CSC 162
DATA STRUCTURES

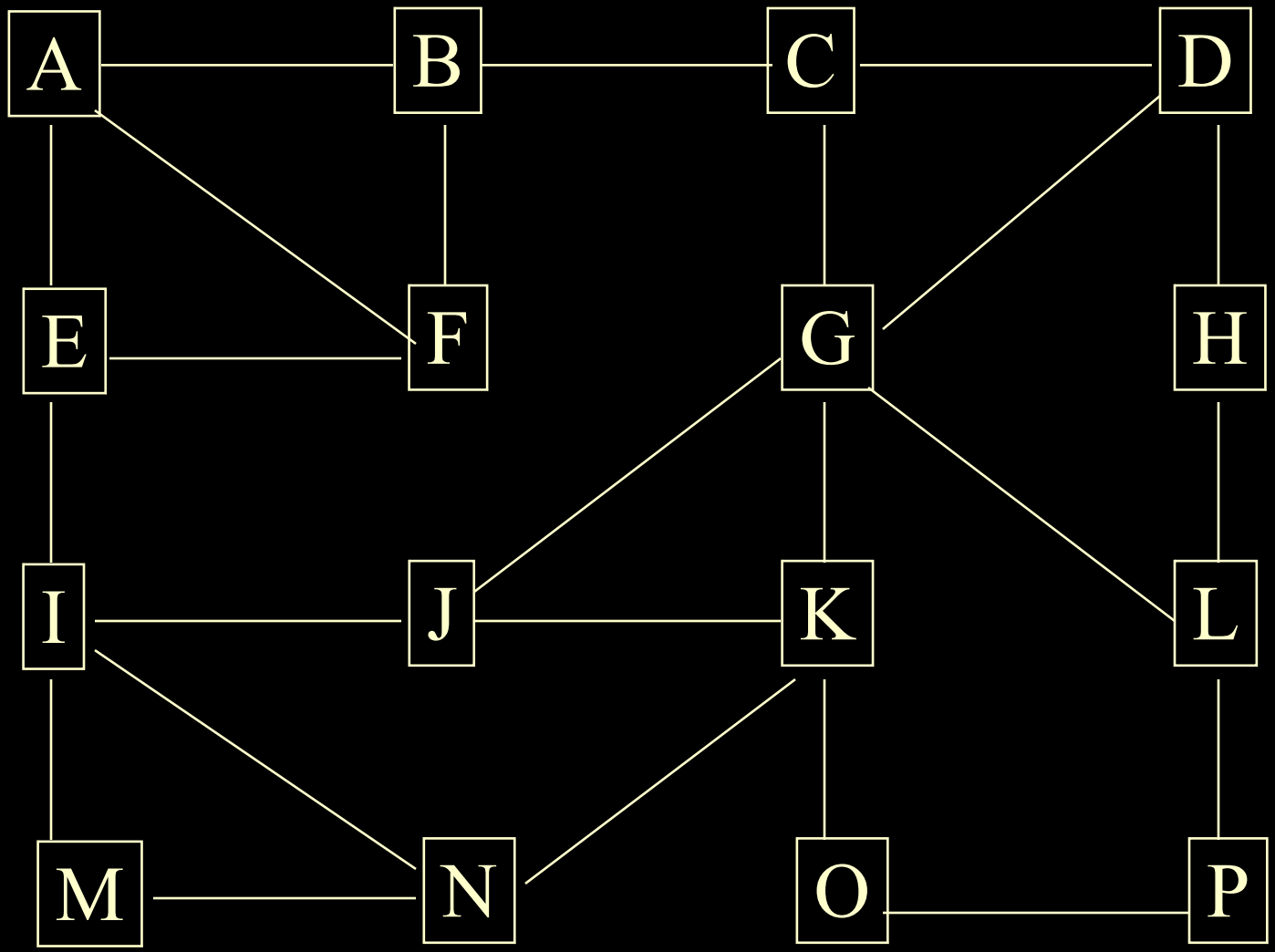
Traversing graphs

- Depth-First Search
  - like a post-order traversal of a tree
- Breadth-First Search
  - Less like tree traversal

Exploring a Maze

A depth-first search (DFS) in an undirected graph G is like wandering in a maze with a string and a can of paint – you can prevent yourself from getting lost.

