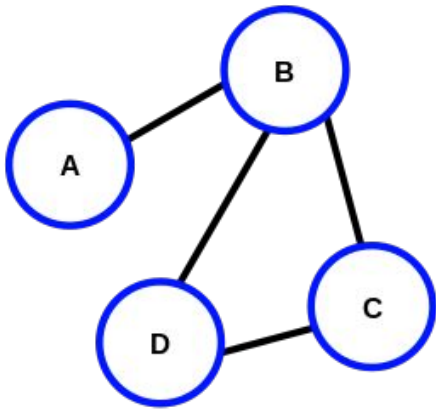
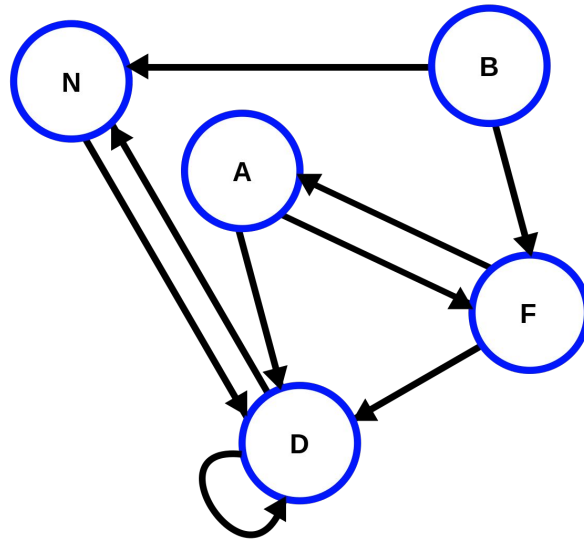


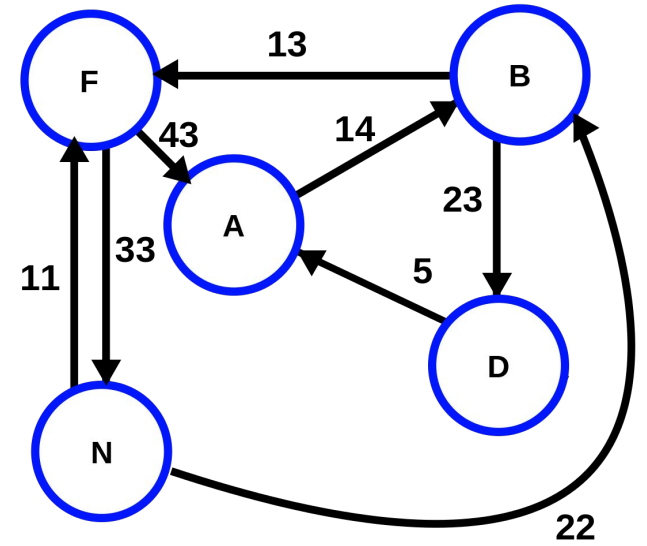
Graphs



**Unweighted,
undirected**

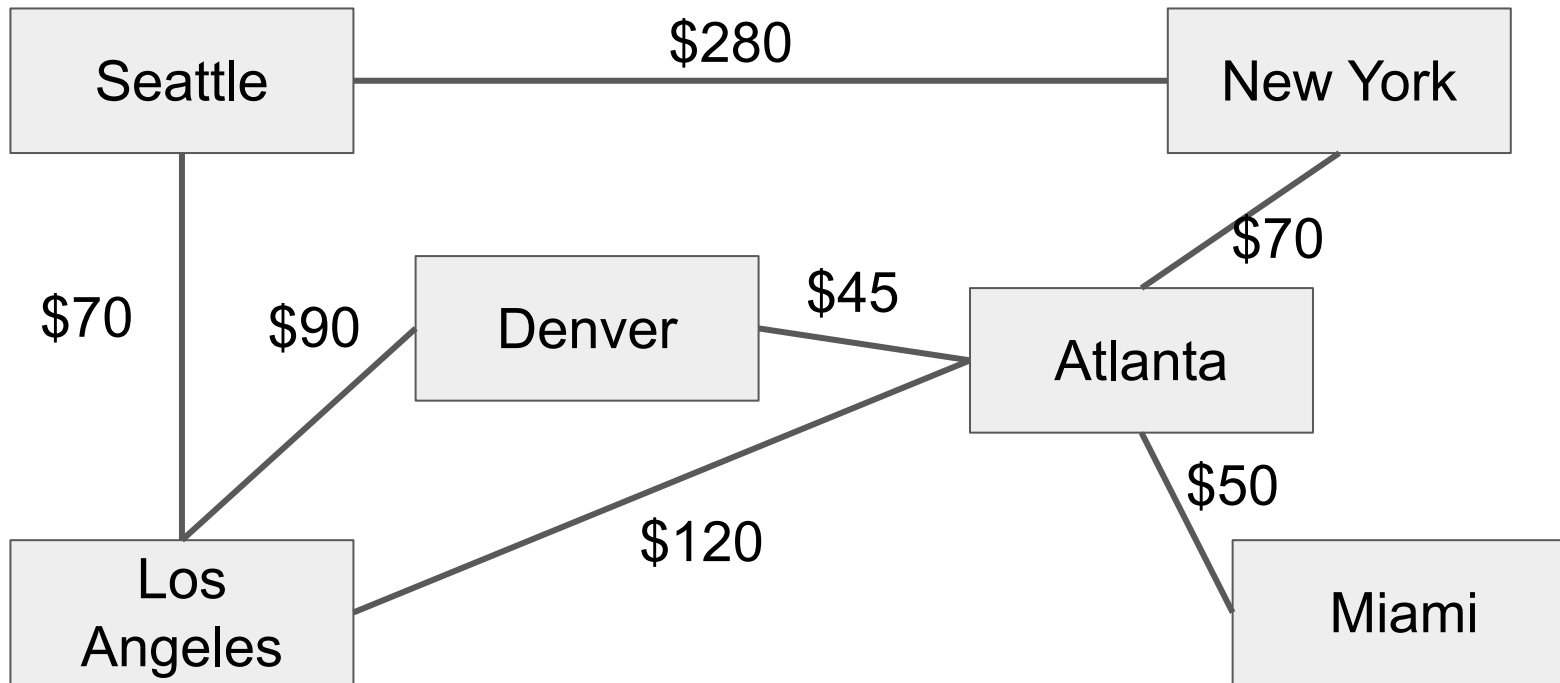


**Unweighted,
directed**



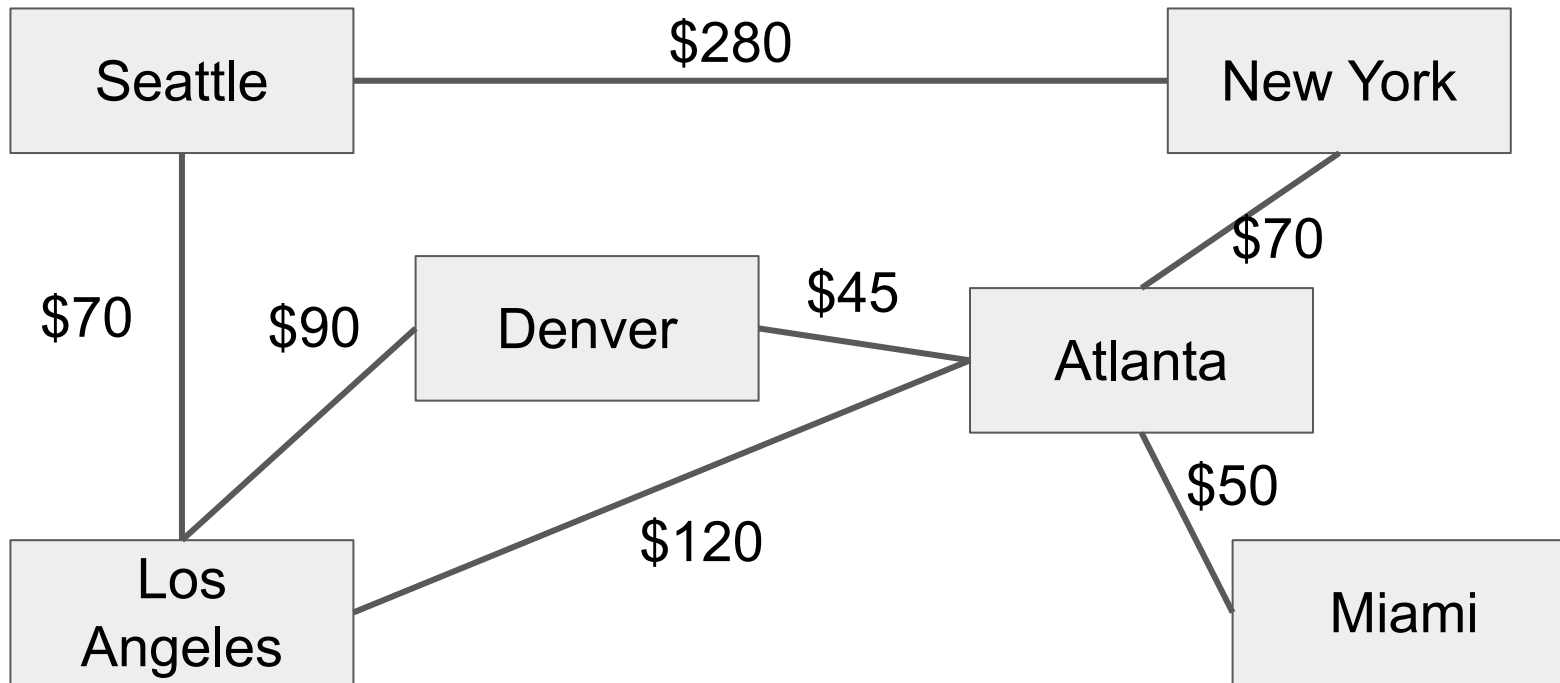
**Weighted,
directed**

Graphs



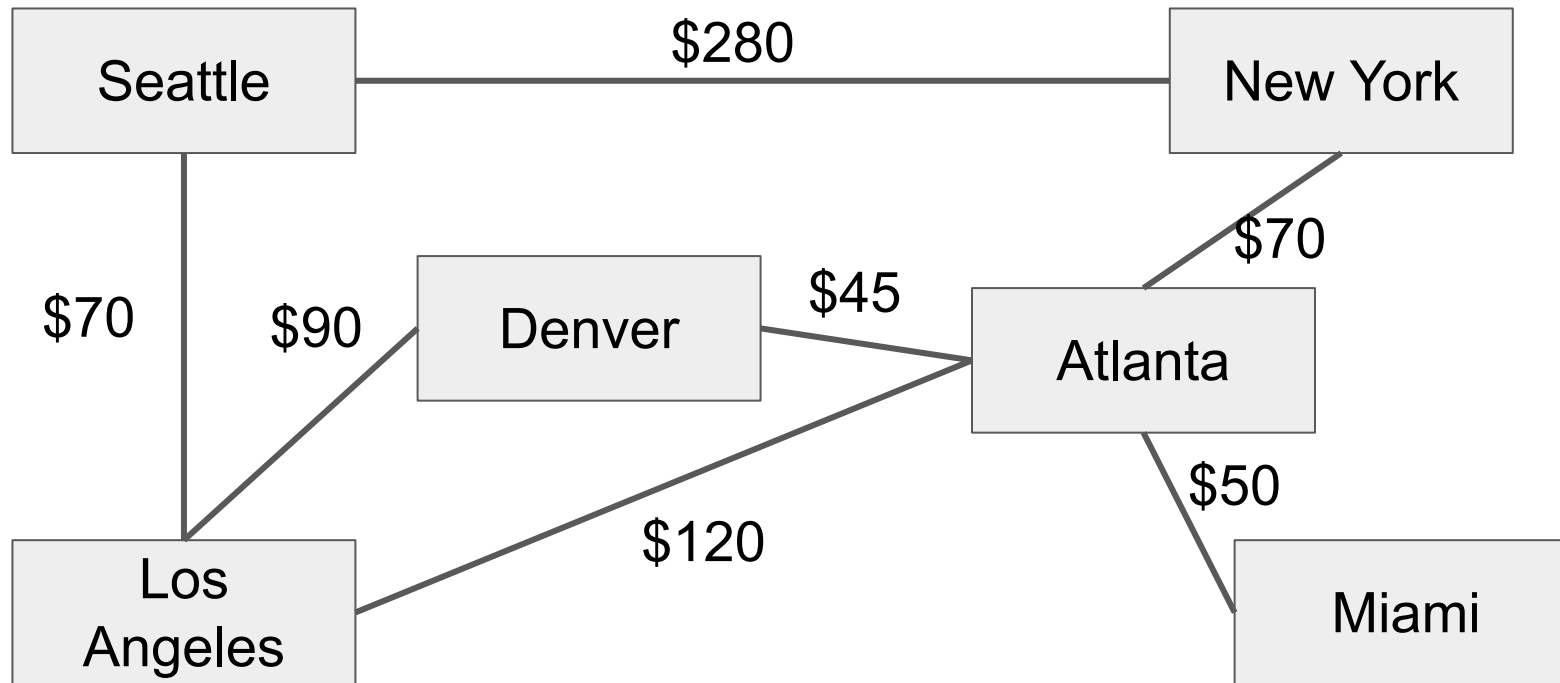
What type of a graph is this?

