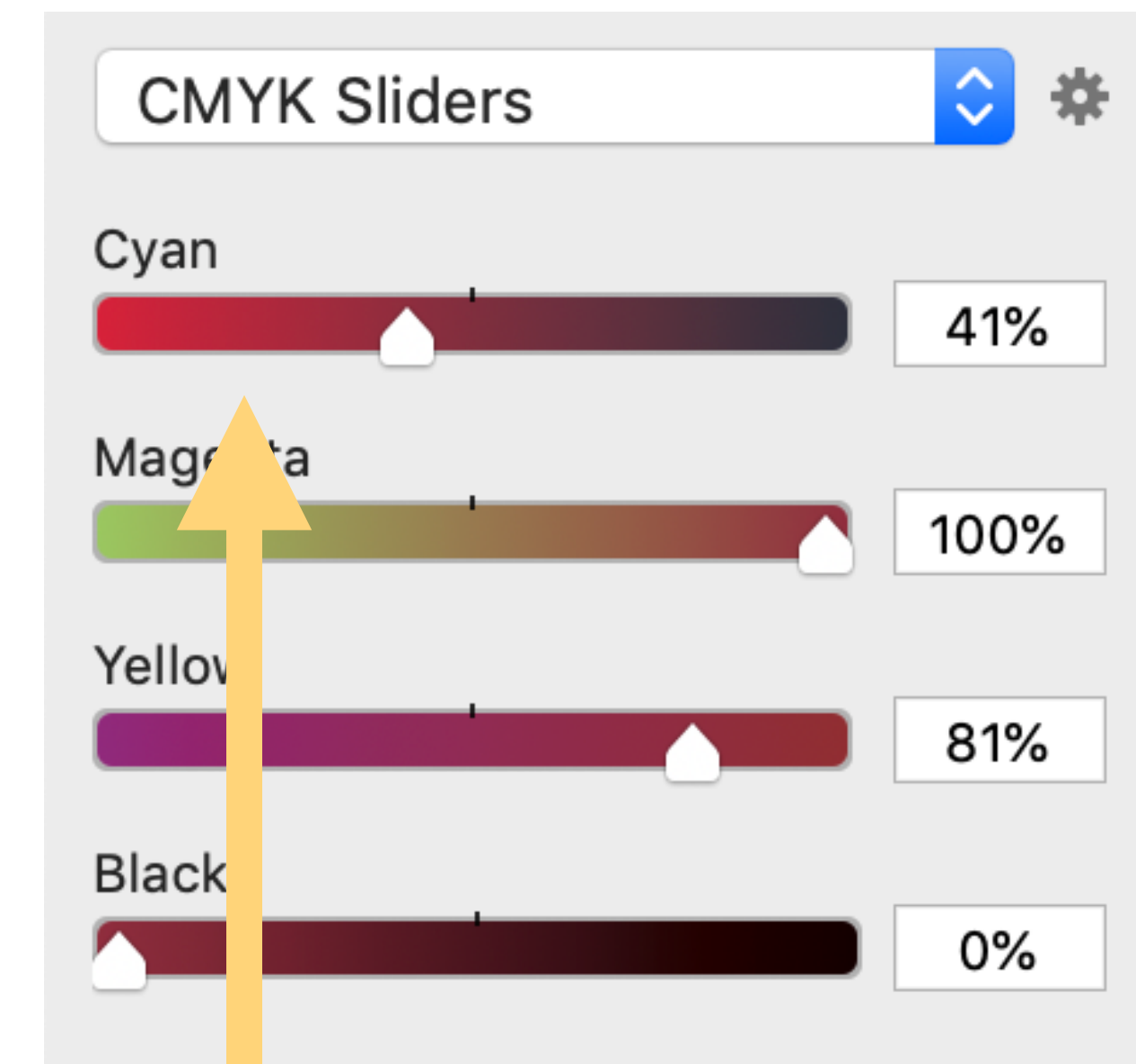
Subtractive color space is better at describing reflected light, e.g., painting, printing.

- Paper has color because it absorbs lights of certain wavelengths.

Pure subtractive color space rarely reflects reality, because it assumes pure absorption. Actual materials scatter lights too. More later.



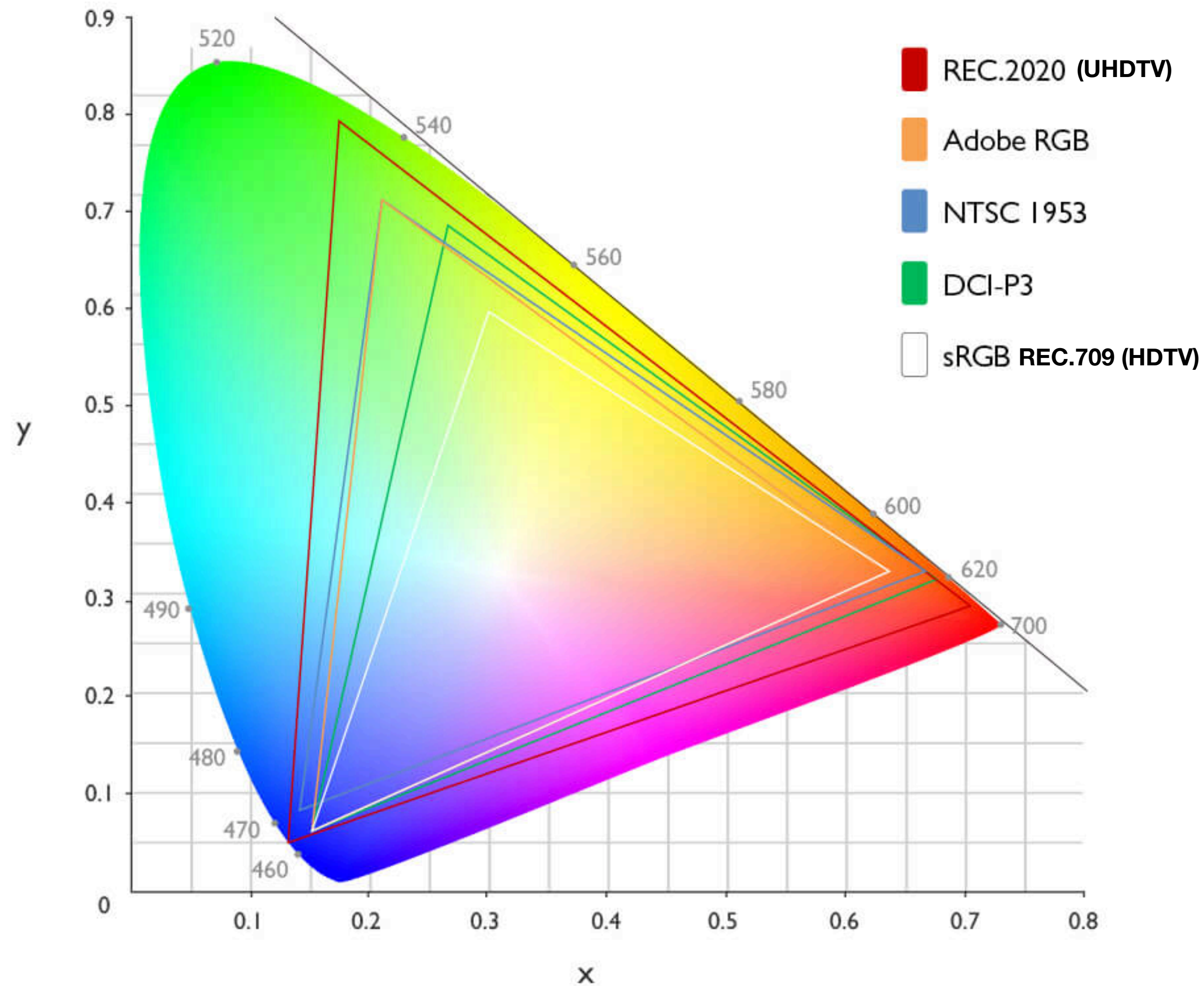
Mixing CMY absorbs all light, generating black.