Example



DFS

1. Start at vertex s

   Tie the end of the string to s and mark “visited” on s

   Make s the current vertex u
2. Travel along an arbitrary edge (u,v)

   unrolling string
3. If edge(u,v) leads to an already visited vertex v

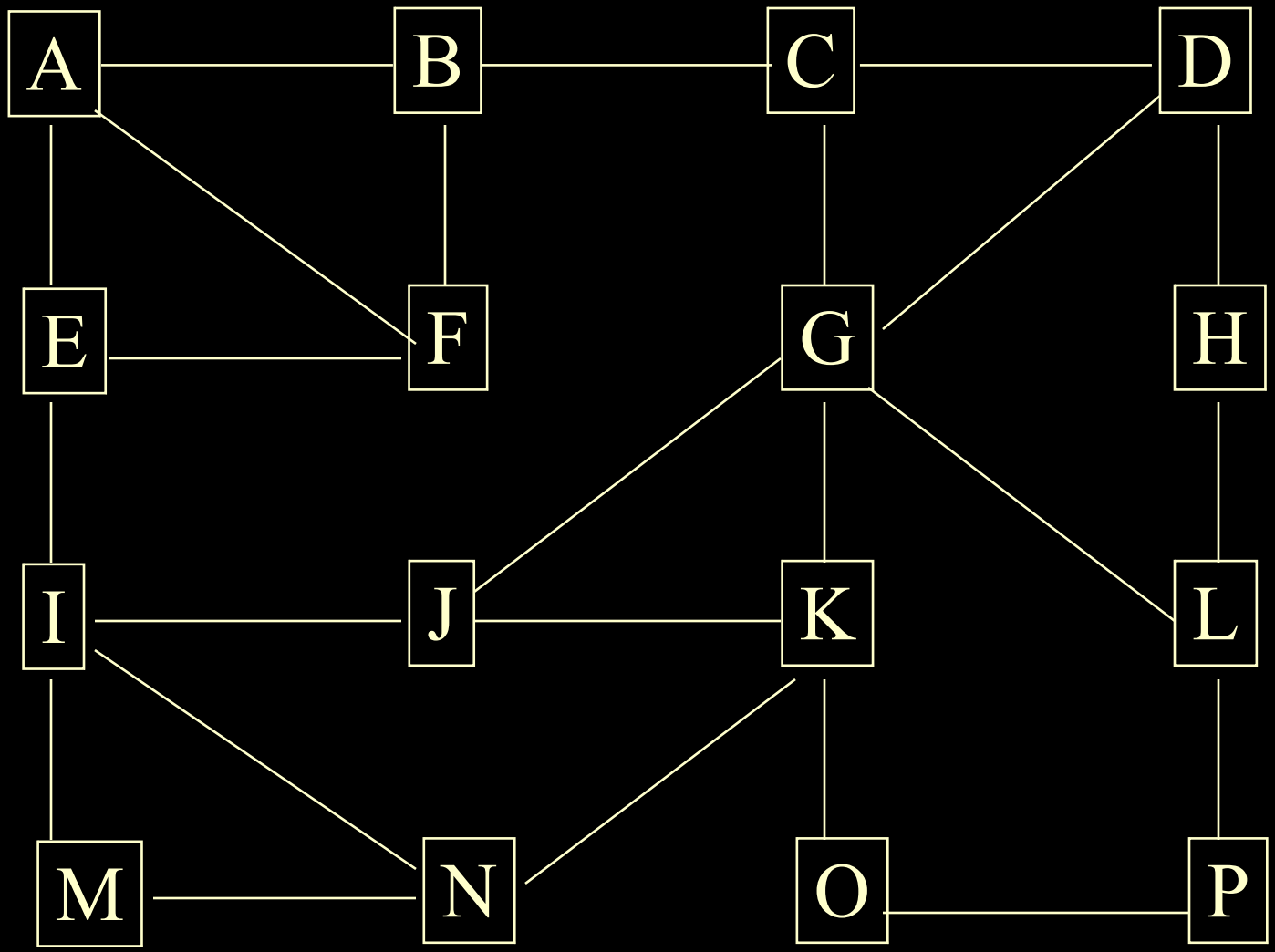
   then return to u

   else mark v as “visited”, set v as current u, repeat @ step 2
4. When all edges lead to visited verticies, backtrack to previous vertex (roll up string) and repeat @ step 2
5. When we backtrack to s and explore all it’s edges we are done