Graphs



Undirected, weighted

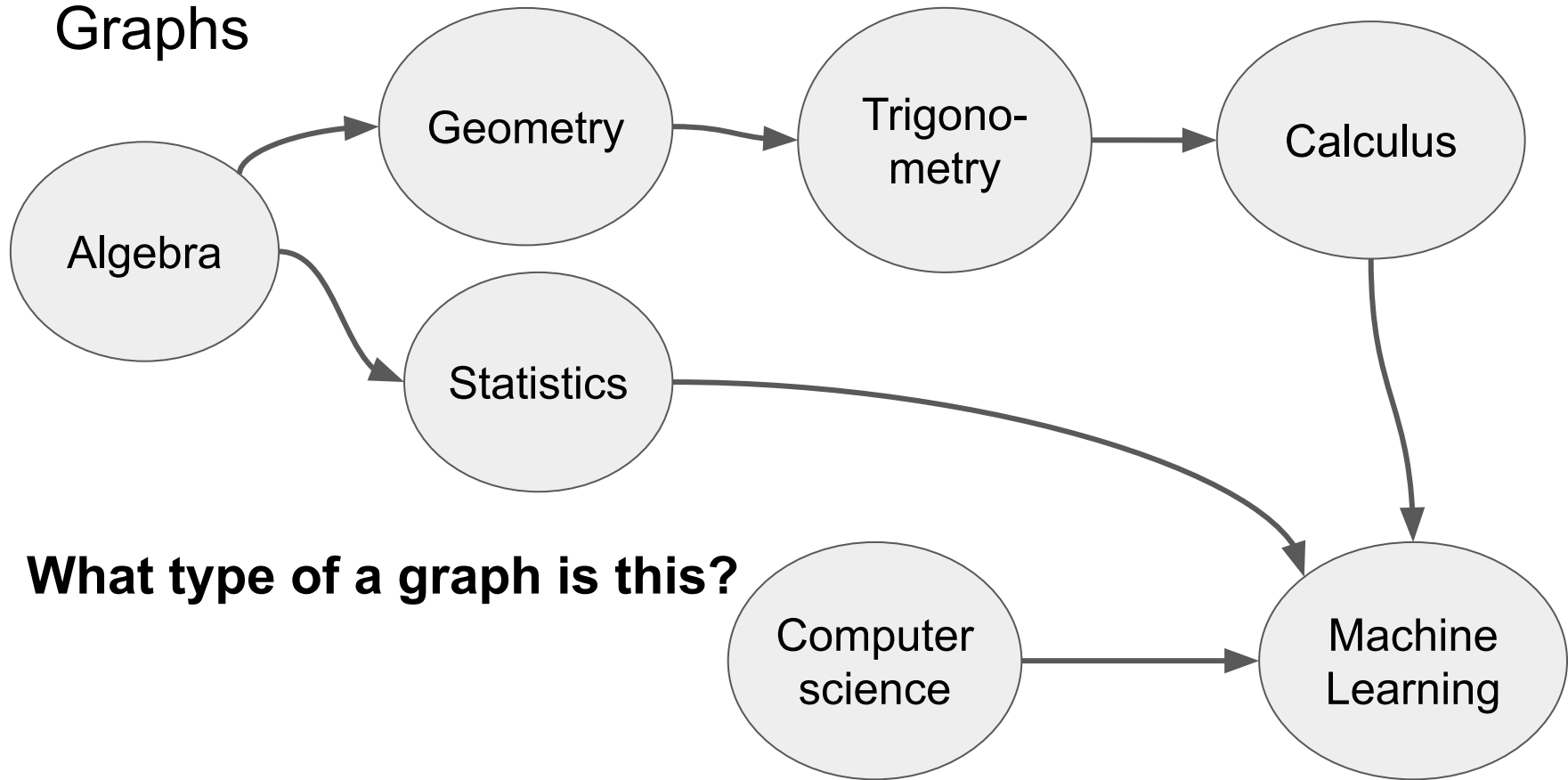
Graphs



Undirected, weighted

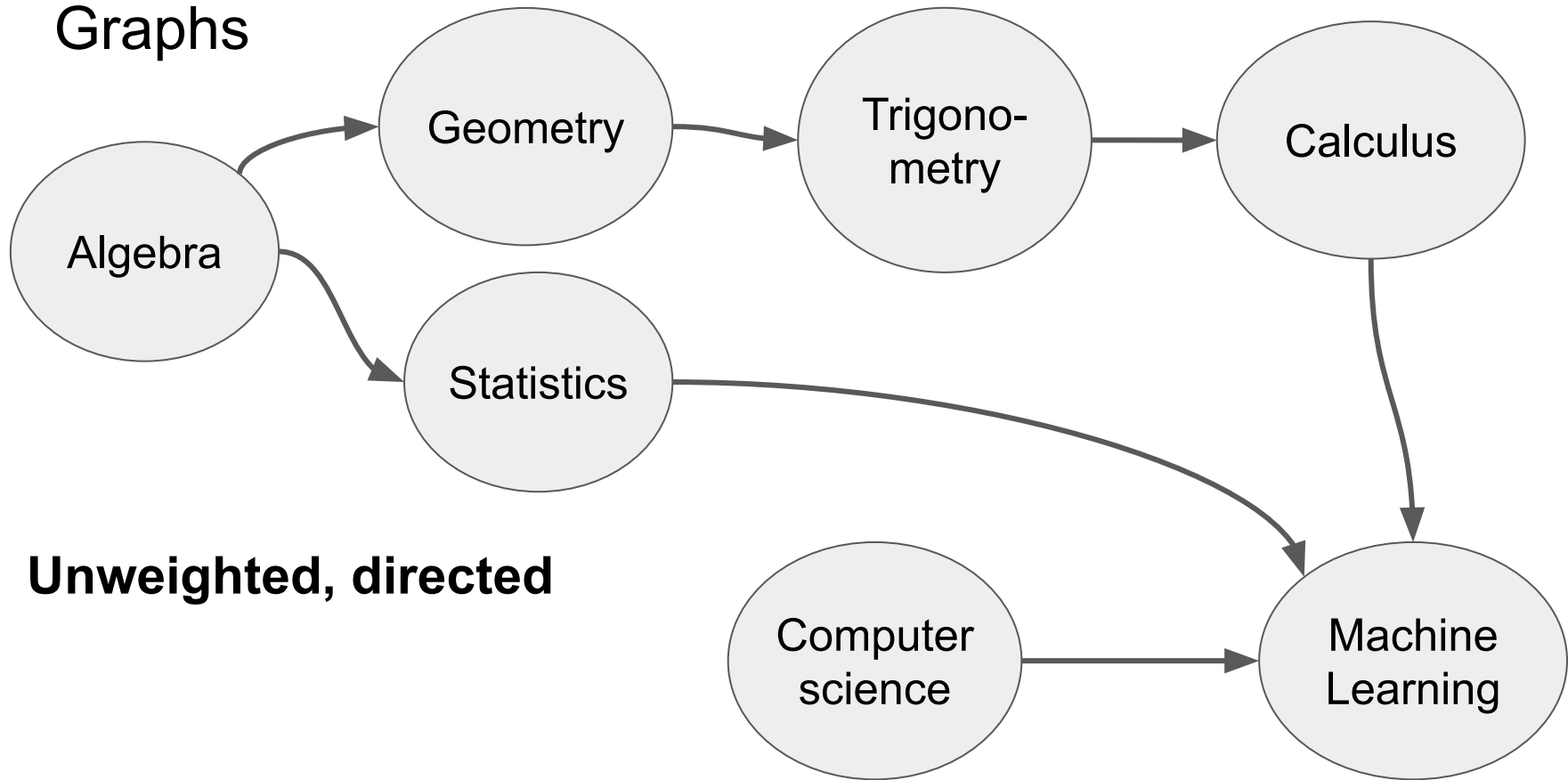
What is the cheapest way to get to Miami from LA?

Graphs



What type of a graph is this?

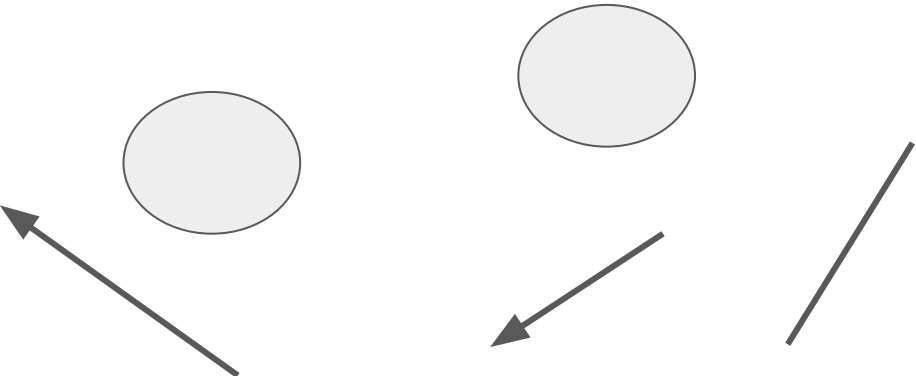
Graphs



Unweighted, directed

Graph representation

- Vertices
- Edges



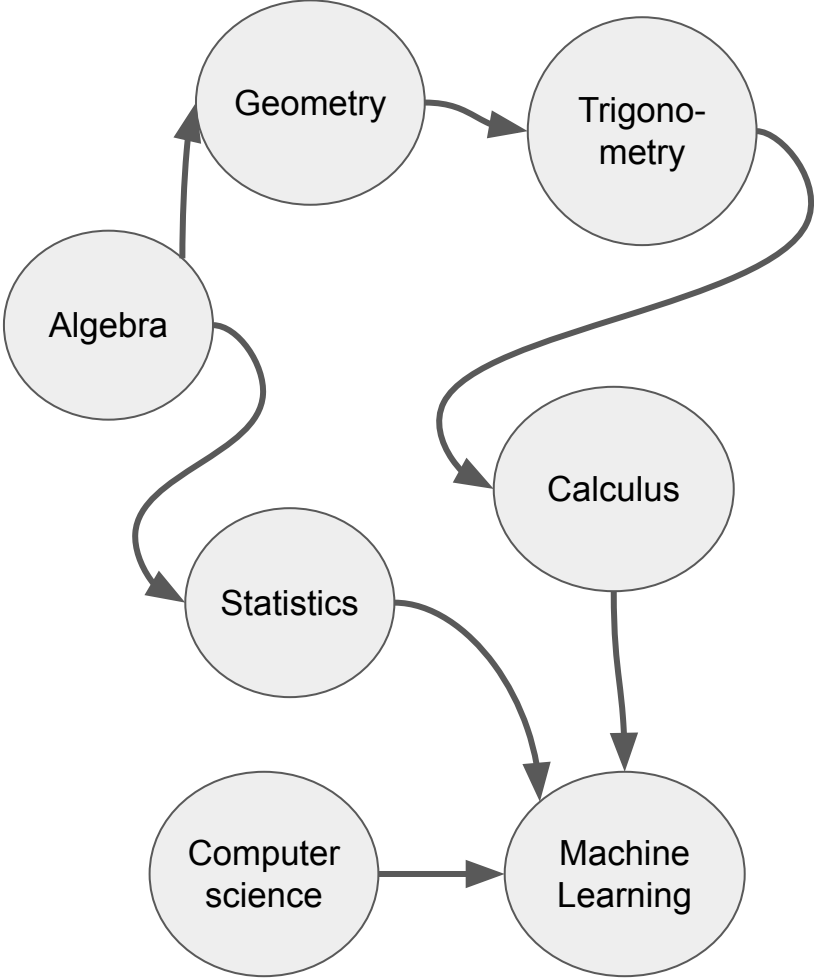
How to connect vertices through edges?

Graph applications examples

- **Networking:** finding the shortest paths, managing the flow
- **Google maps:** building transportation systems
- **Google page ranking:** evaluating pages by the number of references they have
- **Facebook:** friends suggestion algorithm
- **Neural networks**
- **Biological networks**

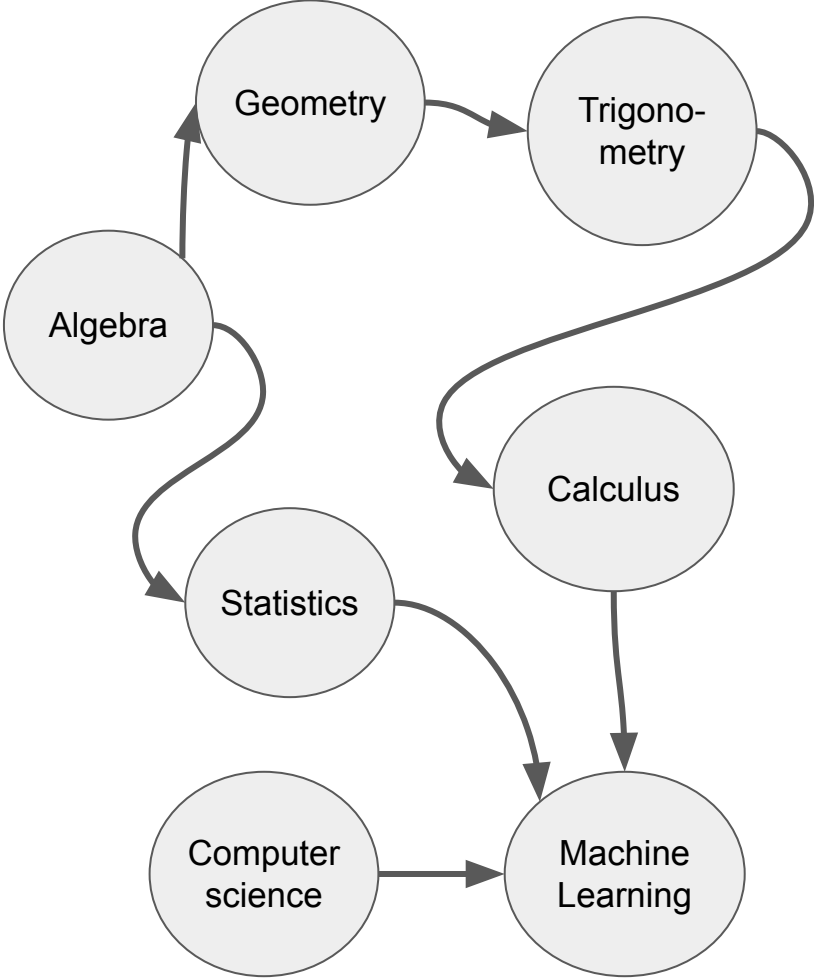
Graph representation: adjacency matrix

| | Alg | Geo | Tri | Cal | Sta | CS | ML |
|-----|-----|-----|-----|-----|-----|----|----|
| Alg | ■ | | | | | | |
| Geo | | ■ | | | | | |
| Tri | | | ■ | | | | |
| Cal | | | | ■ | | | |
| Sta | | | | | ■ | | |
| CS | | | | | | ■ | |
| ML | | | | | | | ■ |



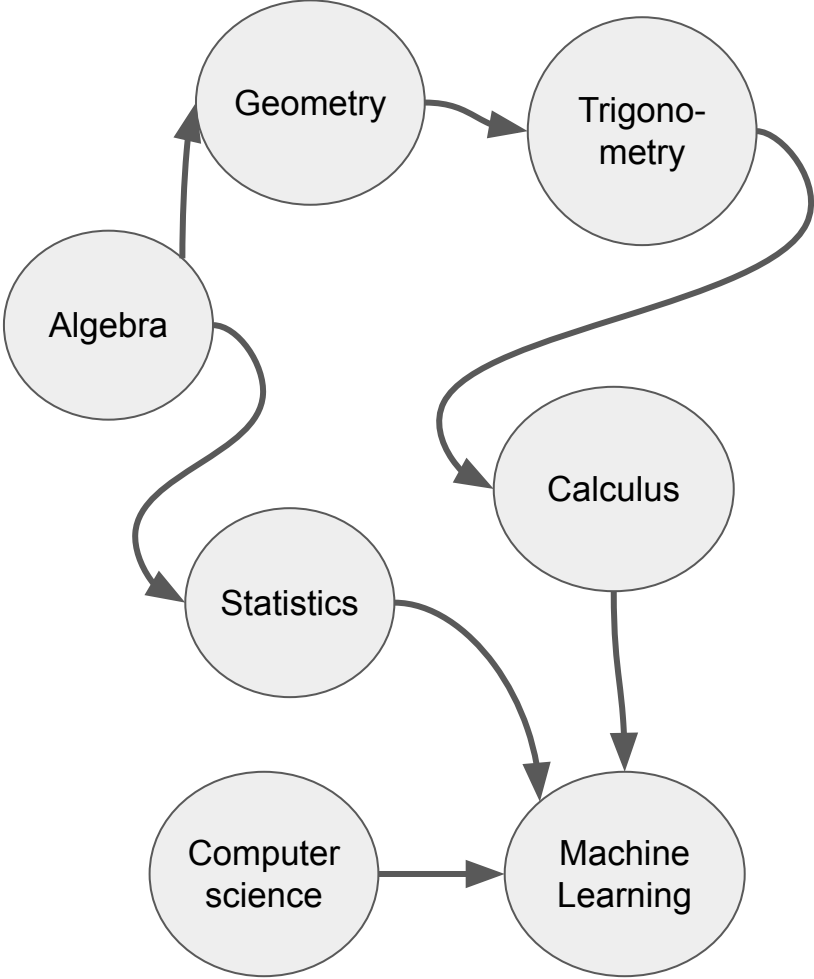
Graph representation: adjacency matrix

| | Alg | Geo | Tri | Cal | Sta | CS | ML |
|-----|-----|-----|-----|-----|-----|----|----|
| Alg | | 1 | | | | | |
| Geo | | | | | | | |
| Tri | | | | | | | |
| Cal | | | | | | | |
| Sta | | | | | | | |
| CS | | | | | | | |
| ML | | | | | | | |



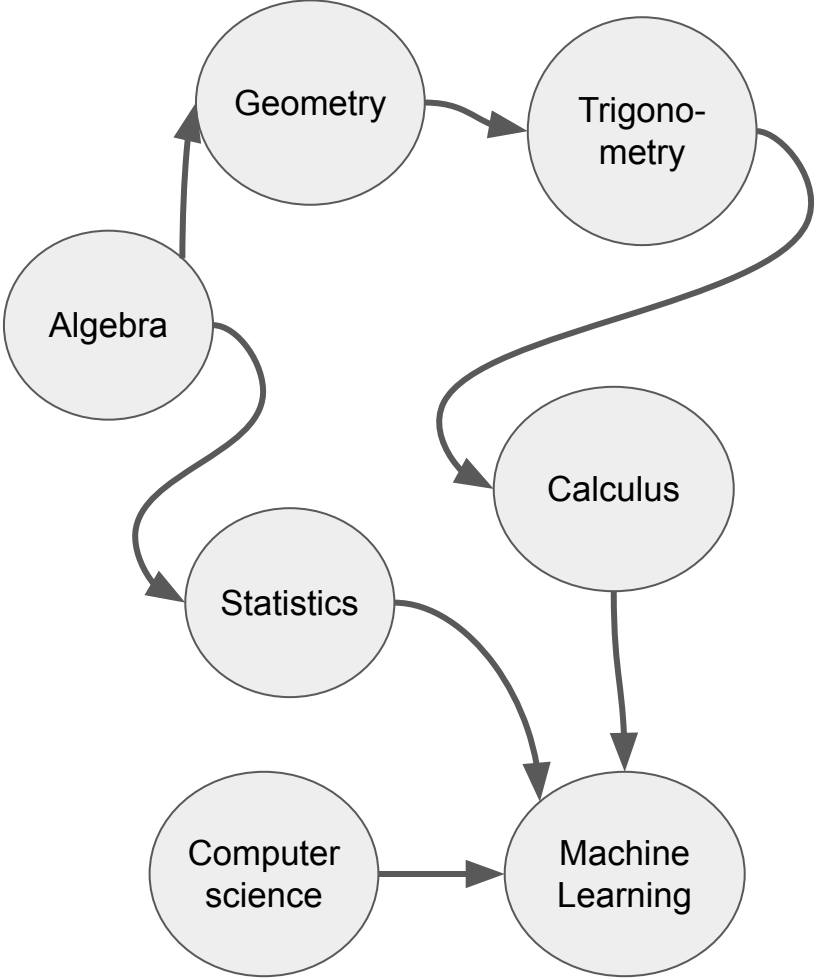
Graph representation: adjacency matrix

| | Alg | Geo | Tri | Cal | Sta | CS | ML |
|-----|-----|-----|-----|-----|-----|----|----|
| Alg | | 1 | 0 | 0 | 1 | 0 | 0 |
| Geo | | | | | | | |
| Tri | | | | | | | |
| Cal | | | | | | | |
| Sta | | | | | | | |
| CS | | | | | | | |
| ML | | | | | | | |