Reducing Cyan adds red.

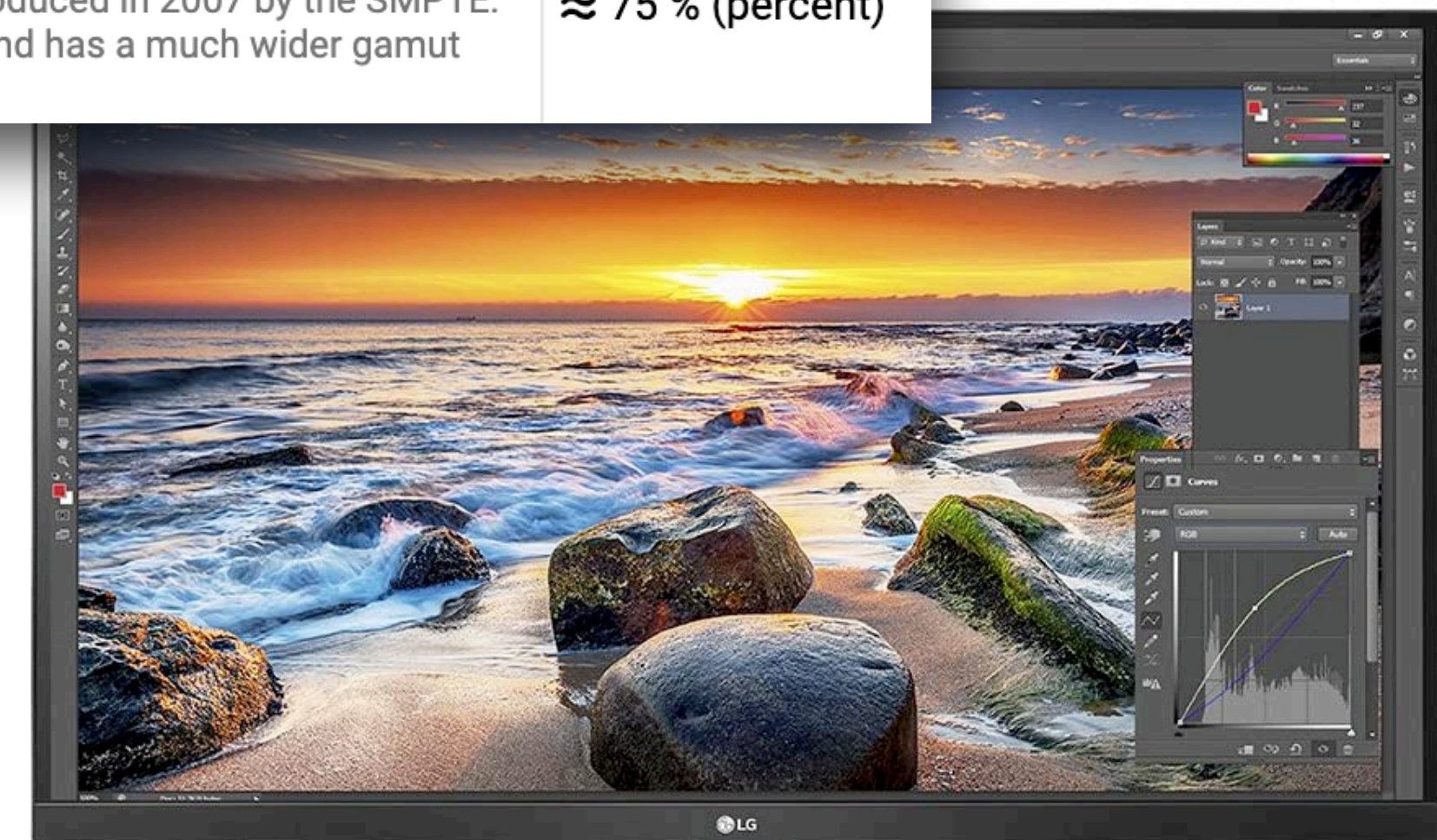


# Color Spaces Designed For Displays



<p><b>sRGB</b></p> <p>sRGB is a color space, developed jointly by Hewlett-Packard and Microsoft in 1996. It is used in different devices such as printers, displays, TV sets, cameras, etc. The sRGB color space covers about 72% of the NTSC color space.</p>	<p>≈ 99 % (percent)</p>
<p><b>Adobe RGB (1998)</b></p> <p>Adobe RGB (1998) is a color space, developed by Adobe Systems in 1998. It has a wider gamut than the sRGB (mainly in the cyan-green range of colors) and is widely used in professional printing.</p>	<p>≈ 73 % (percent)</p>
<p><b>DCI P3</b></p> <p>DCI P3 is a color space, introduced in 2007 by the SMPTE. It is used in digital cinema and has a much wider gamut than the sRGB.</p>	<p>≈ 75 % (percent)</p>

**My monitor LG  
27UK850-W  
(LCD)**





## Mac's ColorSync Utility tool

Profile First Aid Profiles Devices Filters Calculator

Registered ColorSync devices:

Device	Default
▶ Scanners	
▼ Cameras	
Yuhao's iPhone	
▼ Displays	
Color LCD	
▼ DELL P2415Q	<input type="checkbox"/>
HDMI D	<input type="checkbox"/>
HDMI SD	

Color LCD-00AE5BEB-A888-0F3F-4D59-BC4471A4CADC.icc

#	Tag	Data	Size	Description
11	'aarg'	'para'	32	Parametric tone response curve
12	'aabg'	'para'	32	Parametric tone response curve
13	'aagg'	'para'	32	Parametric tone response curve
14	'vcgt'	'vcgt'	48	Apple display video card table
15	'ndin'	'ndin'	62	Apple display native information
16	'chad'	'sf32'	44	Chromatic adaptation matrix
17	'mmod'	'mmod'	40	Apple display make and model informatio

Phosphor Values    Response Curves

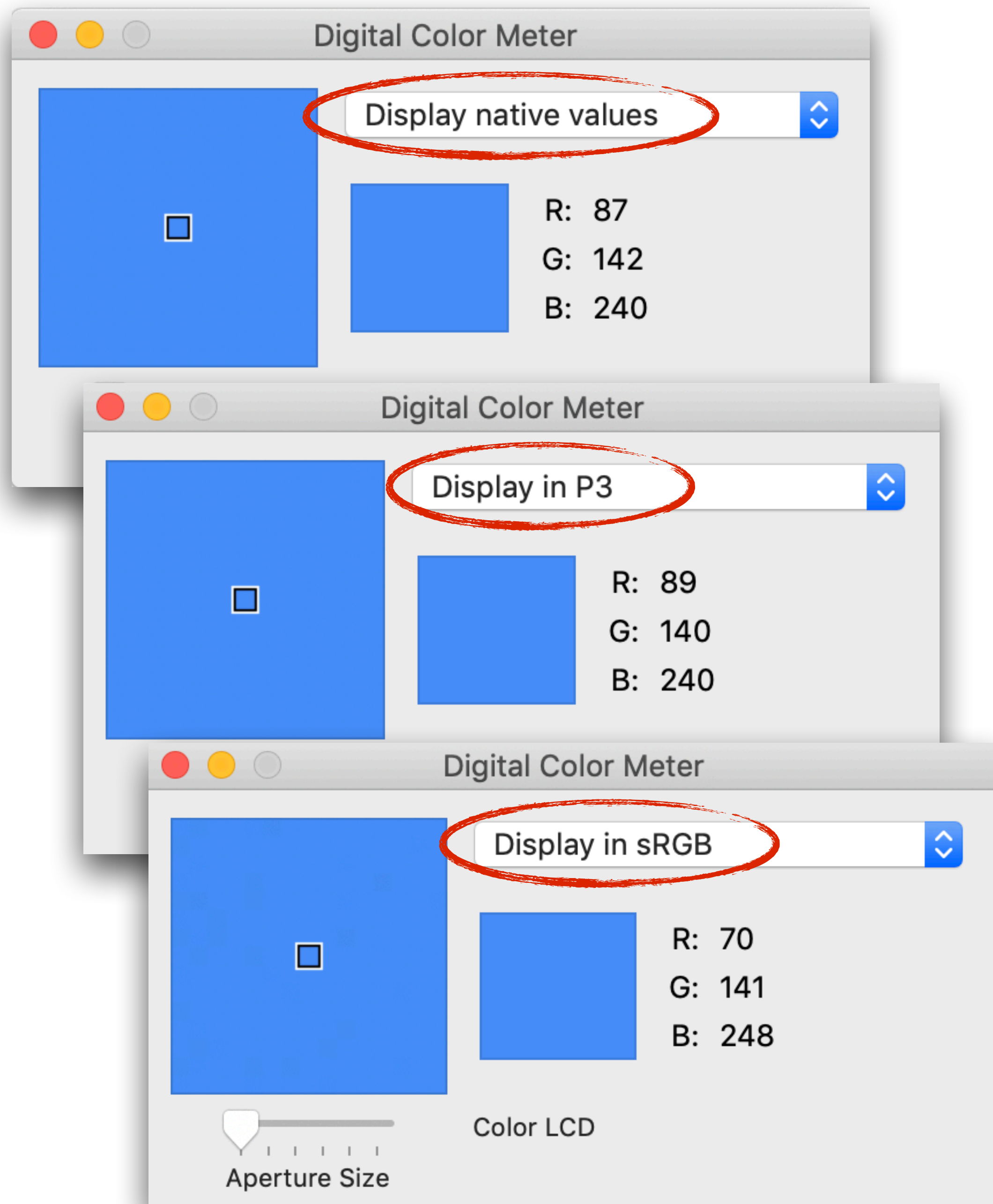
Primaries of the built-in display. Very close to DCI P3 color space.

