Everything is a Function

Chen Ding

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Background

- Turing machine
  - model of computers
  - imperative instructions, rewritable memory
  - sequential, implicit dependence
- Church's Lambda calculus
  - model of abstraction
  - everything is a function
    - the language of mathematics
    - do everything by composing functions
    - no mutable state, no side effects
  - explicit value passing

LC Language

- Source
  - P. J. Landin 1964
  - Matthias Felleisen, now NEU formally Rice
- Syntax

M –> x
| (λ.x M)
| (M M)
| n
| (+ M M)

How is it different from other languages?

Examples

- What does the following LC expressions return

(A) 1

(B) x

(C) (λ.x x)

(D) ((λ.x x) 1)

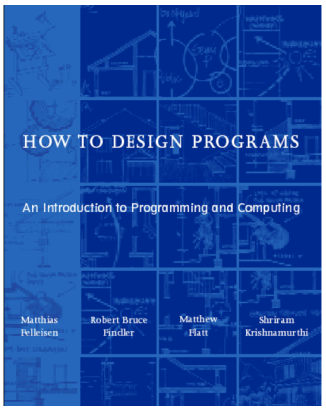
(E) ((λ.f (f 1)) (λ.x x))

Dr. Scheme

http://www.plt-scheme.org/

How to Design Programs

http://www.htdp.org/




```
;; How to design a program
(define (profit ticket-price)
  (- (revenue ticket-price)
     (cost ticket-price)))

(define (revenue ticket-price)
  (*  (attendees ticket-price) ticket-price))

(define (cost ticket-price)
  (+ 180
     (* .04 (attendees ticket-price))))

(define (attendees ticket-price)
  (+ 120
     (* (/ 15 .10) (- 5.00 ticket-price))))
```

```
;; How not to design a program
(define (profit price)
  (- (* (+ 120
           (* (/ 15 .10)
              (- 5.00 price)))
        price)
     (+ 180
        (* .04
           (+ 120
              (* (/ 15 .10)
                 (- 5.00 price)))))))
```

Figure 5: Two ways to express the *profit* program

Functional Programming

- Program to produce functions rather than data
  - what is an object?
- Communicate functions rather data
  - what is communicated on the Internet?
- Example
  - write a function addk
    - given k, produces a unary function that adds k to its input

Binding and Closure

- Variable binding
  - binding happens at the function application
  - use the closest binding under the static scope
- Closure
  - a function and its environment
  - the environment stores the value of all bound variables
- Implementation
  - can functions share a stack?
  - how to model memory?
  - when/how to free memory?
- Examples

(A) $((\lambda.x\ x)\ 1)$

(B) $((\lambda.f\ (f\ 1))\ \ (\lambda.x\ x))$

Church Numerals

- Take f and x, number k is a function that apply f k times to x
  - zero is
  - one is
  - k is
- Operations
  - inc
  - add
  - multiply

Expanding the Language

- Global variables
- Implementing "define"
- Multiple parameters

Control Flow

- Branches
- Recursion

Semantics

How to define the complete meaning of a programming language? How long/short is the definition?

LC Representation in Scheme

```
(define-struct var (name))
(define-struct const (number))
(define-struct proc (param body))
(define-struct app (rator rand))
(define-struct add (left right))
```

1
```
(define one (make-const 1))
```
x
```
(define x (make-var 'x))
```
(λ.x x)
```
(define ifunc (make-proc 'x x))
```
((λ.x x) 1)
```
(make-app ifunc one)
```


Environment

- An evaluation environment is a set of variable bindings

```
(define empty-env
  (lambda ()
    (lambda (var)
      (error 'lookup "variable ~a not found" var))))

(define lookup
  (lambda (var env)
    (env var)))

(define extend
  (lambda (env varN val)
    (lambda (name)
      (if (eq? name varN)
          val
          (env name)))))
```

Closure

- A closure is a function and its evaluation environment

Semantics

- MEval(M, ...)
  - case n:
  - case x:
  - case (λ.x M):
  - case (M_1 M_2):