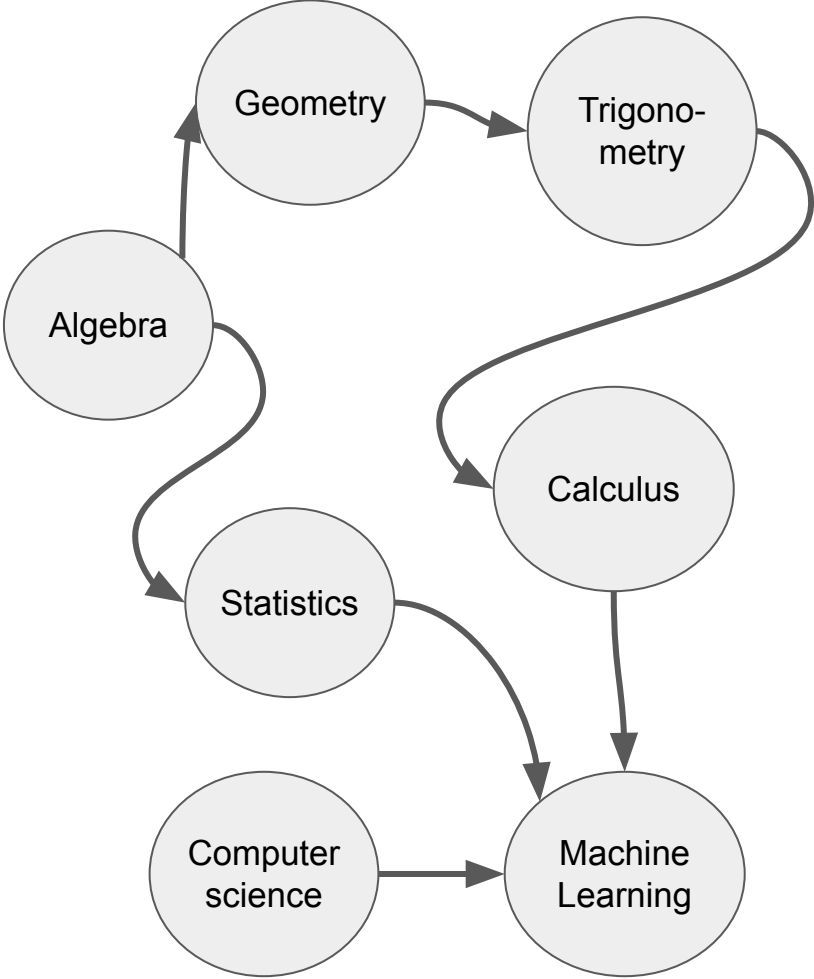
Graph representation: adjacency matrix

| | Alg | Geo | Tri | Cal | Sta | CS | ML |
|-----|-----|-----|-----|-----|-----|----|----|
| Alg | | 1 | 0 | 0 | 1 | 0 | 0 |
| Geo | 0 | | 1 | 0 | 0 | 0 | 0 |
| Tri | | | | | | | |
| Cal | | | | | | | |
| Sta | | | | | | | |
| CS | | | | | | | |
| ML | | | | | | | |



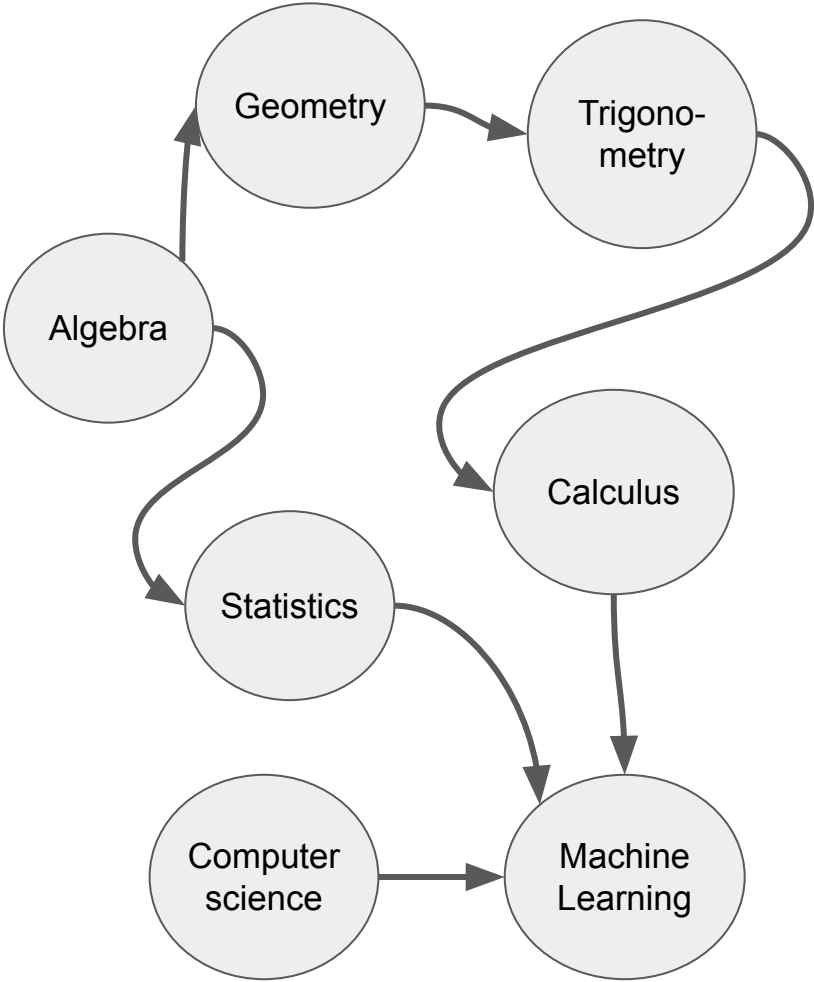
Graph representation: adjacency matrix

| | Alg | Geo | Tri | Cal | Sta | CS | ML |
|-----|-----|-----|-----|-----|-----|----|----|
| Alg | | 1 | 0 | 0 | 1 | 0 | 0 |
| Geo | 0 | | 1 | 0 | 0 | 0 | 0 |
| Tri | 0 | 0 | | 1 | 0 | 0 | 0 |
| Cal | | | | | | | |
| Sta | | | | | | | |
| CS | | | | | | | |
| ML | | | | | | | |



Graph representation: adjacency matrix

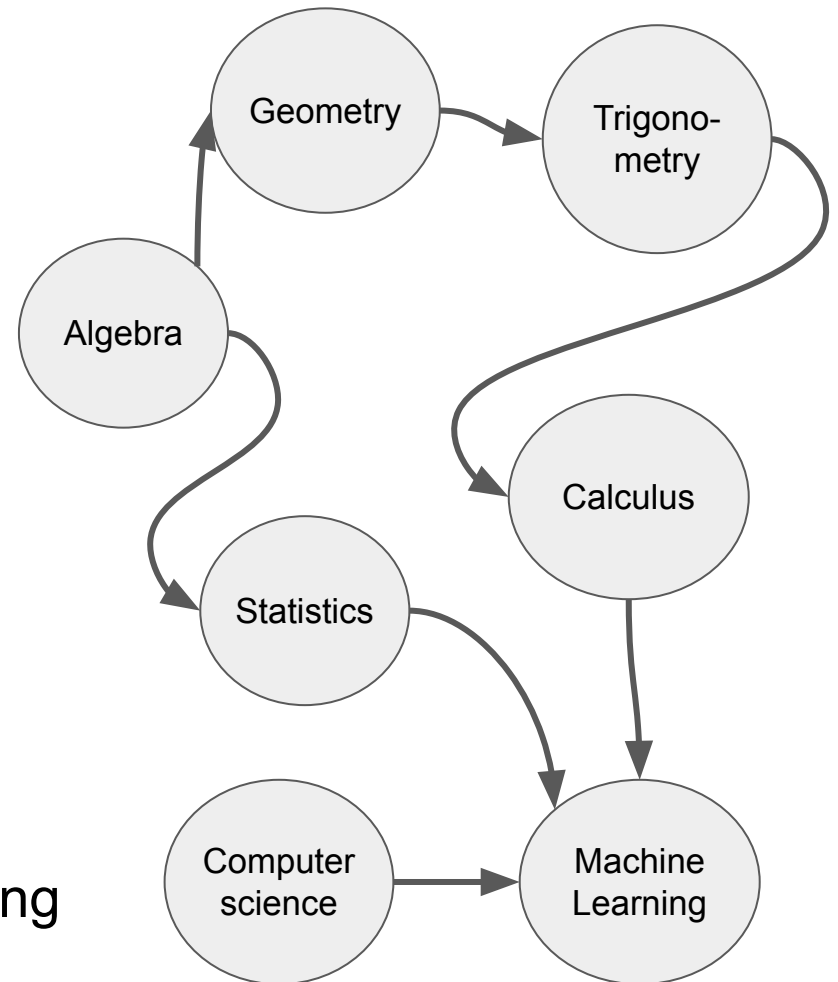
| | Alg | Geo | Tri | Cal | Sta | CS | ML |
|-----|-----|-----|-----|-----|-----|----|----|
| Alg | | 1 | 0 | 0 | 1 | 0 | 0 |
| Geo | 0 | | 1 | 0 | 0 | 0 | 0 |
| Tri | 0 | 0 | | 1 | 0 | 0 | 0 |
| Cal | 0 | 0 | 0 | | 0 | 0 | 1 |
| Sta | 0 | 0 | 0 | 0 | | 0 | 1 |
| CS | 0 | 0 | 0 | 0 | 0 | | 1 |
| ML | 0 | 0 | 0 | 0 | 0 | 0 | |



Graph representation: adjacency list

keys **values**

- **Algebra:** Geometry, Statistics
- **Geometry:** Trigonometry
- **Trigonometry:** Calculus
- **Calculus:** Machine Learning
- **Statistics:** Machine Learning
- **Computer Science:** Machine learning



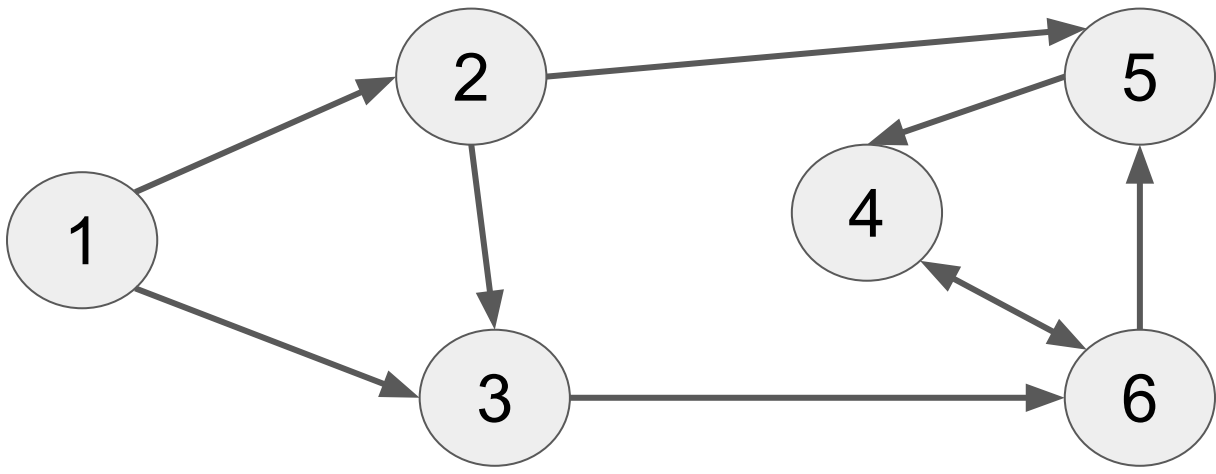
Graph traversals

- The process of visiting each vertex in a graph.
 - Checking, updating vertexes during the visit.
- Is there a challenge compared to tree traversals?
 - A different structure: node can be visited multiple times
- Traversal algorithms:
 - Breadth first search
 - Depth first search

Traversal algorithms

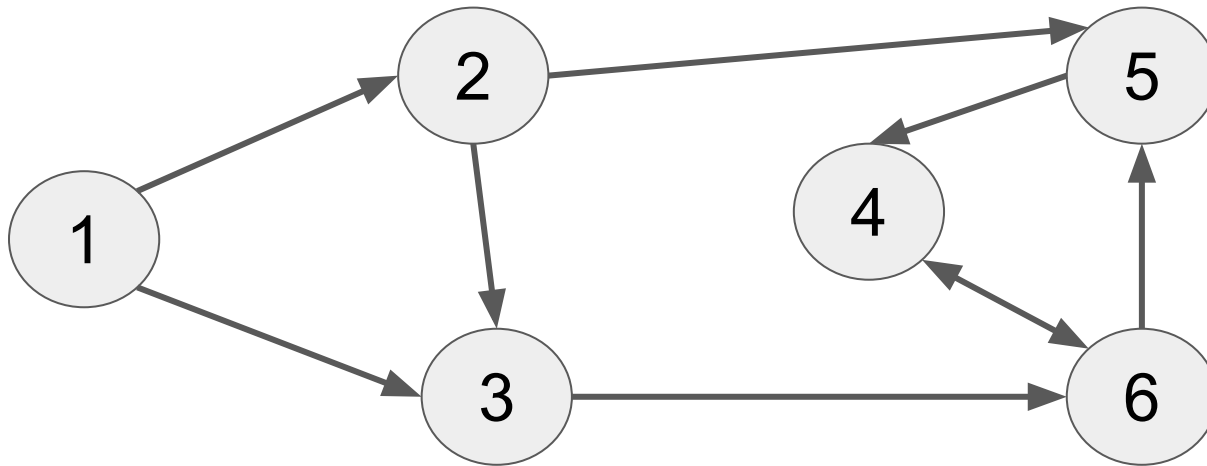
- DFS: visiting a child before siblings
 - Implementation?
 - Adding children into a stack
- BFS: visiting a sibling before visiting children
 - Implementation?
 - Adding children into a queue
- **Practice: implement BFS and DFS for a graph**

Depth-first search

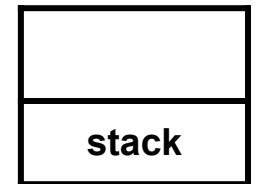


dfs(node=1, graph)