Red	x:	0.6797
	y:	0.3203
Green	x:	0.2647
	y:	0.6904
Blue	x:	0.1504
	y:	0.0547
White	x:	0.3125
	y:	0.3291

The built-in display of my MacBook

The external Dell display



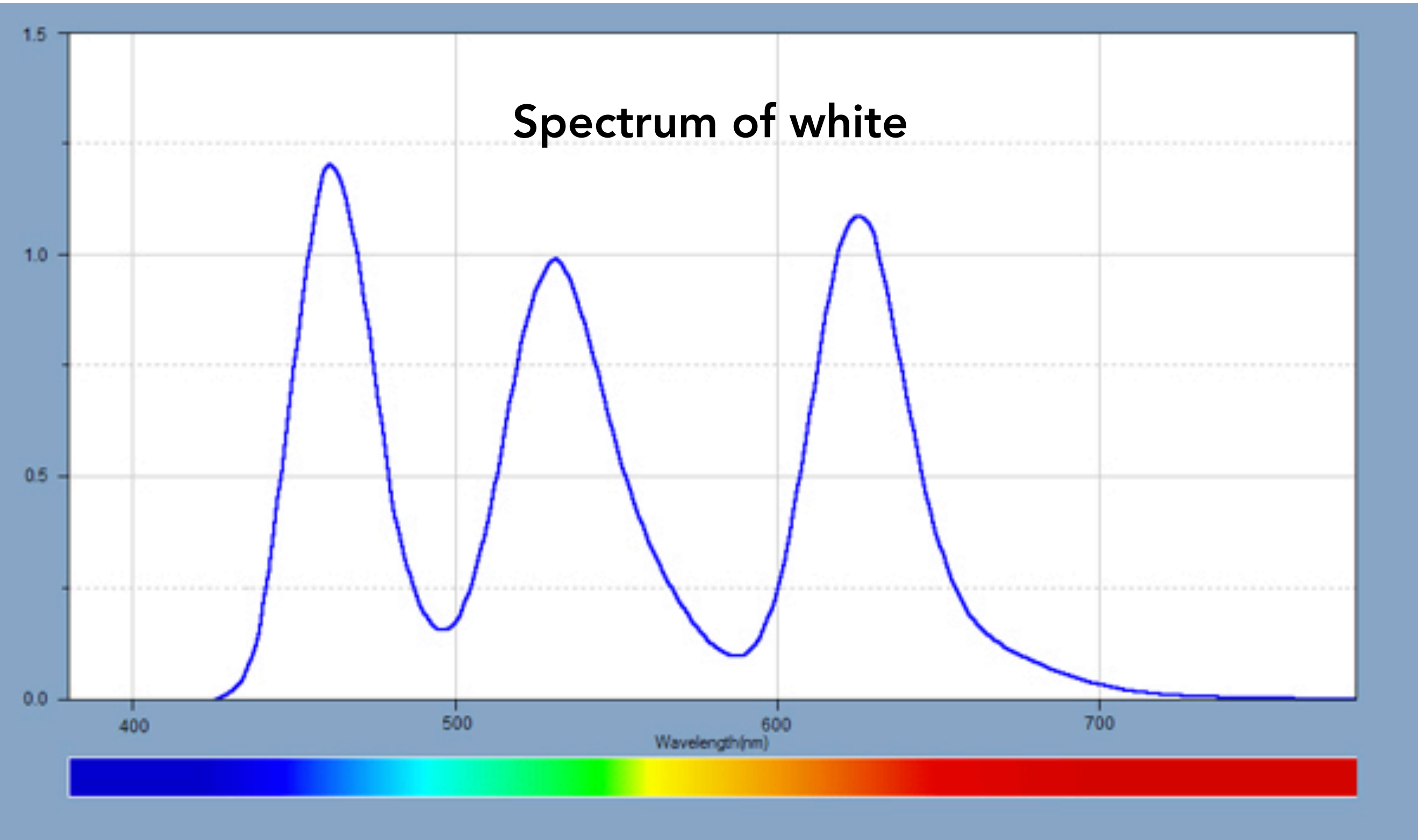
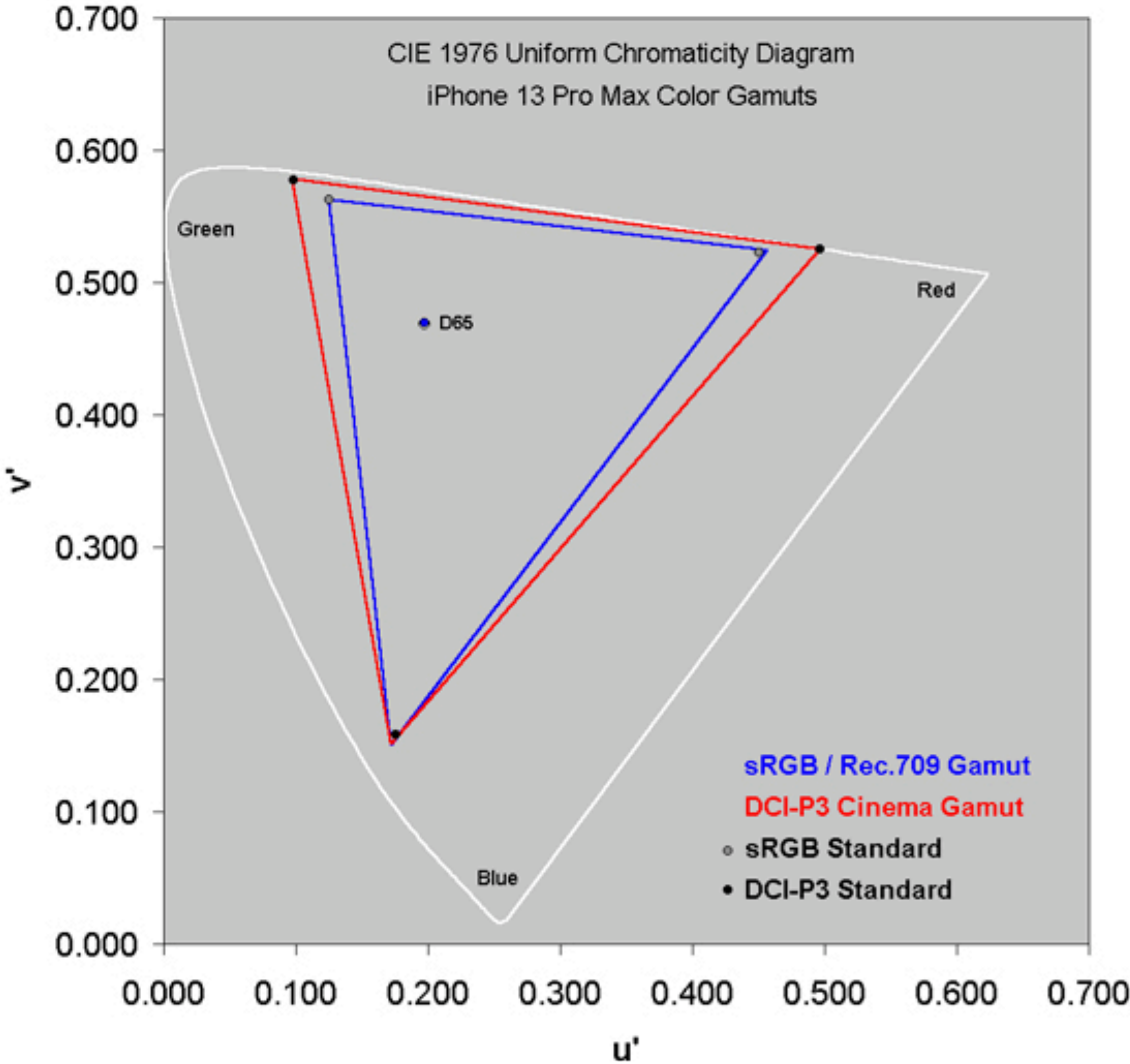