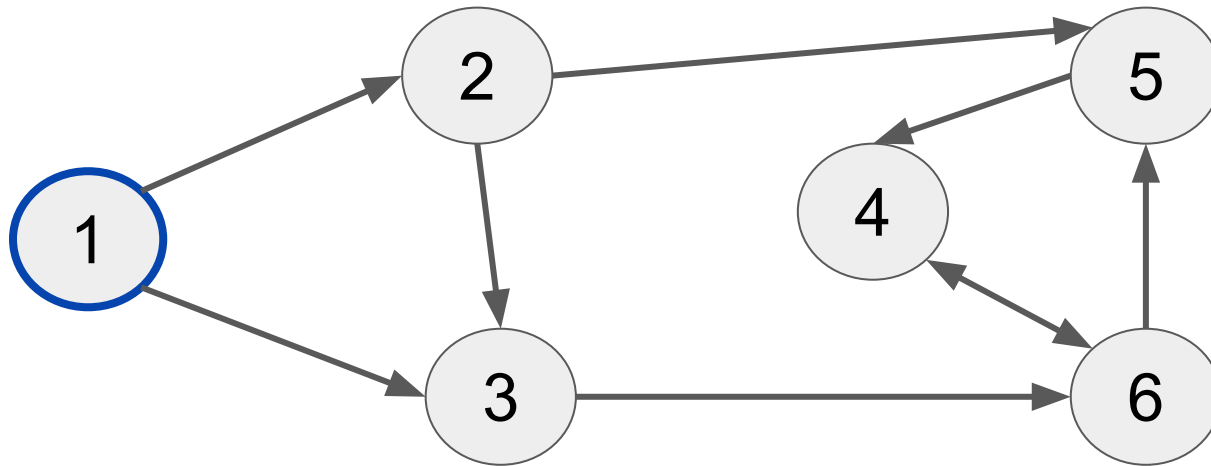
Depth-first search



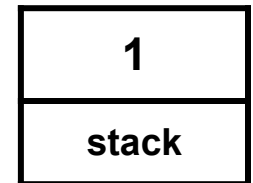
dfs(node=1, graph)



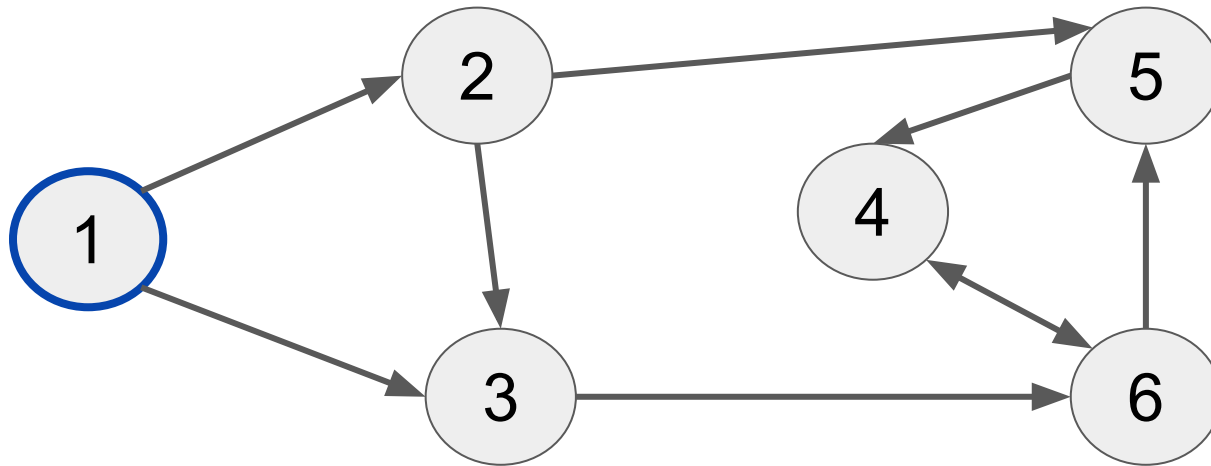
Depth-first search



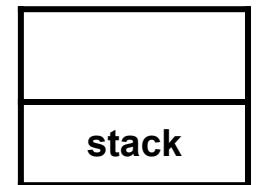
dfs(node=1, graph)



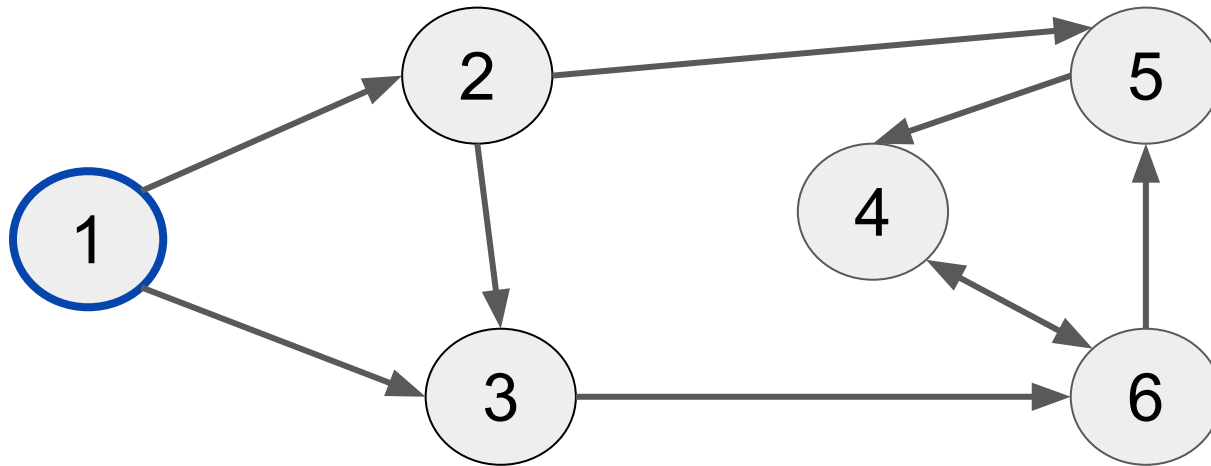
Depth-first search



dfs(node=1, graph)



Depth-first search

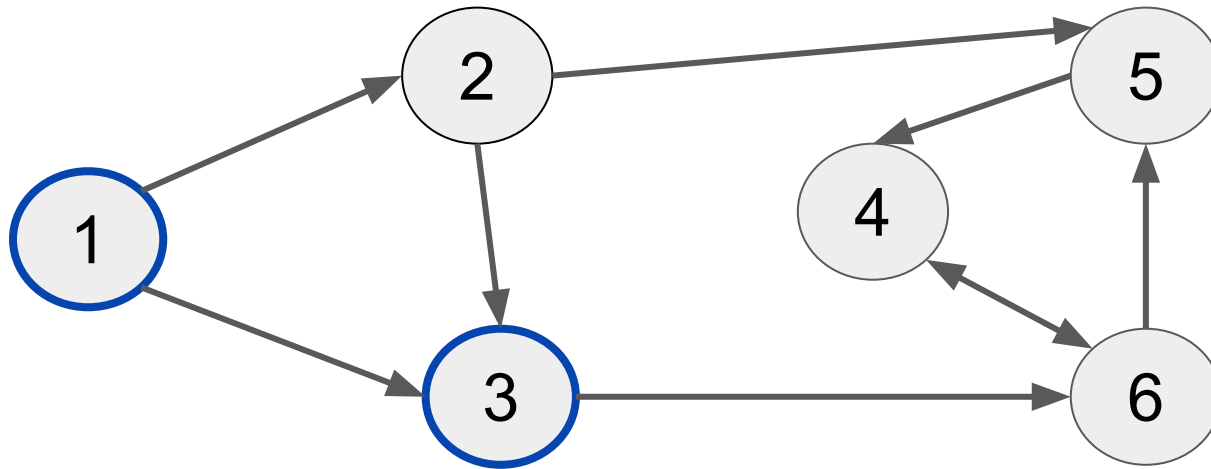


dfs(node=1, graph)

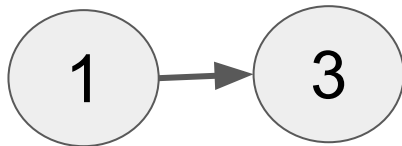


| |
|-------|
| 3 |
| 2 |
| stack |

Depth-first search

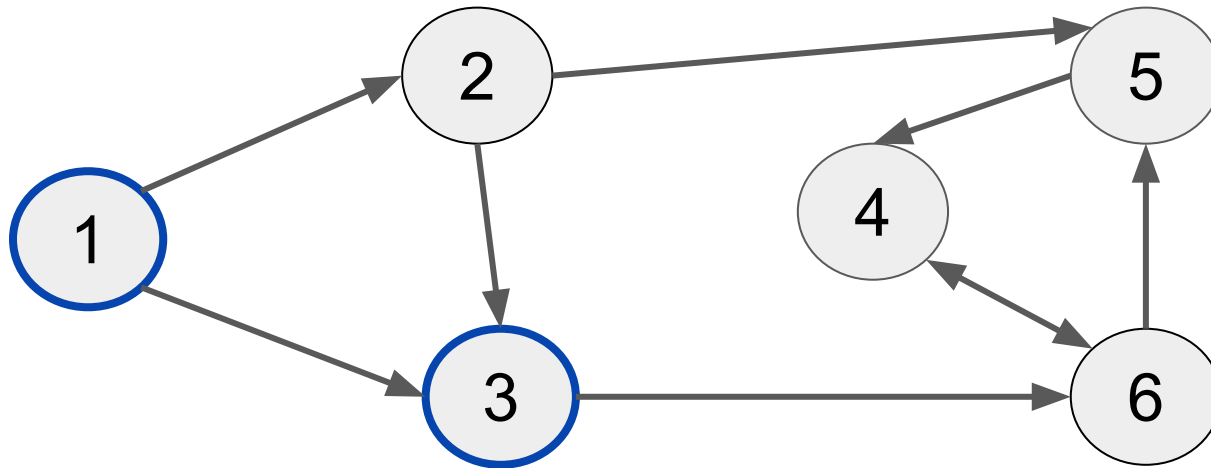


dfs(node=1, graph)

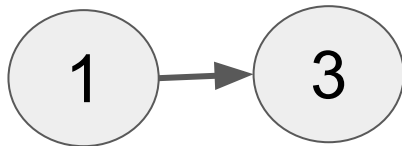


| |
|-------|
| 2 |
| stack |

Depth-first search

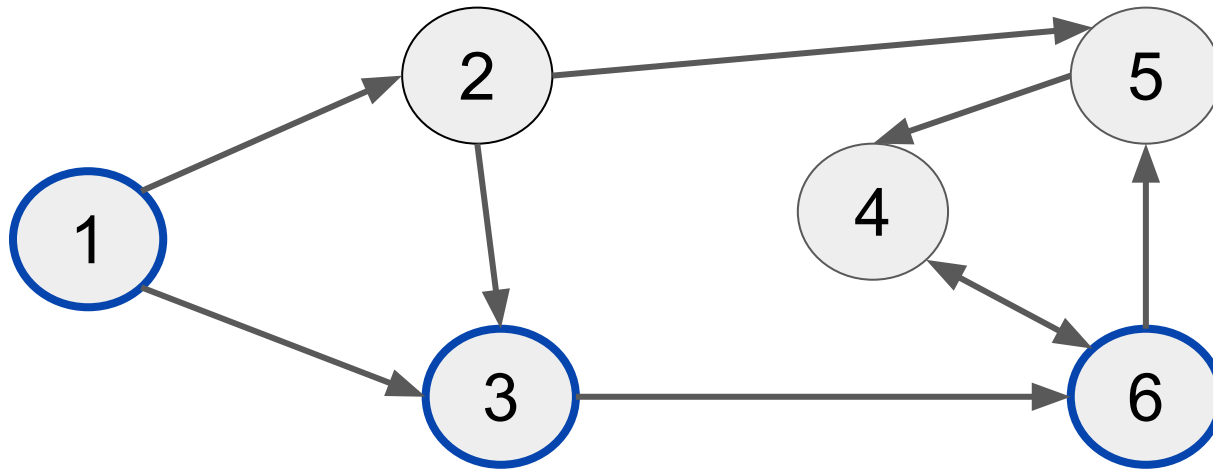


dfs(node=1, graph)

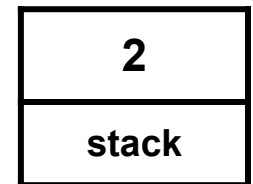
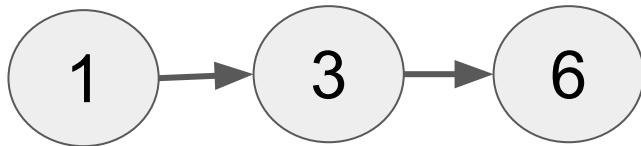


| |
|-------|
| 6 |
| 2 |
| stack |

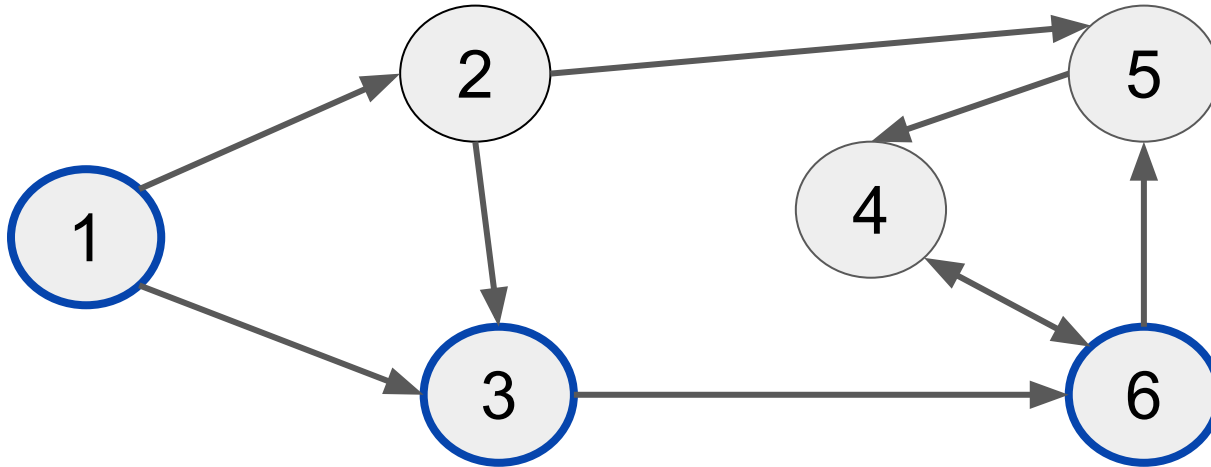
Depth-first search



dfs(node=1, graph)



Depth-first search

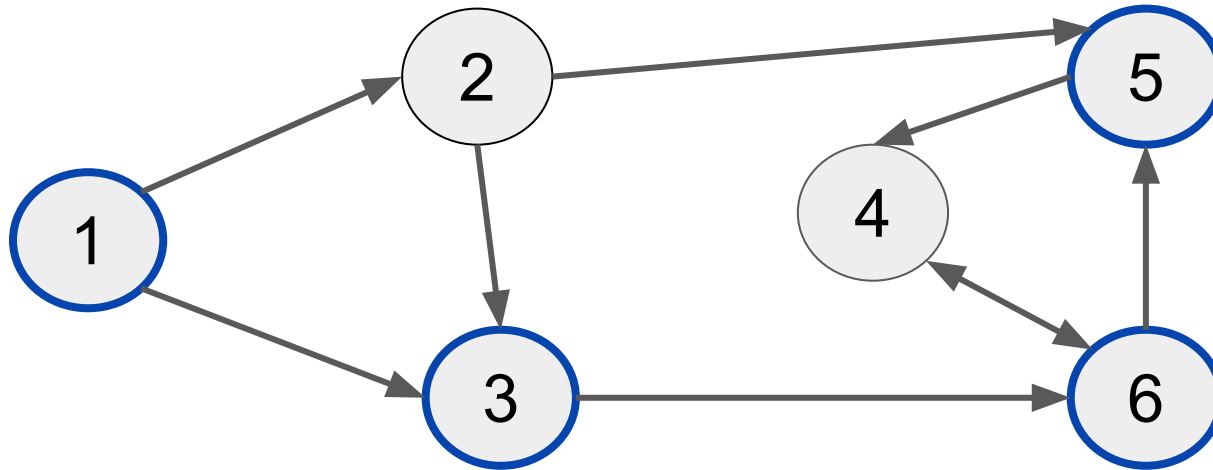


dfs(node=1, graph)