Display native color space rarely matches exactly with an existing color space (manufacturing constraints).

The same pixel shown in three different color spaces. The RGB values in the display native space roughly match the values in the DCI P3 color space, but are very different from the values in the sRGB color space.

# iPhone13 Pro Max measurement data

Matches the sRGB and DCI P3 gamuts pretty well, but not perfectly.

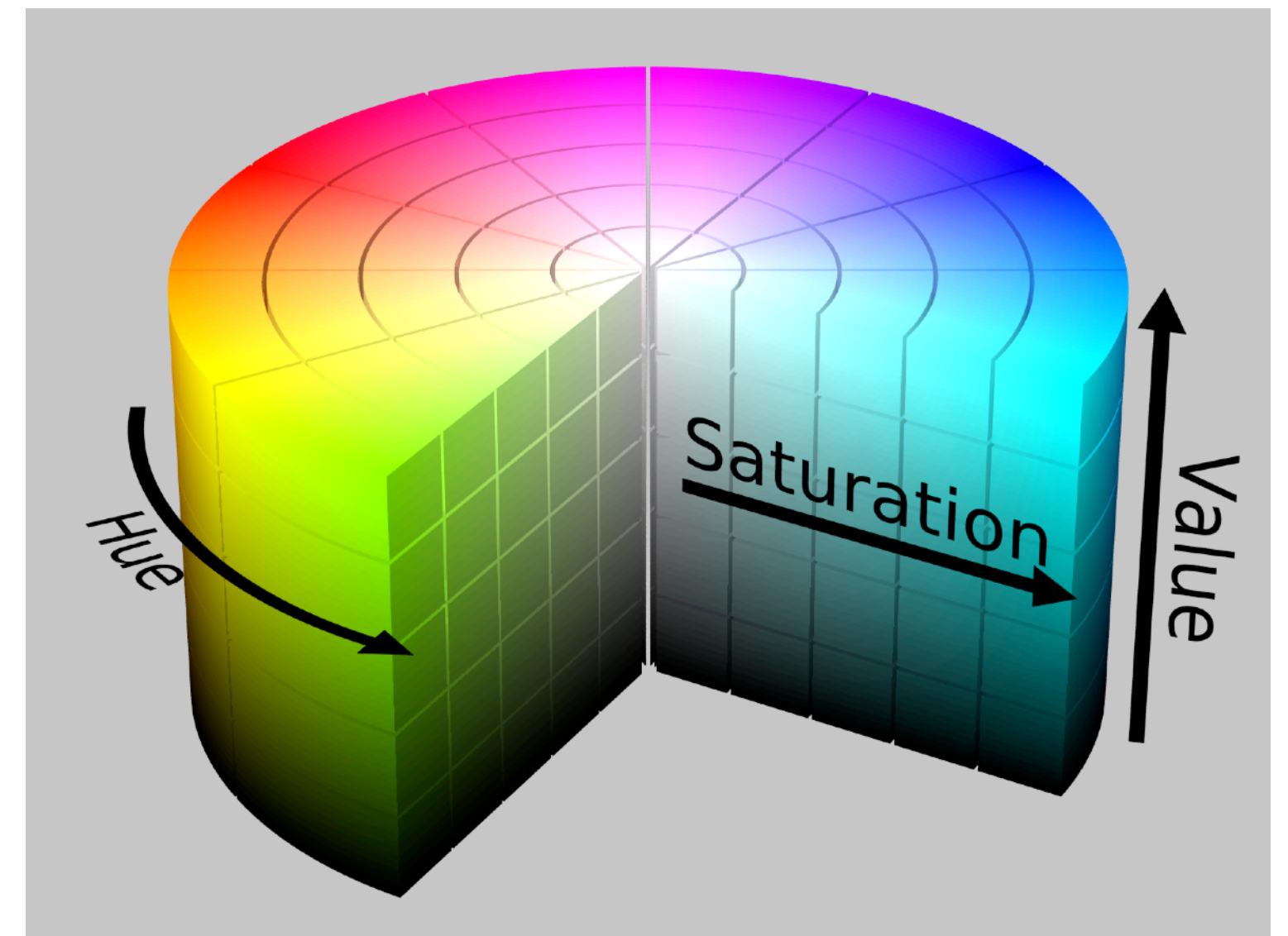
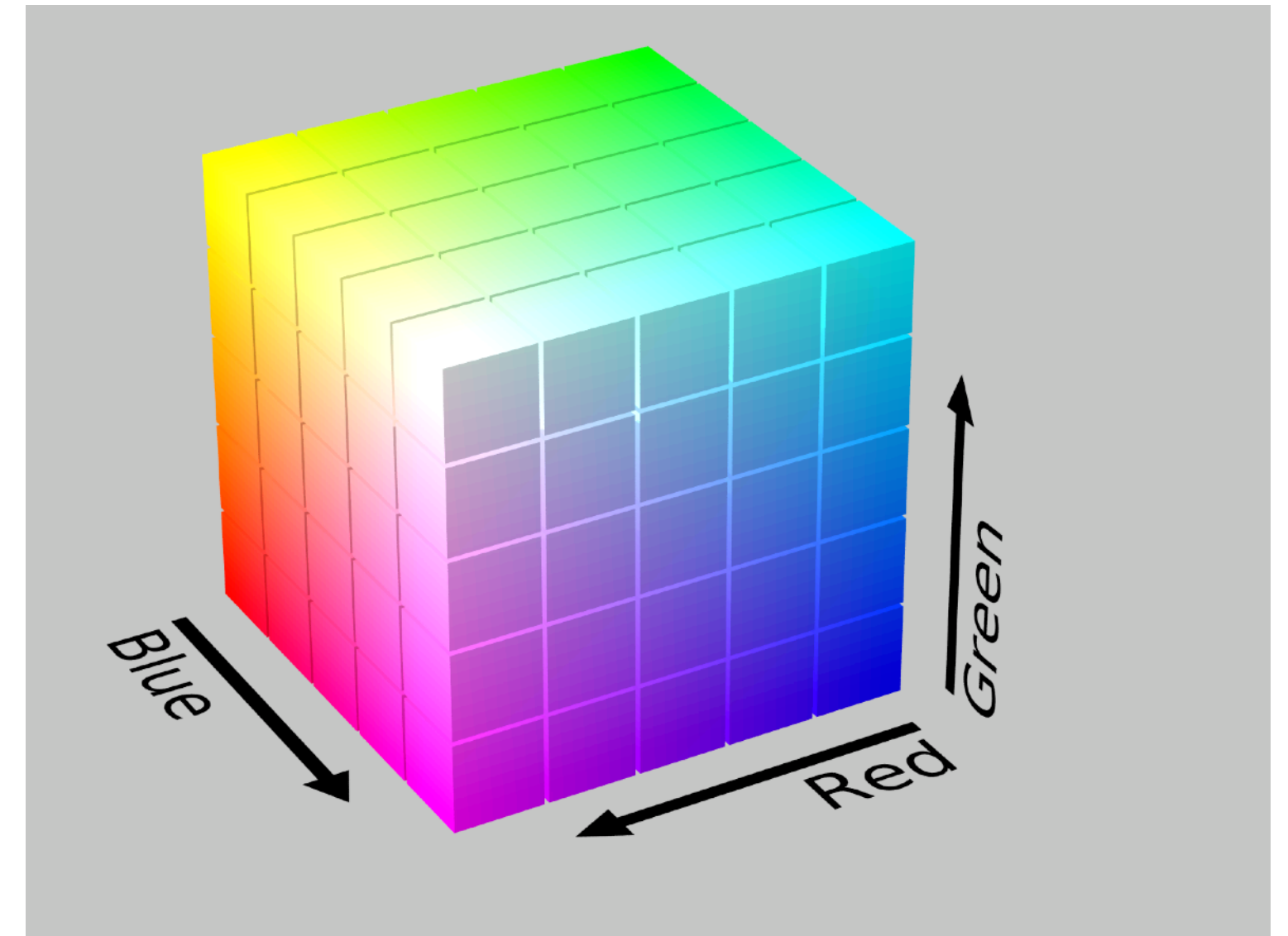