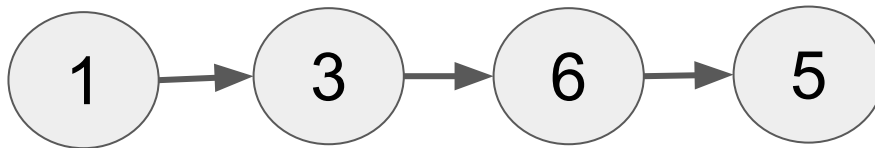


| |
|-------|
| 5 |
| 4 |
| 2 |
| stack |

Depth-first search

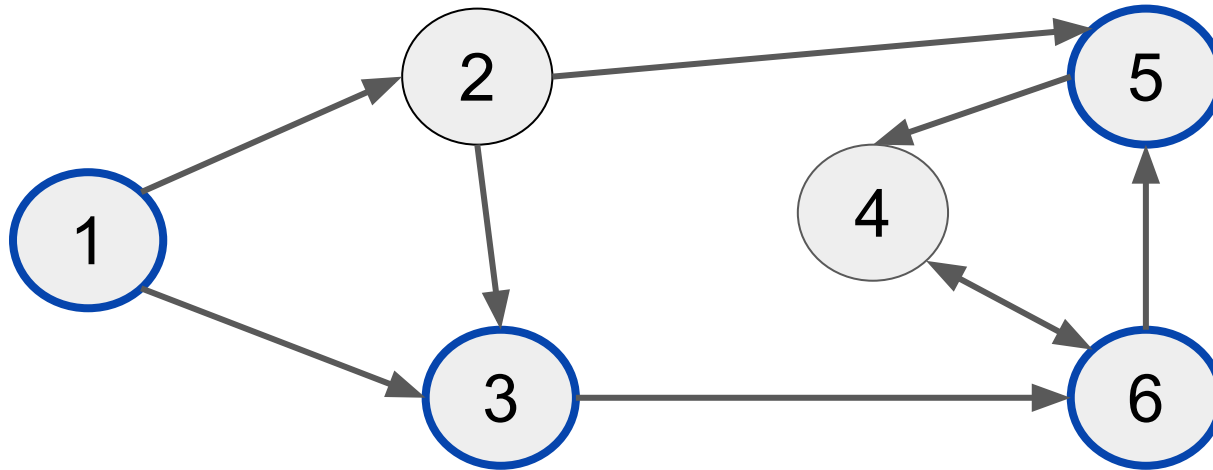


dfs(node=1, graph)

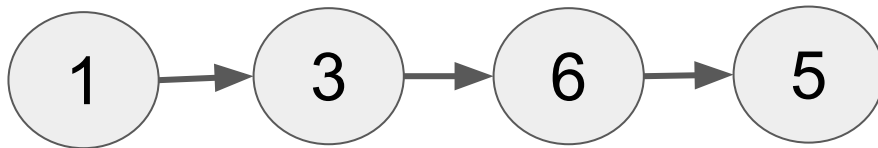


| |
|-------|
| 4 |
| 2 |
| stack |

Depth-first search

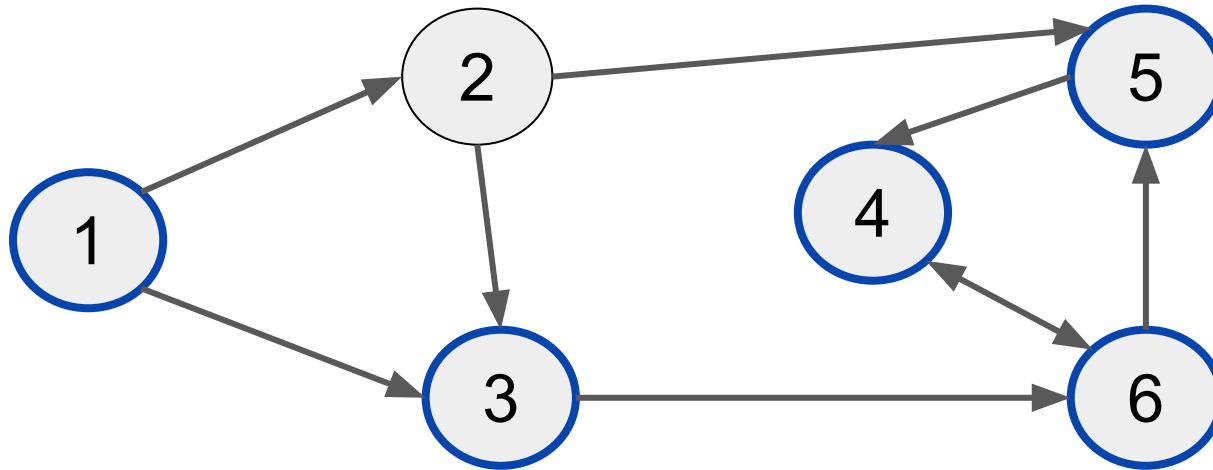


dfs(node=1, graph)

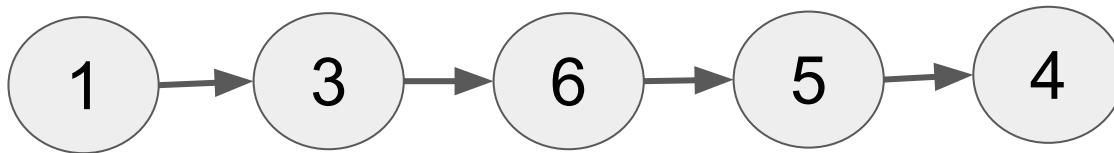


| |
|-------|
| 4 |
| 4 |
| 2 |
| stack |

Depth-first search

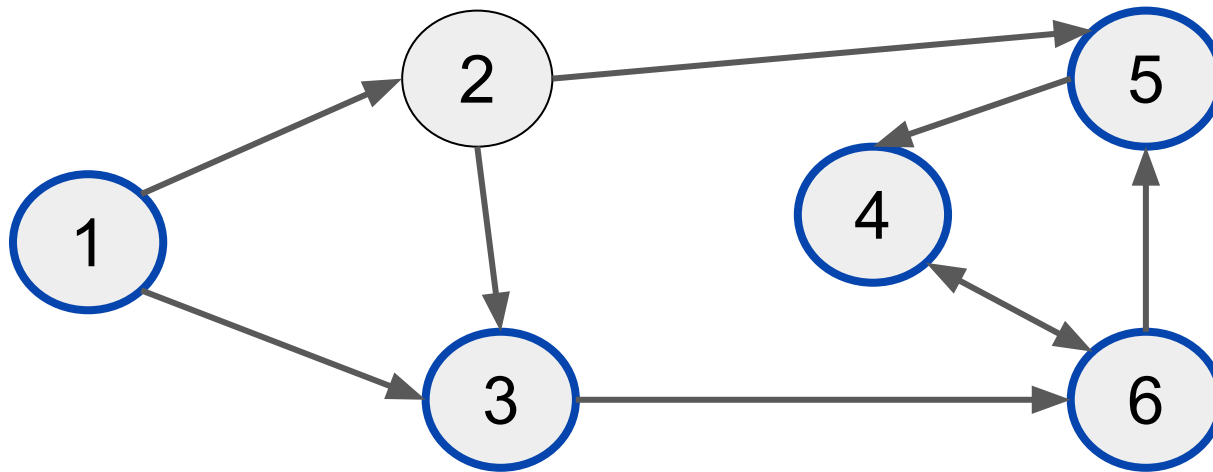


dfs(node=1, graph)

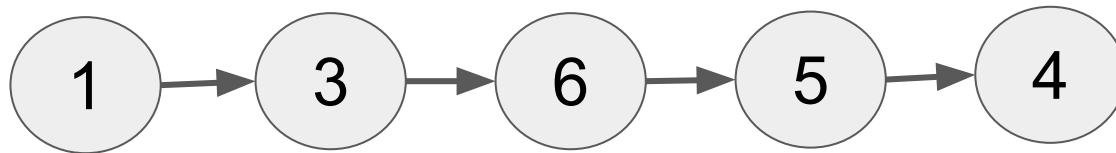


| |
|-------|
| 4 |
| 2 |
| stack |

Depth-first search

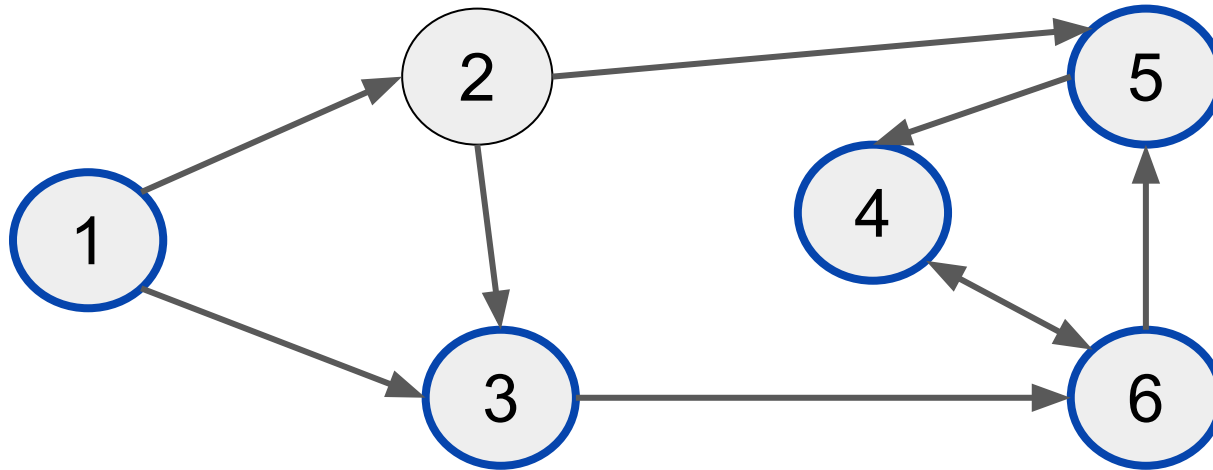


dfs(node=1, graph)

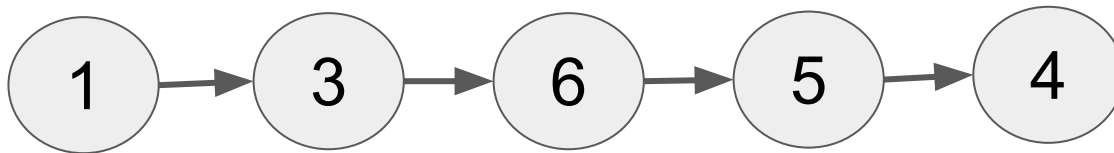


| |
|-------|
| 6 |
| 4 |
| 2 |
| stack |

Depth-first search

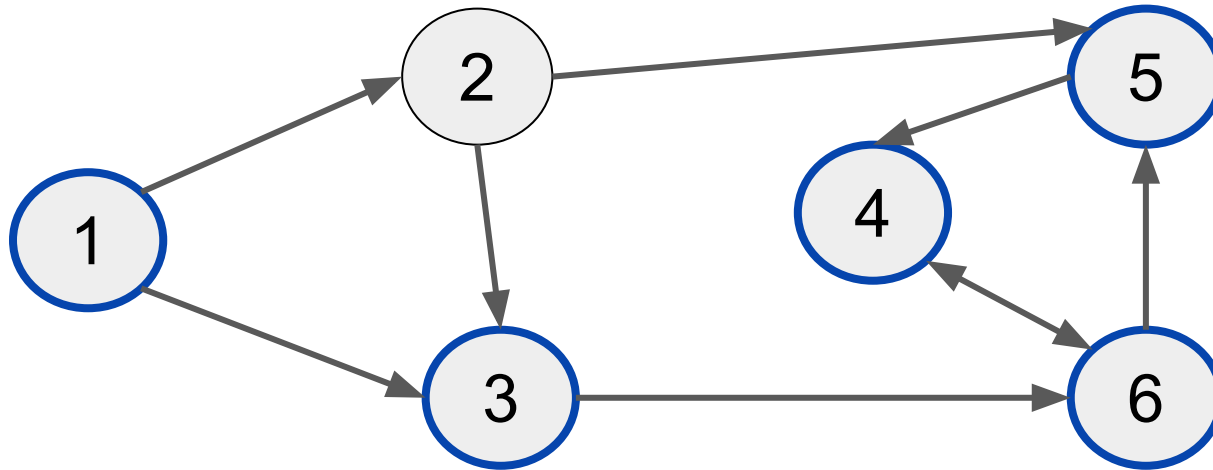


dfs(node=1, graph)

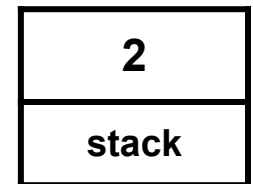
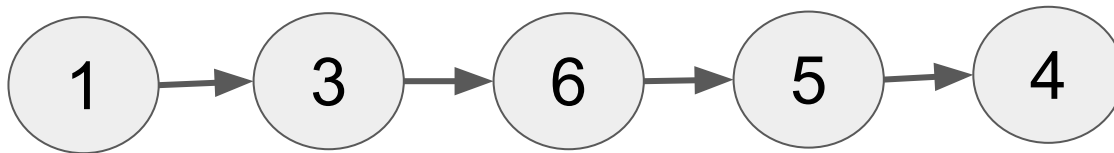


| |
|-------|
| 4 |
| 2 |
| stack |

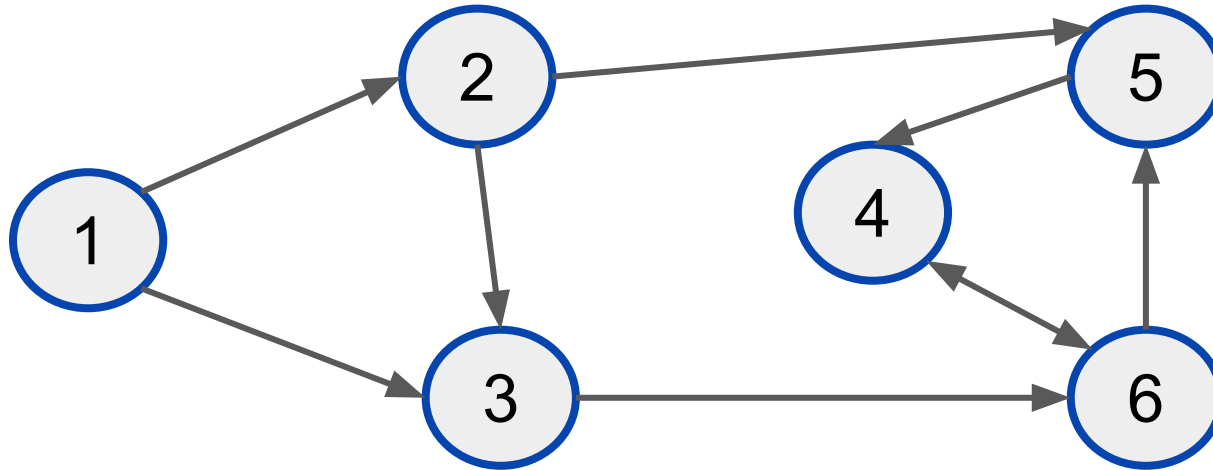
Depth-first search



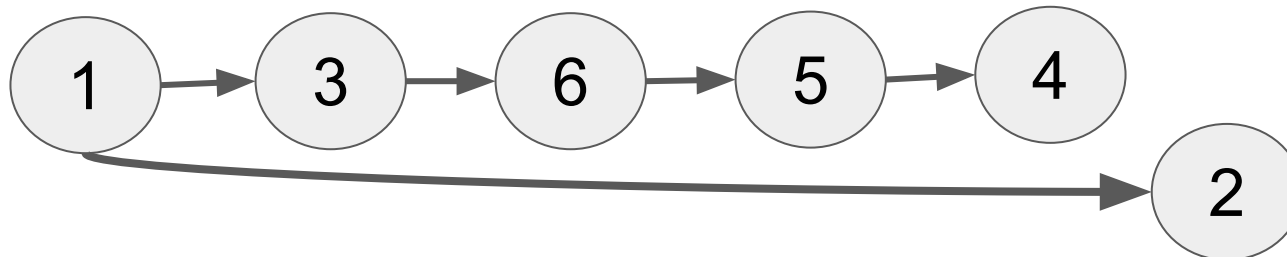
dfs(node=1, graph)



Depth-first search

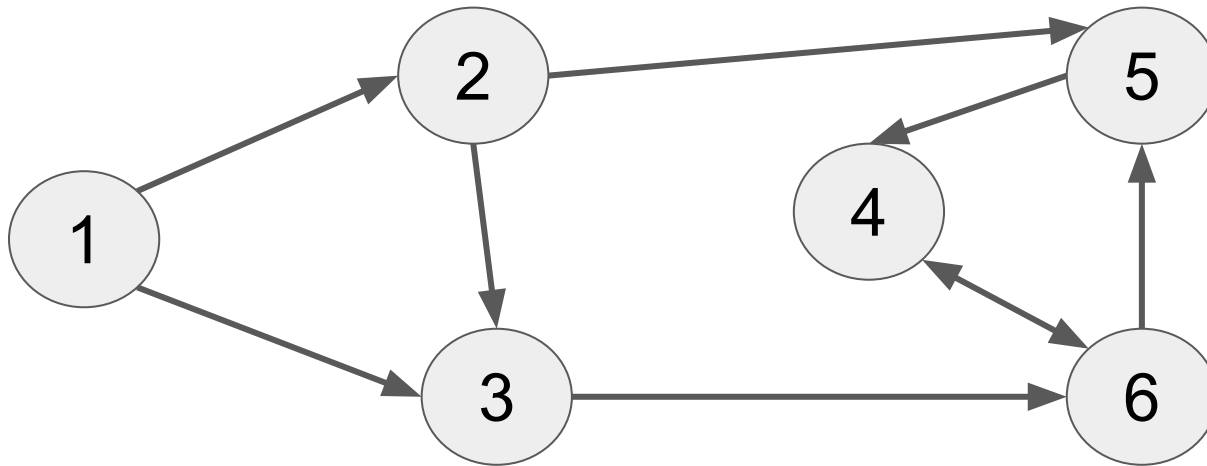


dfs(node=1, graph)



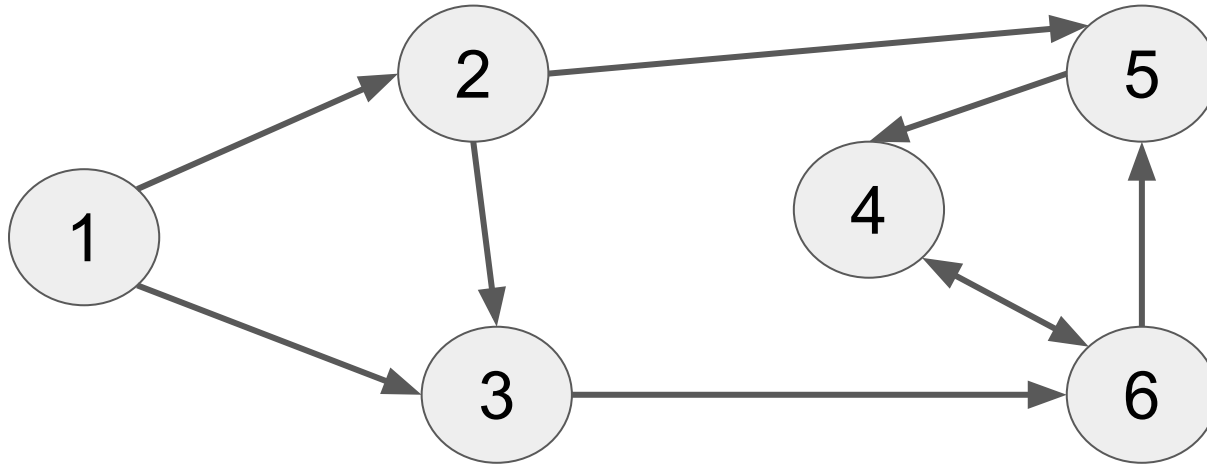
stack

Breadth-first search



bfs(node=1, graph)

Breadth-first search

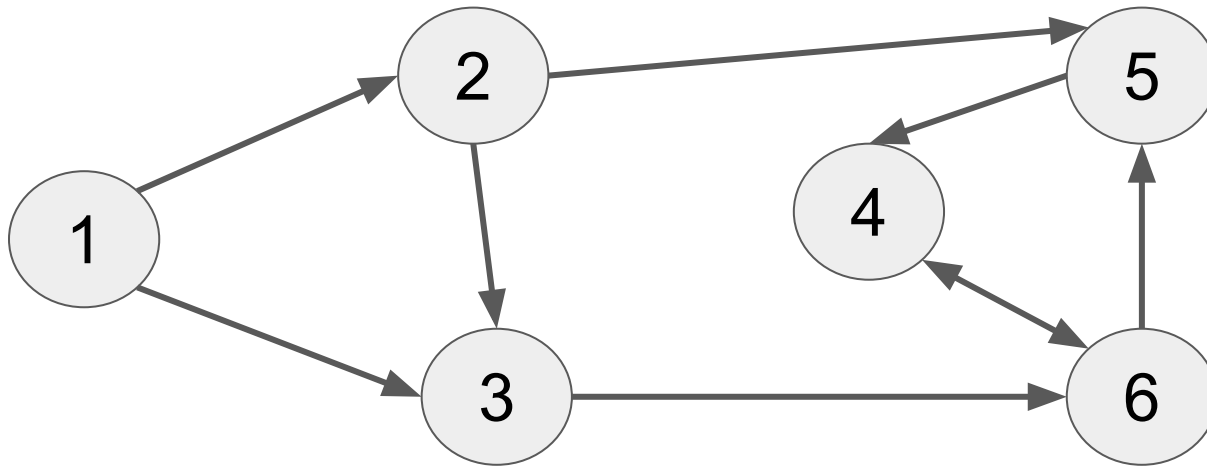


bfs(node=1, graph)

| | |
|-------|------|
| front | rear |
|-------|------|

Queue

Breadth-first search

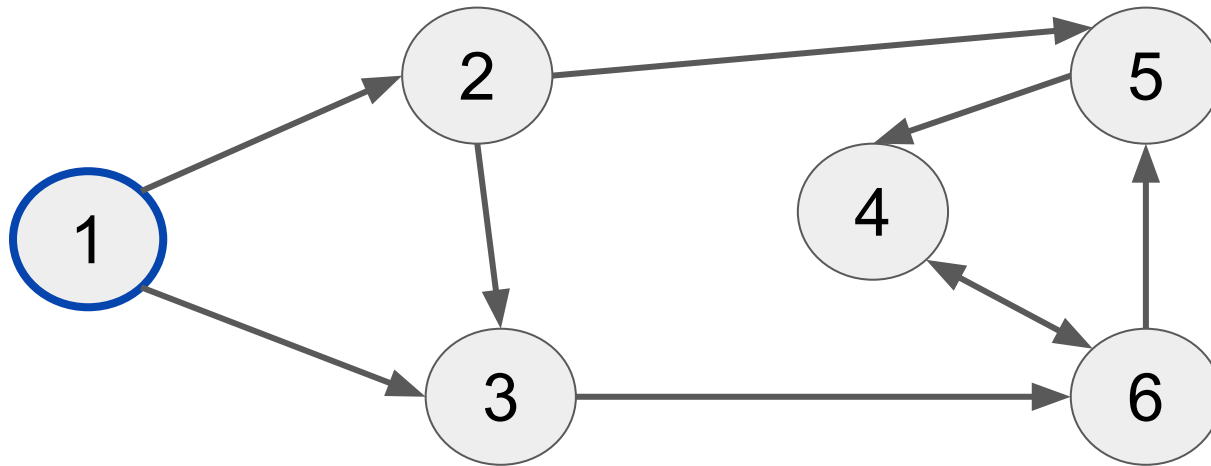


bfs(node=1, graph)

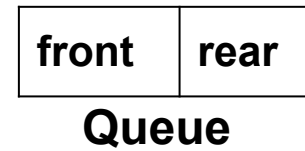
| | | |
|-------|---|------|
| front | 1 | rear |
|-------|---|------|

Queue

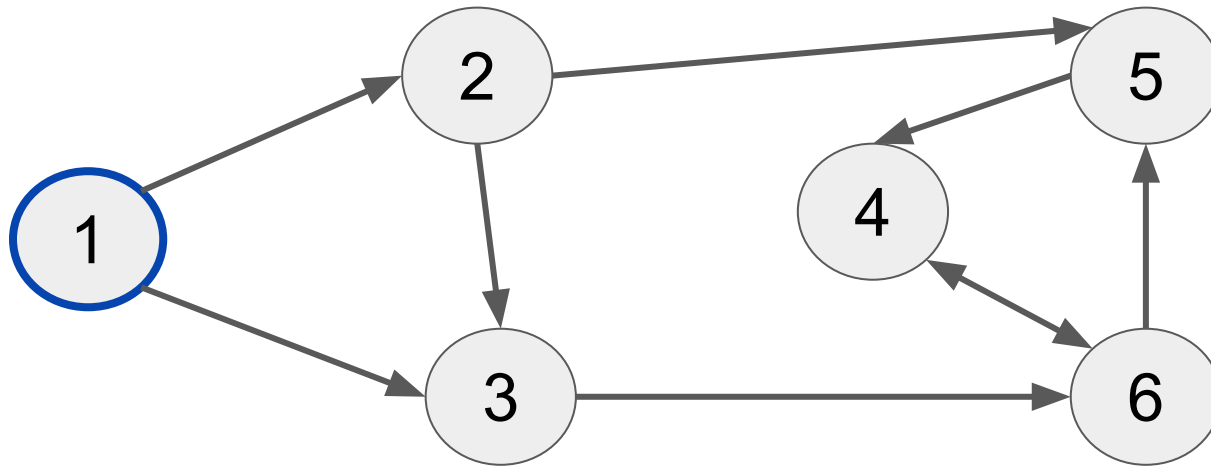
Breadth-first search



bfs(node=1, graph)



Breadth-first search



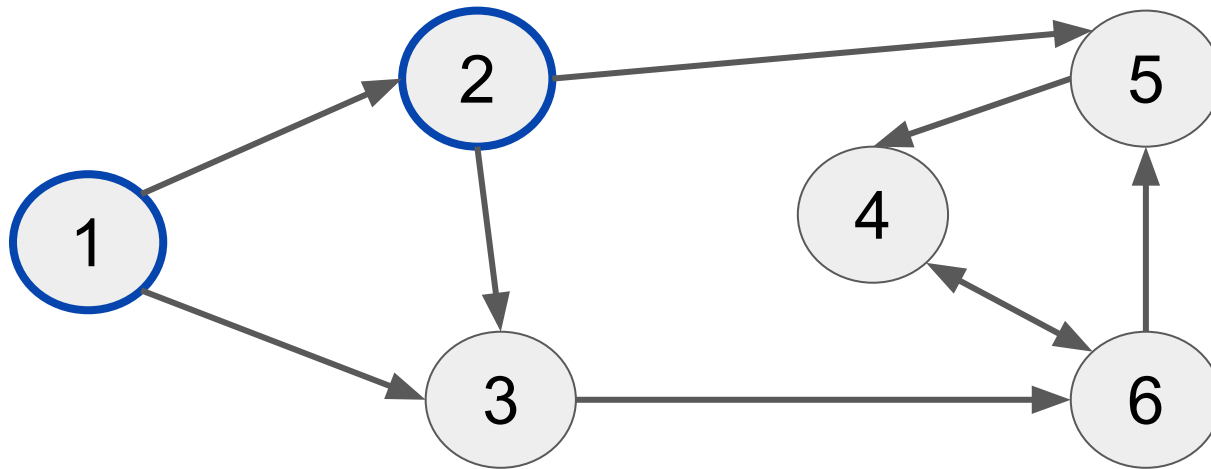
bfs(node=1, graph)



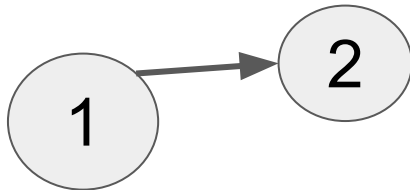
| | | | |
|-------|---|---|------|
| front | 2 | 3 | rear |
|-------|---|---|------|

Queue

Breadth-first search



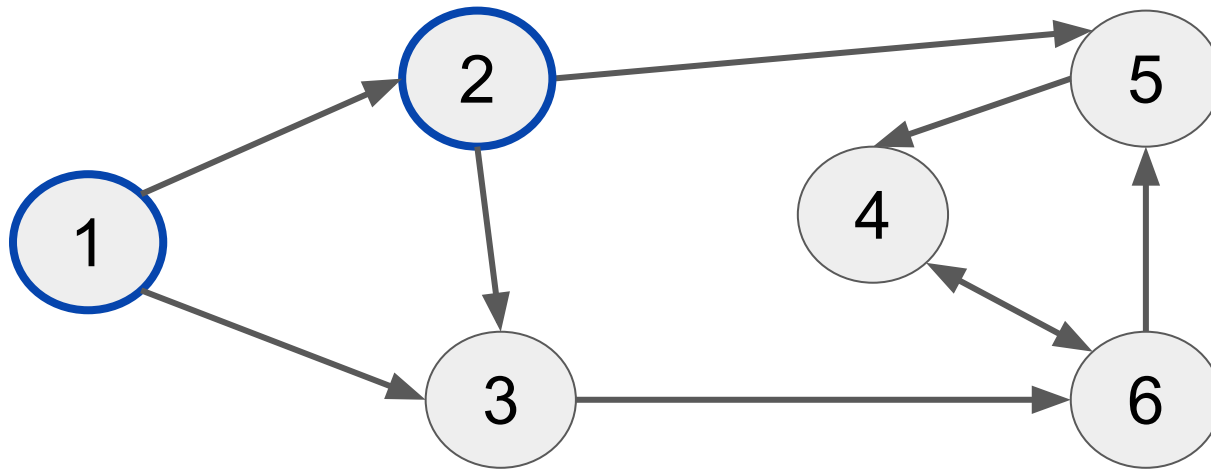
bfs(node=1, graph)



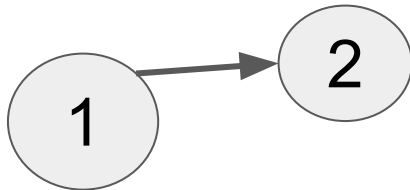
| | | |
|-------|---|------|
| front | 3 | rear |
|-------|---|------|

Queue

Breadth-first search



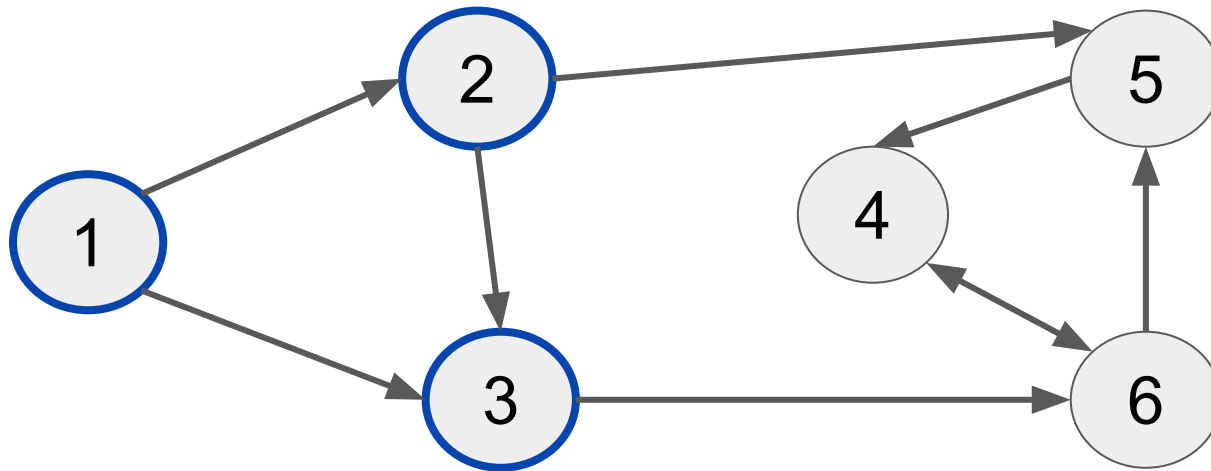
bfs(node=1, graph)



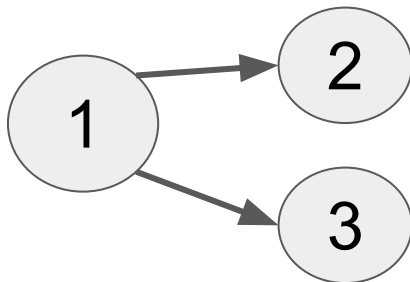
| | | | | |
|-------|---|---|---|------|
| front | 3 | 5 | 3 | rear |
|-------|---|---|---|------|