# Perceptual Color Space

HSB/HSV/HSL: intuitive representation of RGB space

- Hue: "nuances" of color
- Saturation: purity
- Value/brightness/lightness: amount of color

HSB/HSV/HSL don't change the gamut; they just map a cube to a cylinder.



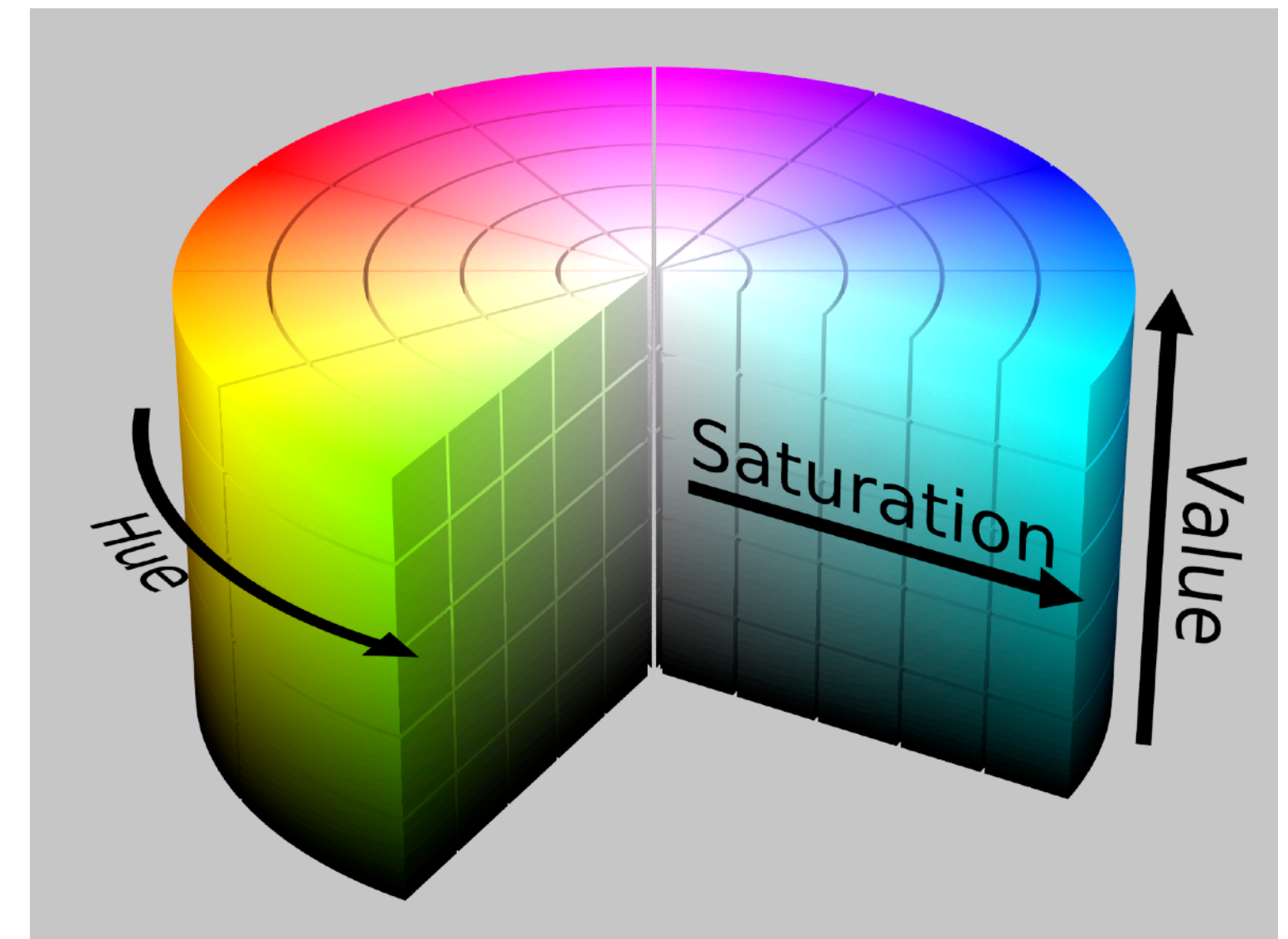
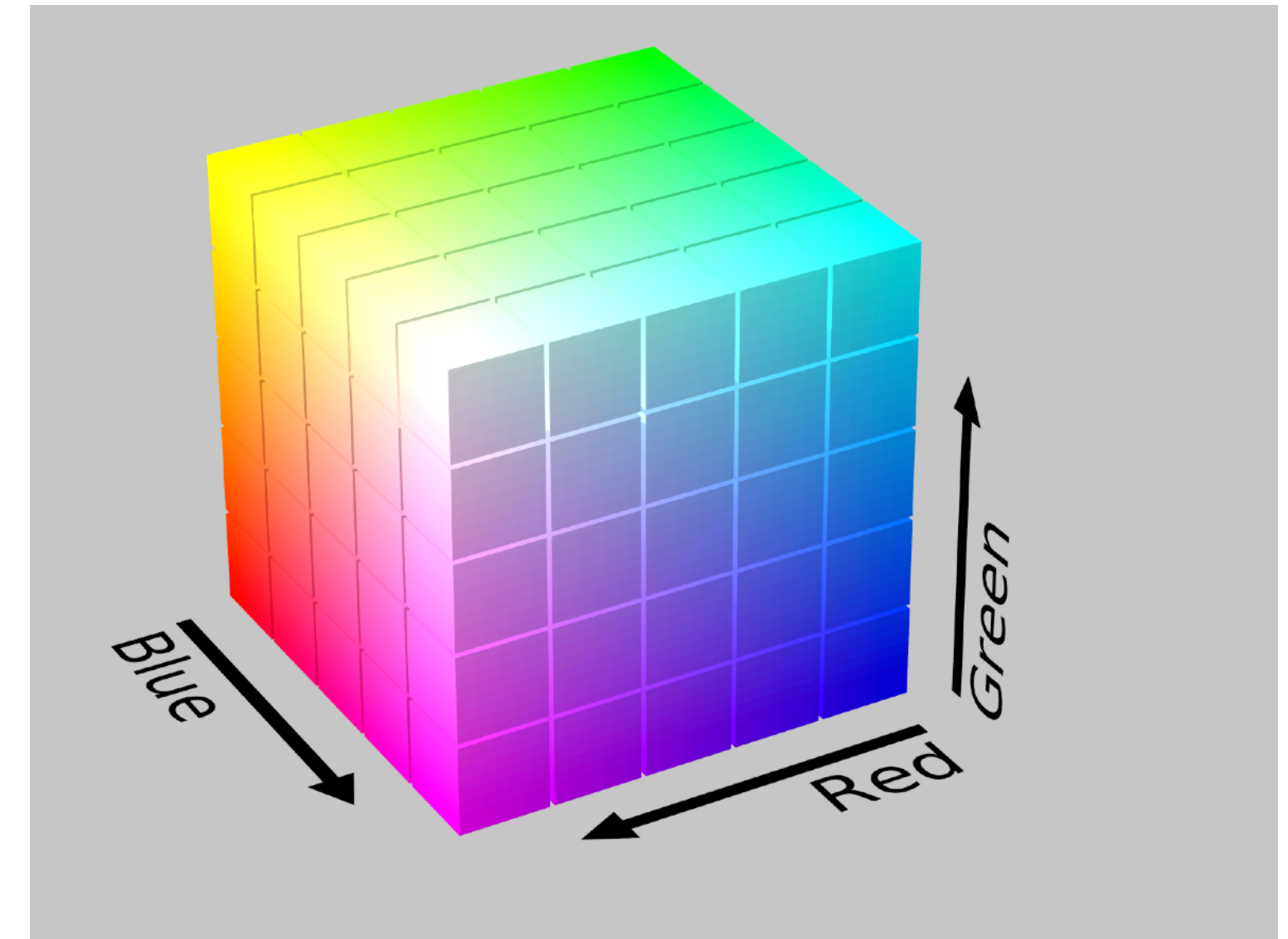
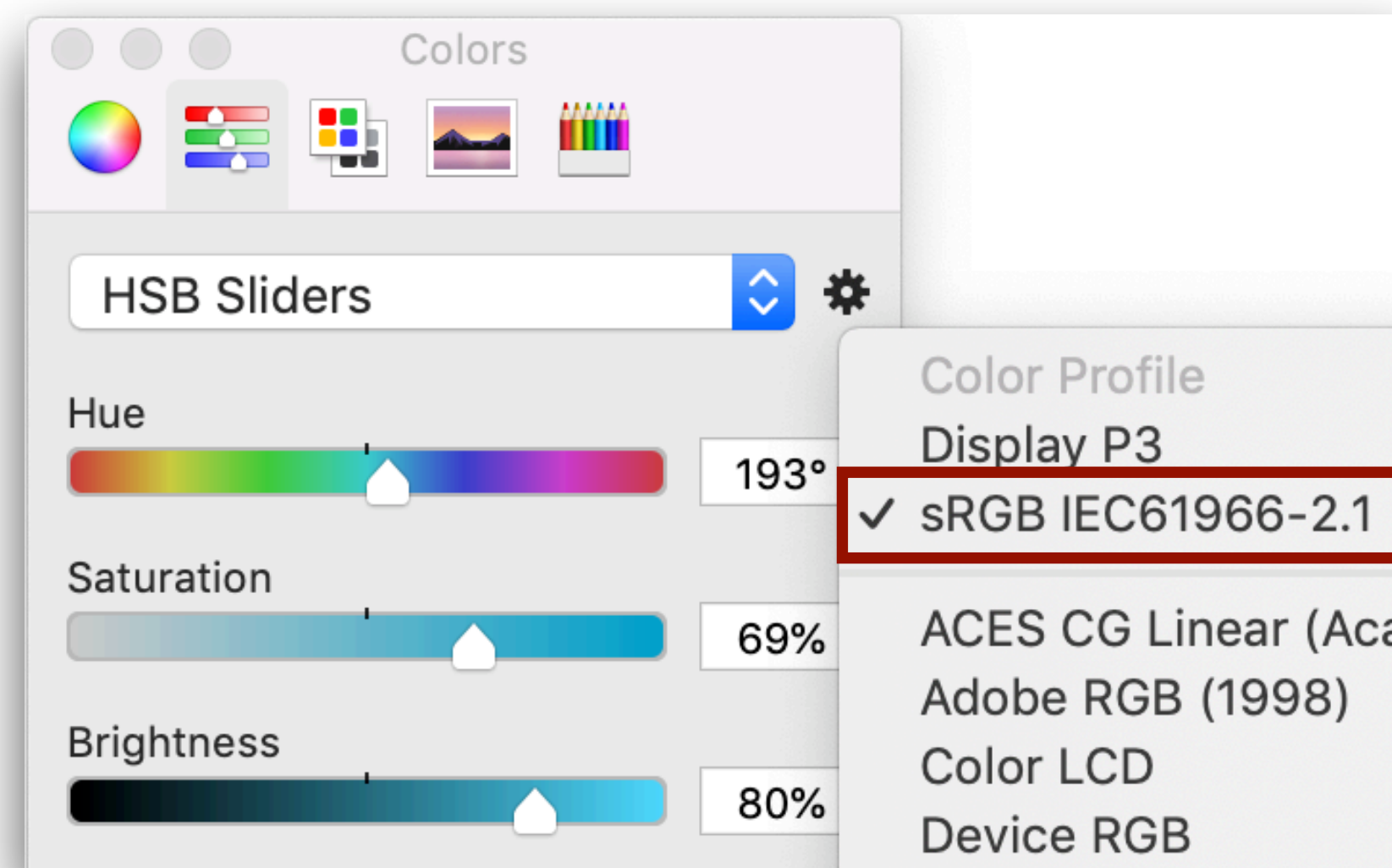


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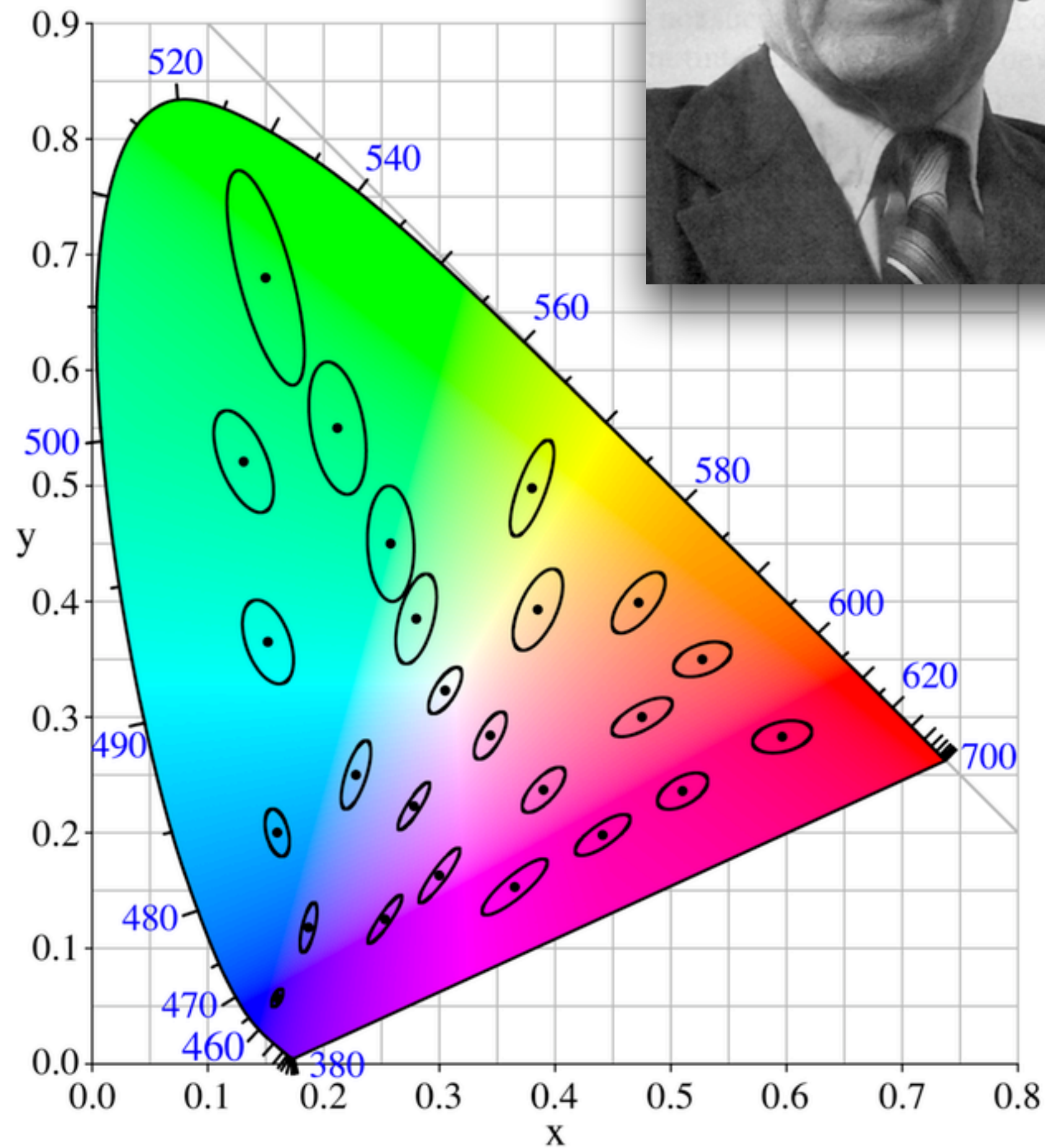




David L. MacAdam  
1910 — 1998  
Kodak and UR



# MacAdam Ellipses



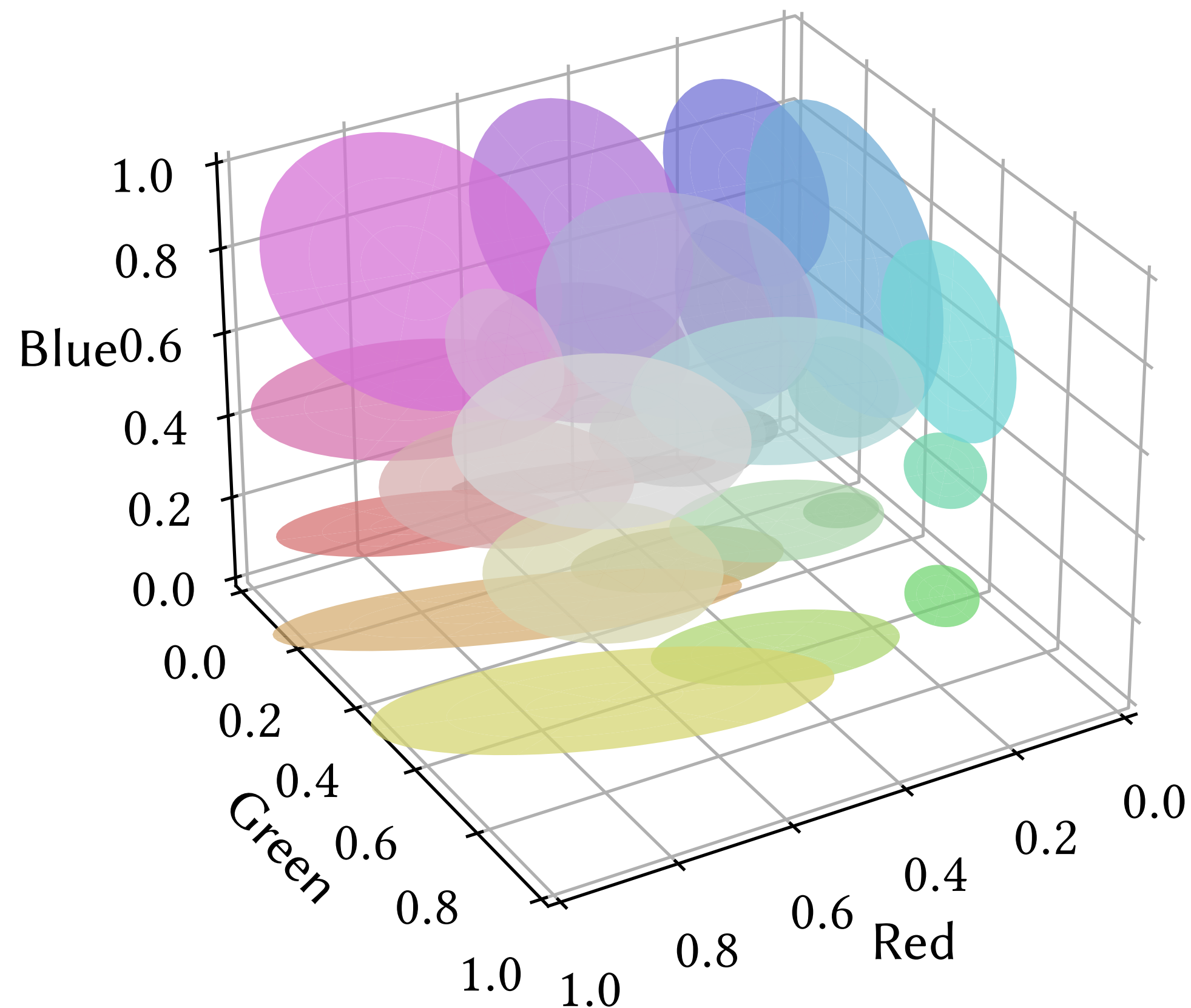
Distances in the XYZ color space (and any RGB color space) are not proportional to the actual color difference.

Colors in the same MacAdam ellipses (in chromaticity plot) are perceptually the same.

Good discussions of how MacAdam initially derived the ellipses: <https://sites.google.com/site/malscolourscience/macadam>.

# MacAdam Ellipses

The ellipsoids in the linear sRGB space



Distances in the XYZ color space (and any RGB color space) are not proportional to the actual color difference.

Colors in the same MacAdam ellipses (in chromaticity plot) are perceptually the same.

Ellipses are ellipsoids in 3D color space.