Queue

Breadth-first search



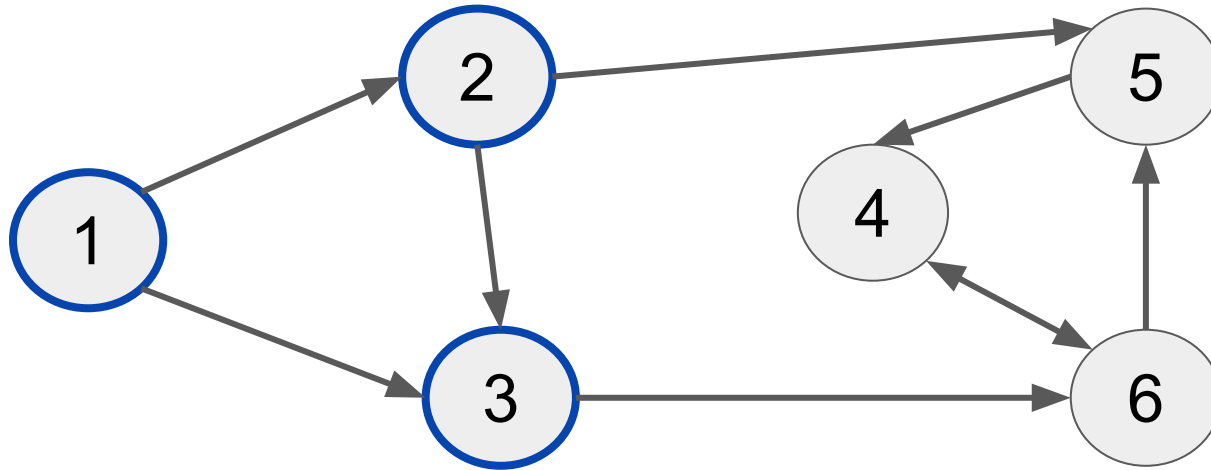
bfs(node=1, graph)



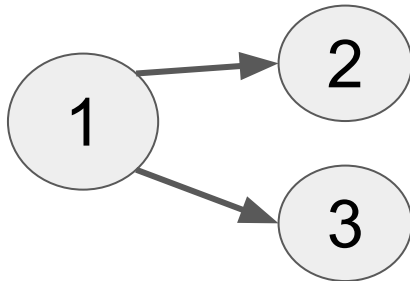
| | | | |
|-------|---|---|------|
| front | 5 | 3 | rear |
|-------|---|---|------|

Queue

Breadth-first search



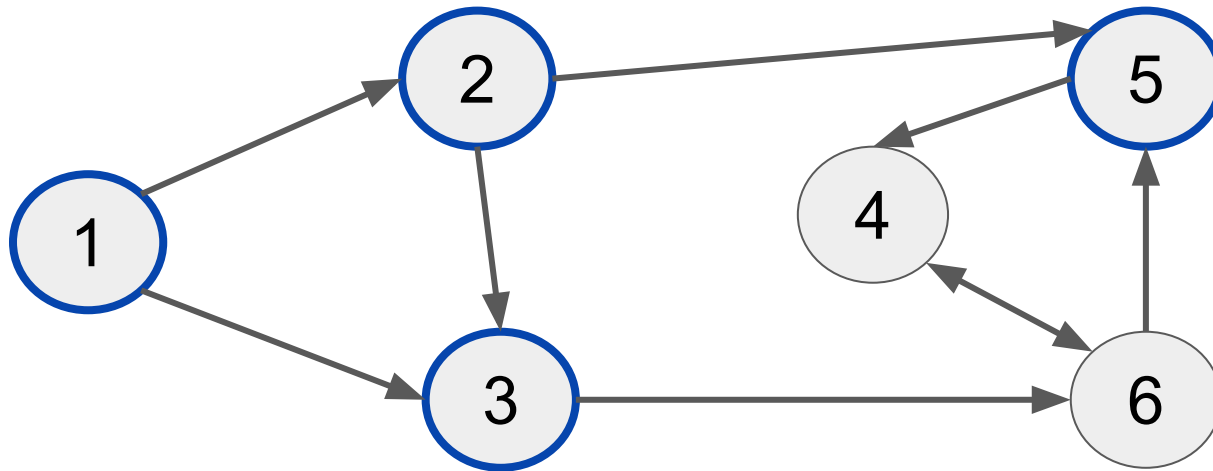
bfs(node=1, graph)



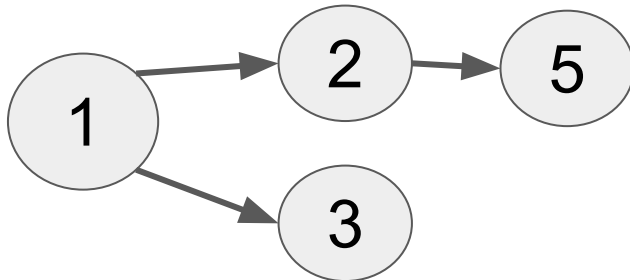
| | | | | |
|-------|---|---|---|------|
| front | 5 | 3 | 6 | rear |
|-------|---|---|---|------|

Queue

Breadth-first search



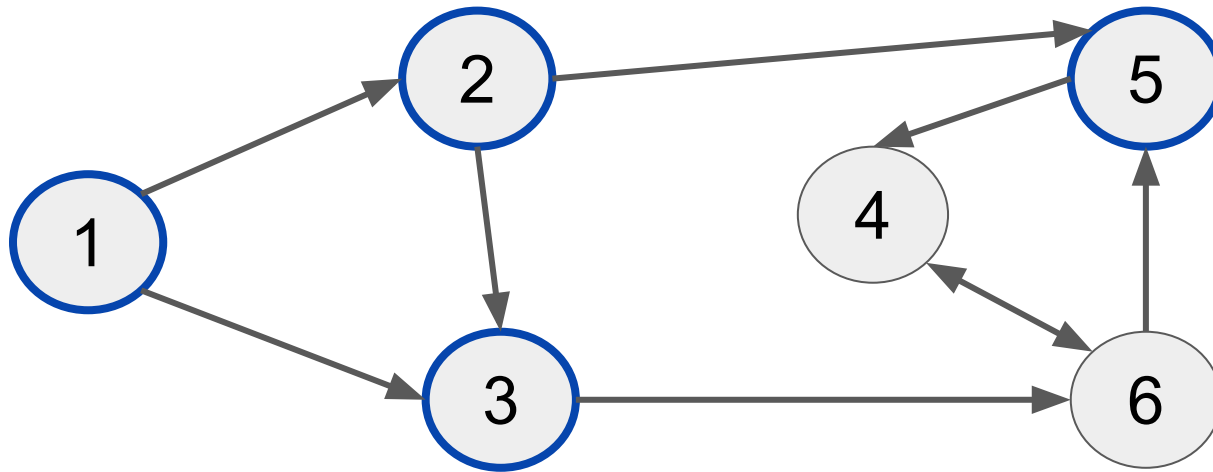
bfs(node=1, graph)



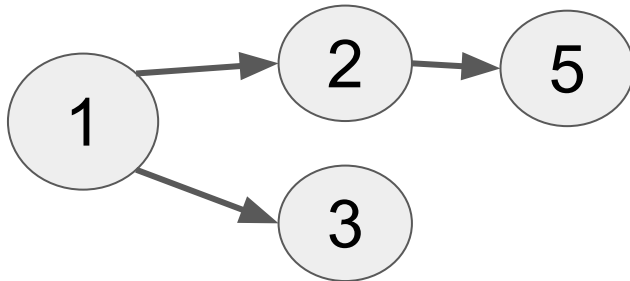
| | | | |
|-------|---|---|------|
| front | 3 | 6 | rear |
|-------|---|---|------|

Queue

Breadth-first search



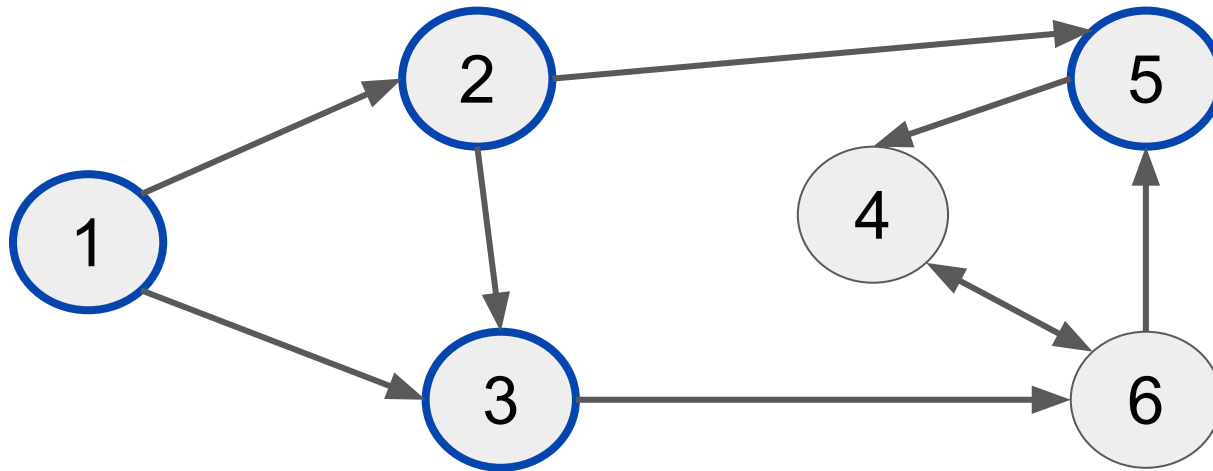
bfs(node=1, graph)



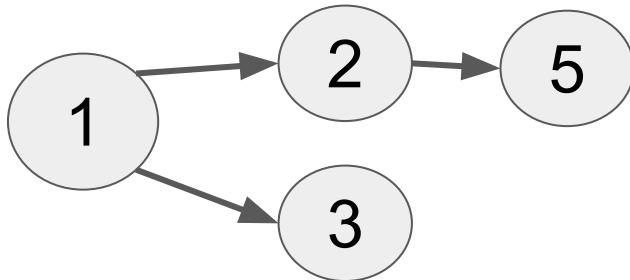
| | | | | |
|-------|---|---|---|------|
| front | 3 | 6 | 4 | rear |
|-------|---|---|---|------|

Queue

Breadth-first search



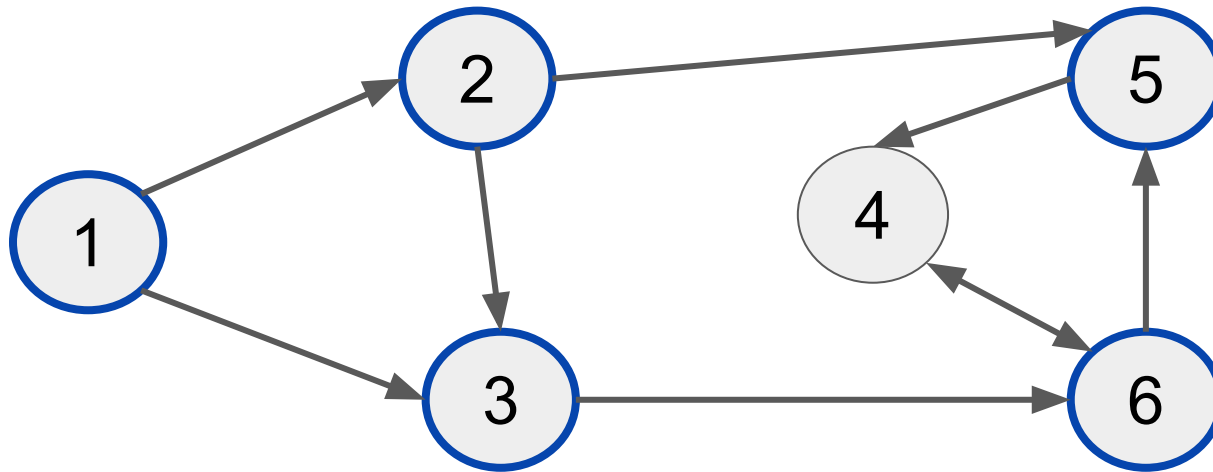
bfs(node=1, graph)



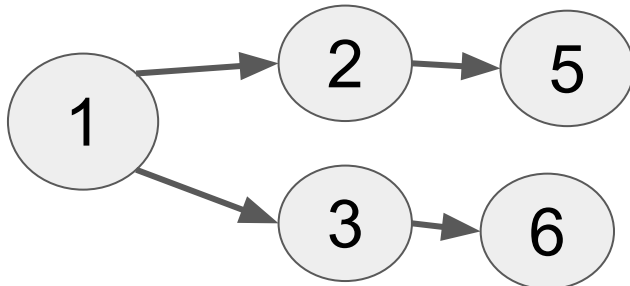
| | | | |
|-------|---|---|------|
| front | 6 | 4 | rear |
|-------|---|---|------|

Queue

Breadth-first search



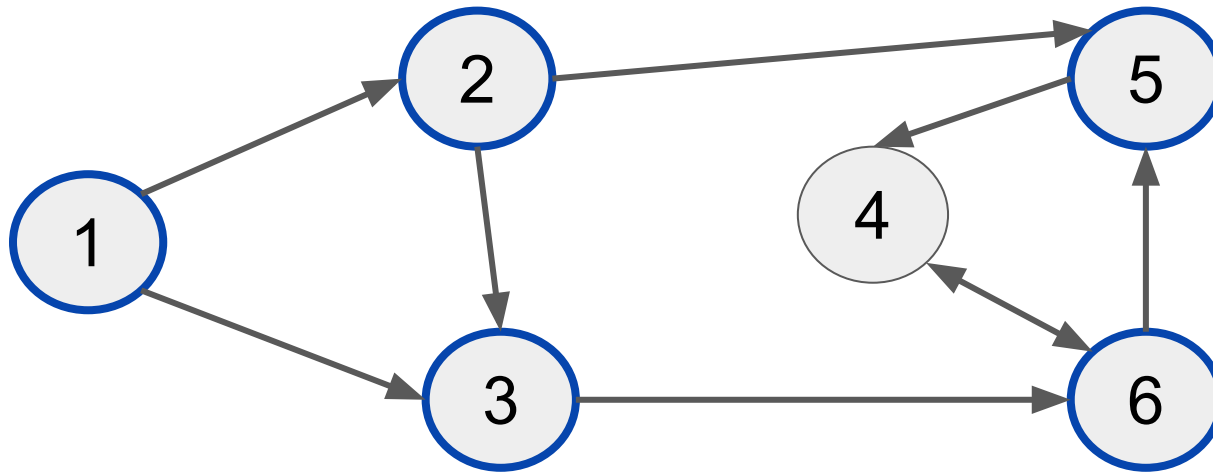
bfs(node=1, graph)



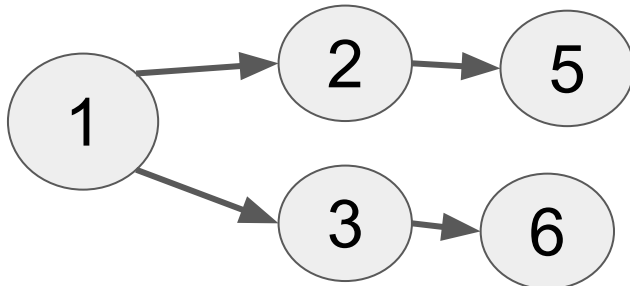
| | | |
|-------|---|------|
| front | 4 | rear |
|-------|---|------|

Queue

Breadth-first search



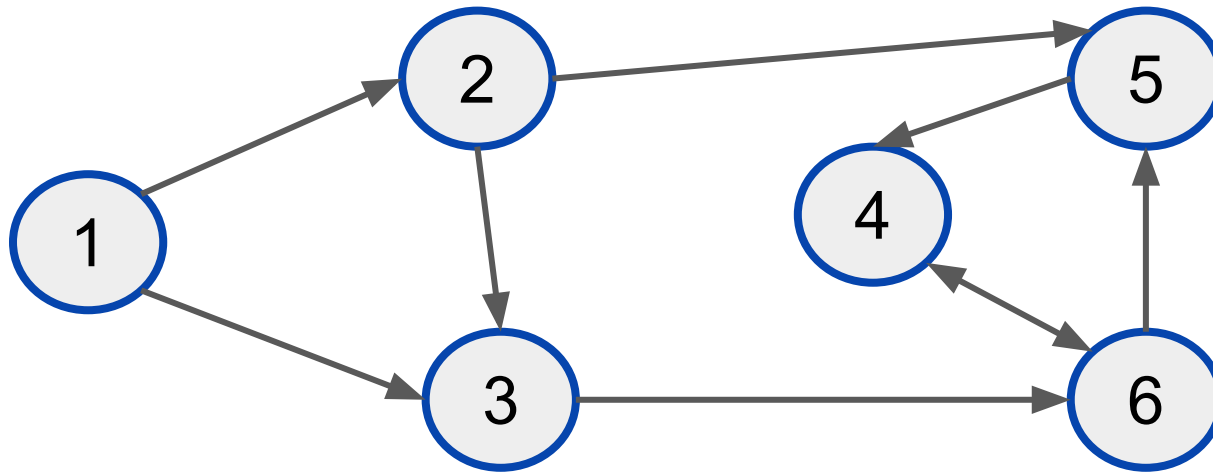
bfs(node=1, graph)



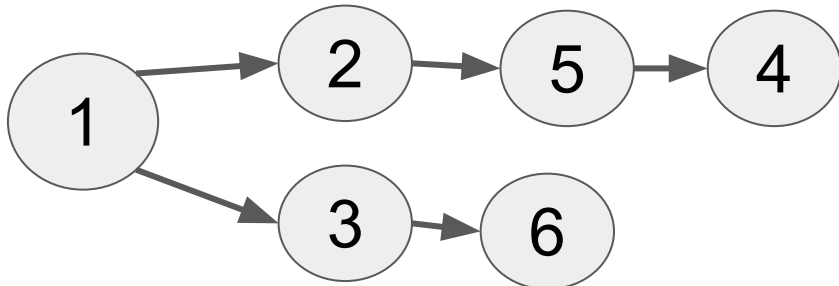
| | | | | |
|-------|---|---|---|------|
| front | 4 | 4 | 5 | rear |
|-------|---|---|---|------|

Queue

Breadth-first search



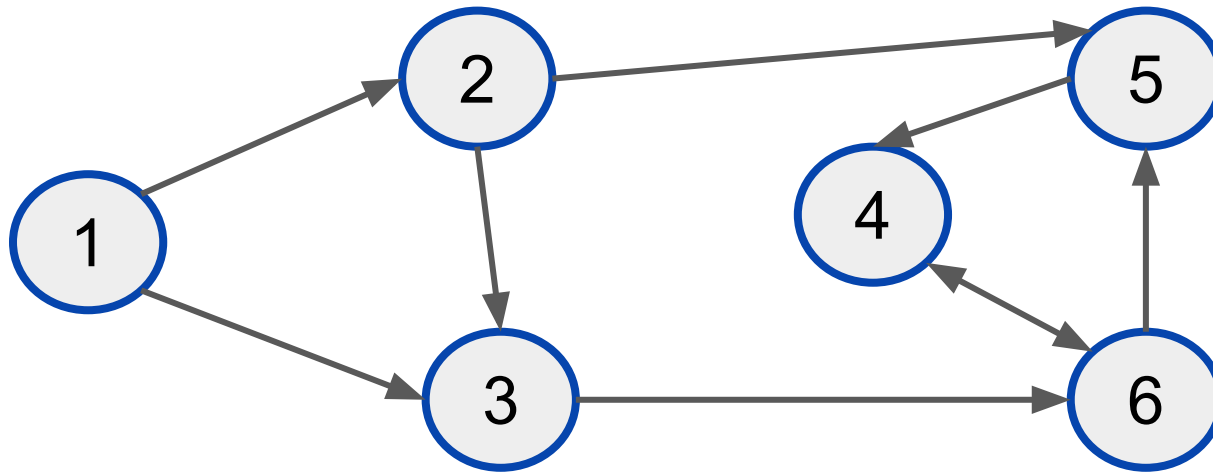
bfs(node=1, graph)



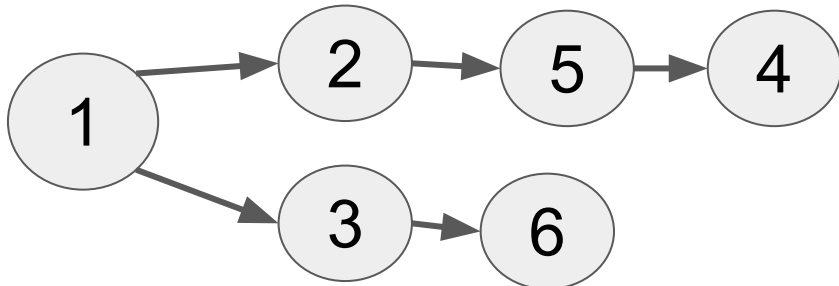
| | | | |
|-------|---|---|------|
| front | 4 | 5 | rear |
|-------|---|---|------|

Queue

Breadth-first search



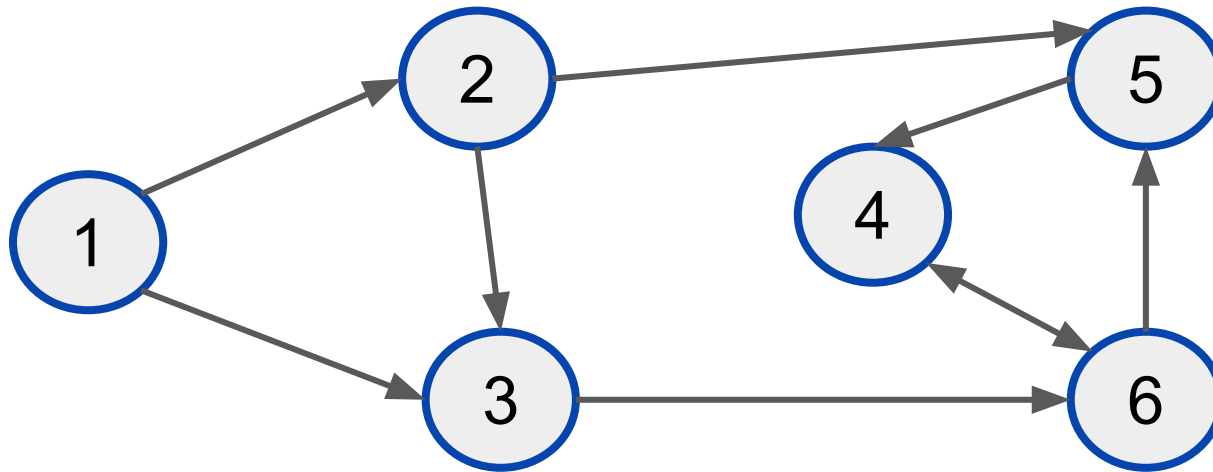
bfs(node=1, graph)



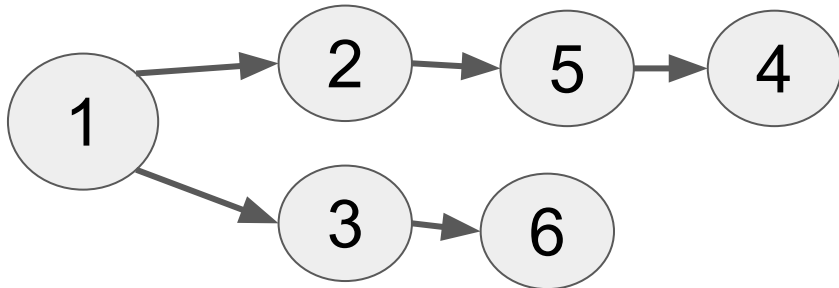
| | | | | |
|-------|---|---|---|------|
| front | 4 | 5 | 6 | rear |
|-------|---|---|---|------|

Queue

Breadth-first search



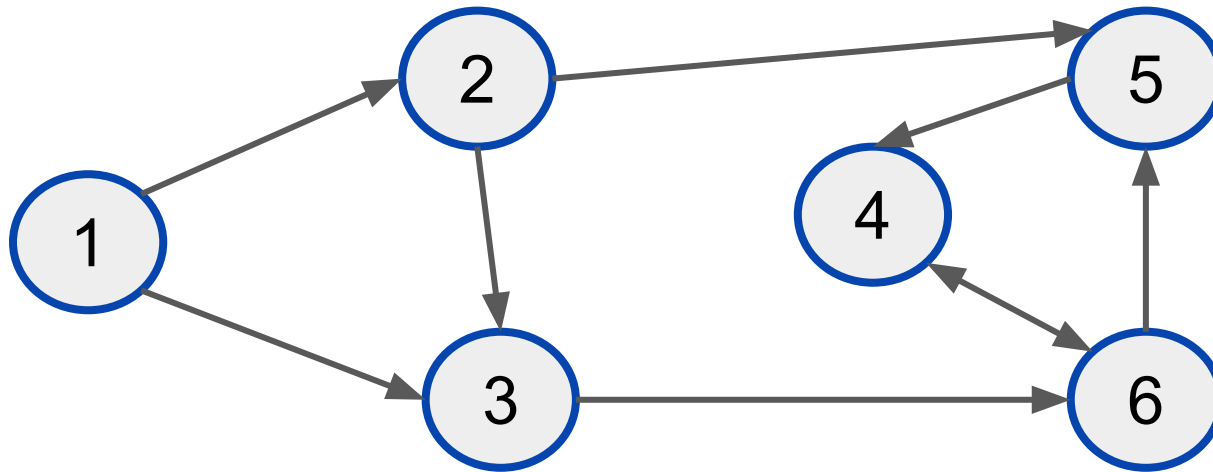
bfs(node=1, graph)



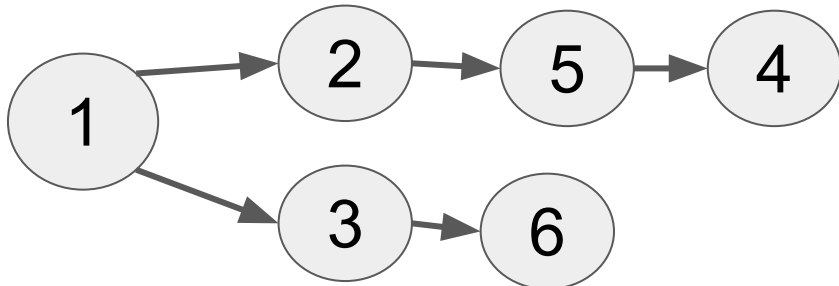
| | | | |
|-------|---|---|------|
| front | 5 | 6 | rear |
|-------|---|---|------|

Queue

Breadth-first search



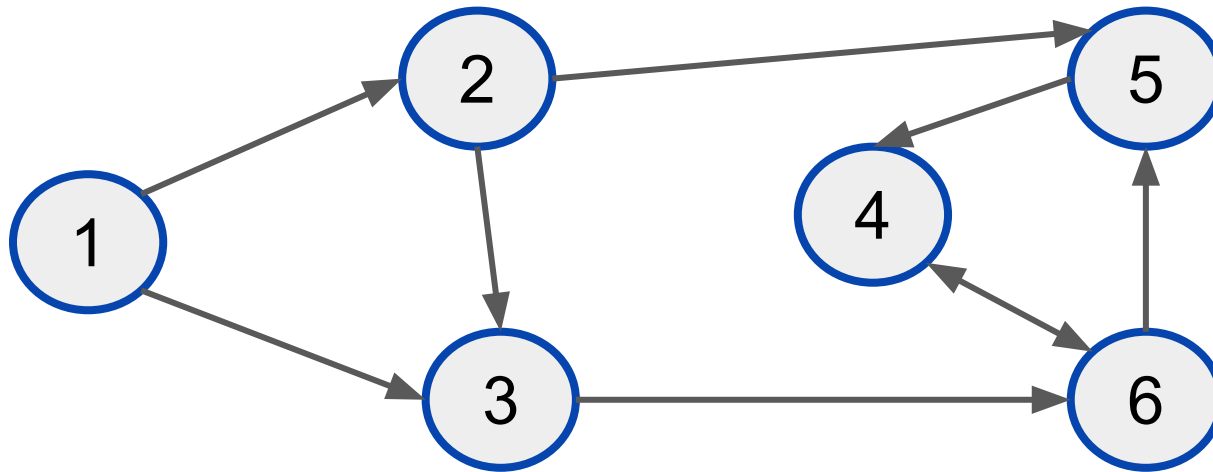
bfs(node=1, graph)



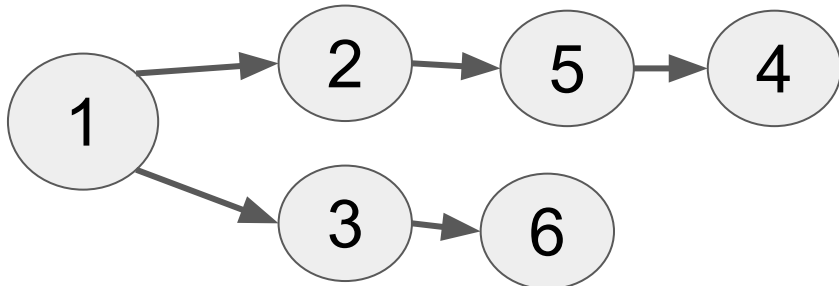
| | | |
|-------|---|------|
| front | 6 | rear |
|-------|---|------|

Queue

Breadth-first search



bfs(node=1, graph)

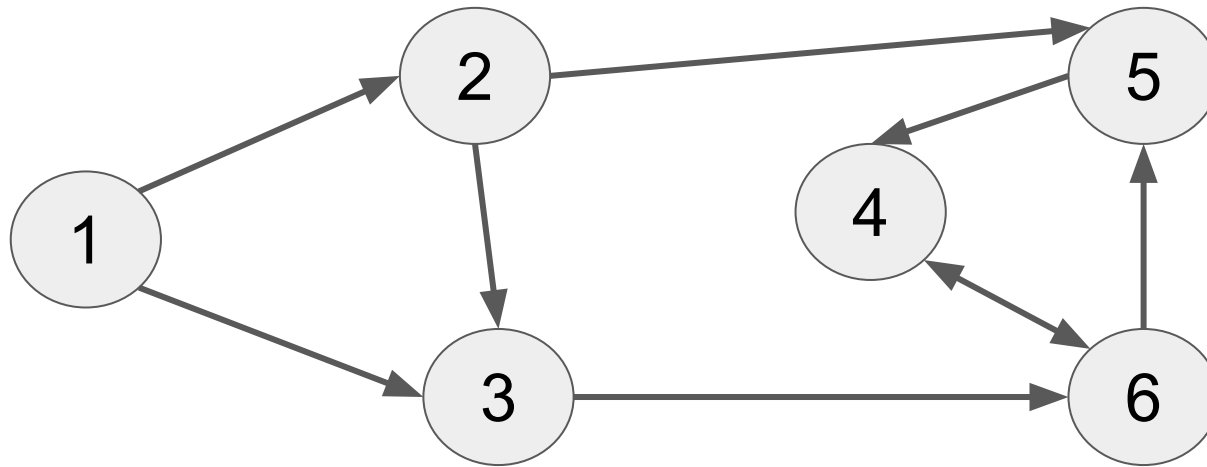


| | |
|-------|------|
| front | rear |
|-------|------|

Queue

Practice problem

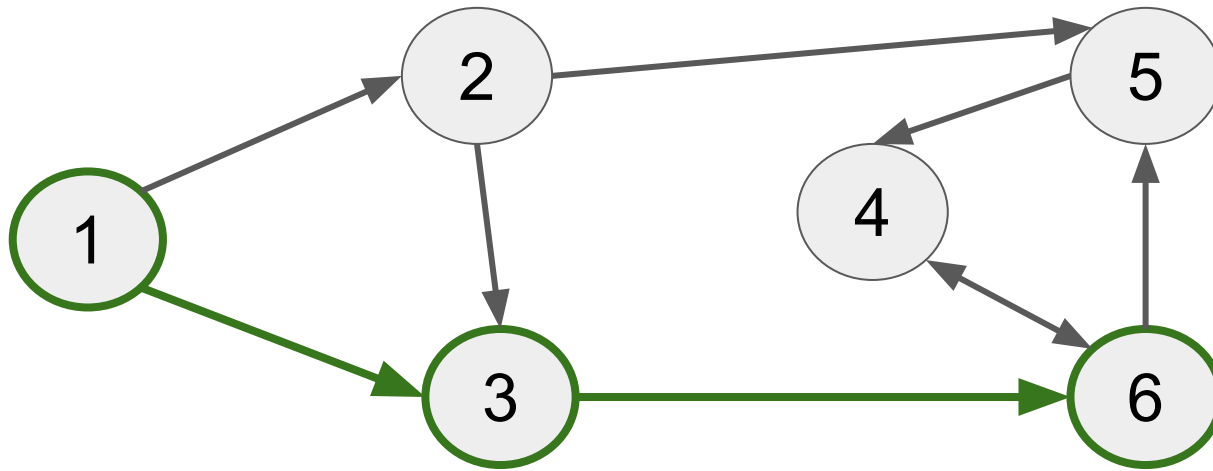
- Given a non-weighted directed graph with adjacency list:
 - Find the shortest path between from SRC to DST



bfs(node=1, graph)

Practice problem

- Given a non-weighted directed graph with adjacency list:
 - Find the shortest path between from SRC to DST



bfs(node=1, graph)