# Psychophysical Studies to Derive the Ellipsoids

Press SPACE to start.  
Run 0/90

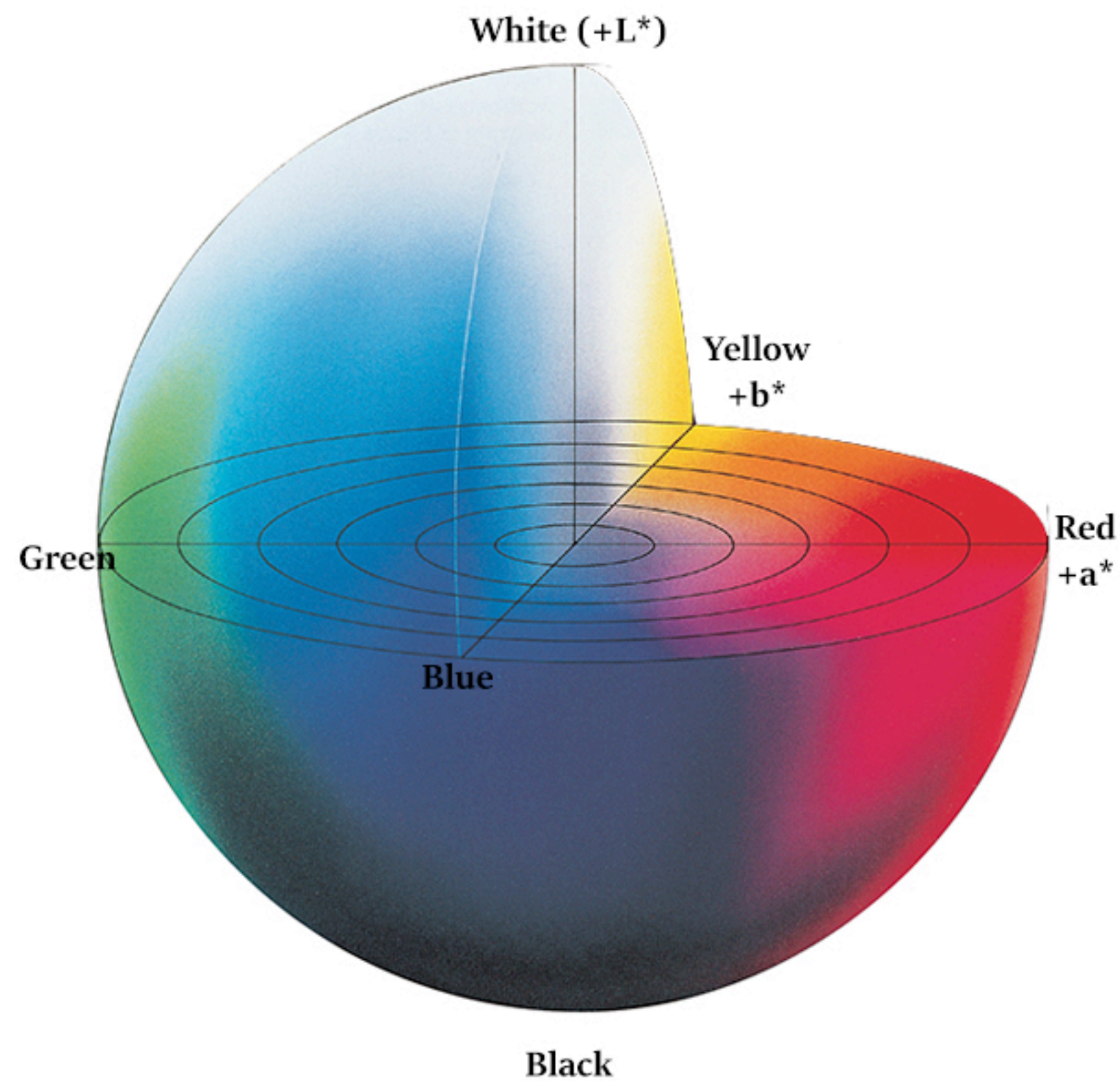
# Psychophysical Studies to Derive the Ellipsoids

Press SPACE to start.  
Run 0/90



# "Perceptually Uniform" Color Spaces

CIELAB (CIE 1976 L\*a\*b\*)



$$X = X_n f^{-1} \left( \frac{L^* + 16}{116} + \frac{a^*}{500} \right)$$

$$Y = Y_n f^{-1} \left( \frac{L^* + 16}{116} \right)$$

$$Z = Z_n f^{-1} \left( \frac{L^* + 16}{116} - \frac{b^*}{200} \right)$$

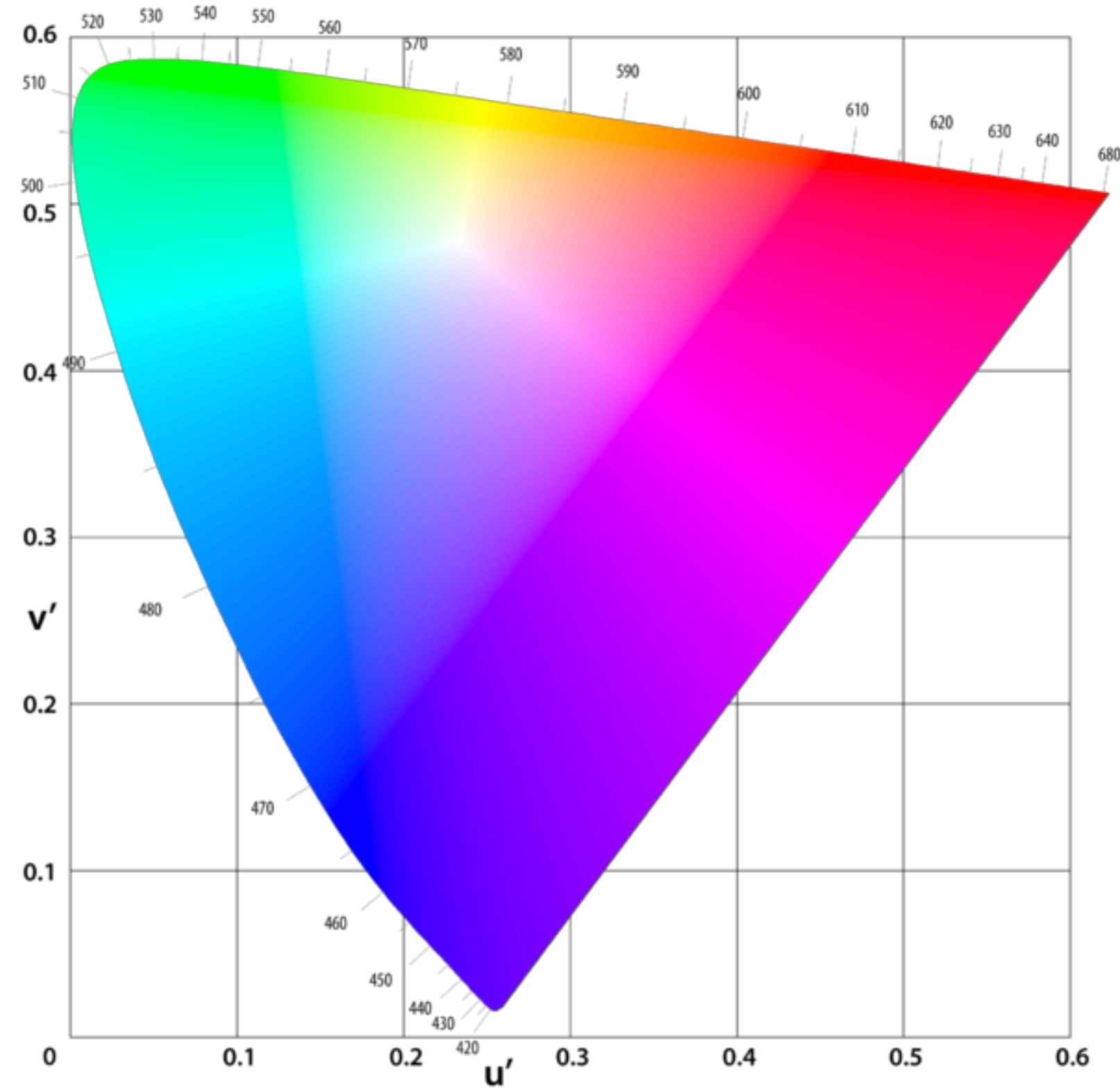
$$f^{-1}(t) = \begin{cases} t^3, & t > \delta \\ 3\delta^2(t - \frac{4}{29}), & t \leq \delta \end{cases}$$

$$\delta = \frac{6}{29}$$

$[X_n, Y_n, Z_n]$  is the brightest point in the scene

# "Perceptually Uniform" Color Spaces

CIELUV (CIE 1976 L\*u\*v\*)



$$L^* = 116f\left(\frac{Y}{Y_n}\right) - 16 \quad (1.23)$$

$$u^* = 13L^*(u' - u'_n) \quad (1.24)$$

$$v^* = 13L^*(v' - v'_n) \quad (1.25)$$

$$f(x) = \begin{cases} x^{\frac{1}{3}} & x > 0.008856 \\ 7.787x + \frac{16}{116} & x \leq 0.008856 \end{cases} \quad (1.26)$$

$$u' = \frac{4X}{X + 15Y + 3Z} \quad (1.27)$$

$$v' = \frac{9Y}{X + 15Y + 3Z} \quad (1.28)$$

$$u'_n = \frac{4X_n}{X_n + 15Y_n + 3Z_n} \quad (1.29)$$

$$v'_n = \frac{9Y_n}{X_n + 15Y_n + 3Z_n} \quad (1.30)$$