DFS Pseudocode (labels edges)

```
DFS( Vertex v)
    for each edge incident on v do:
        if edge e is unexplored then
            let w be the other endpoint of e
            if vertex w is unexplored then
                label e as a discovery edge
                recursively call DFS(w)
            else
                label e as a backedge
```

Example



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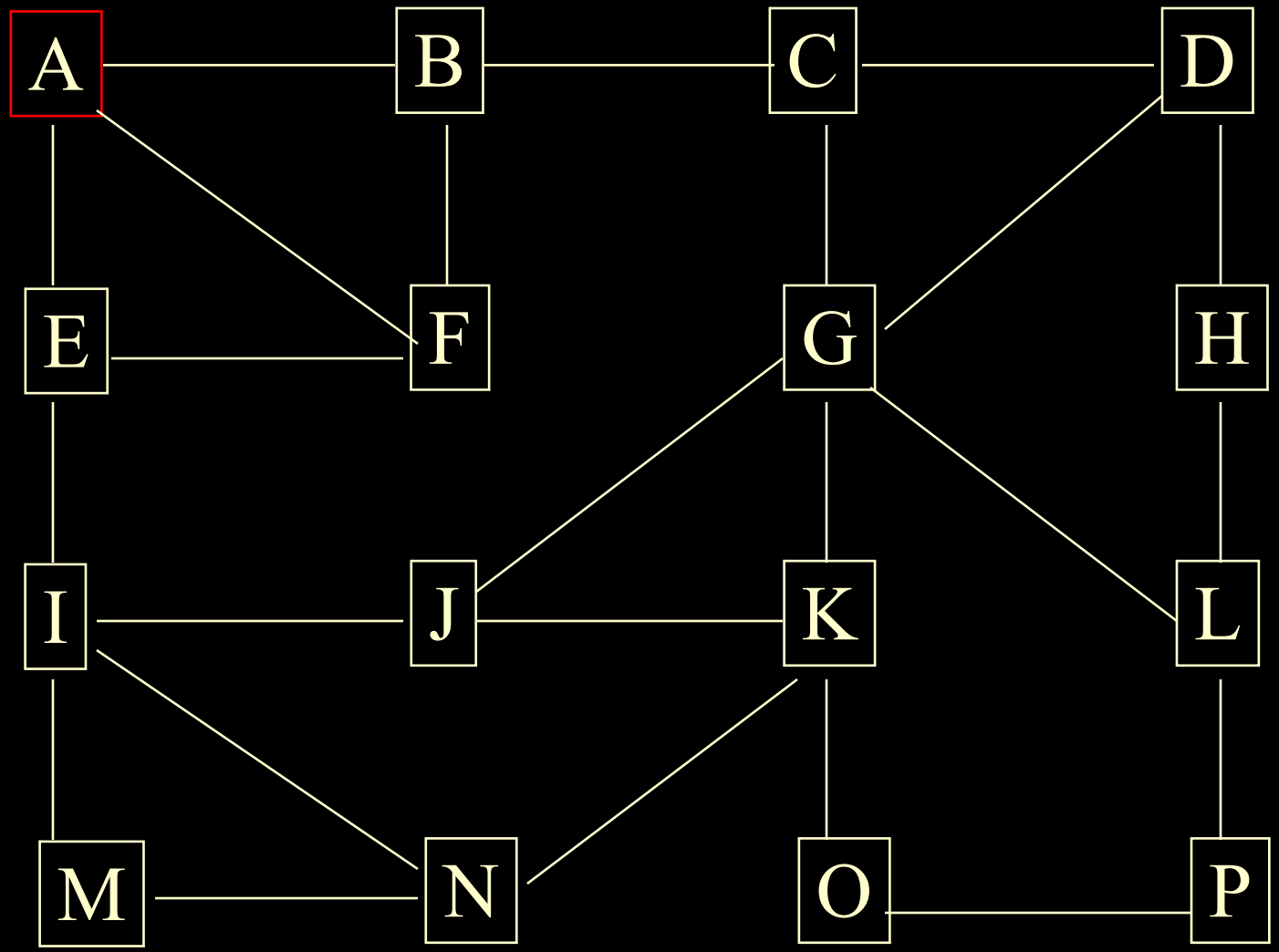

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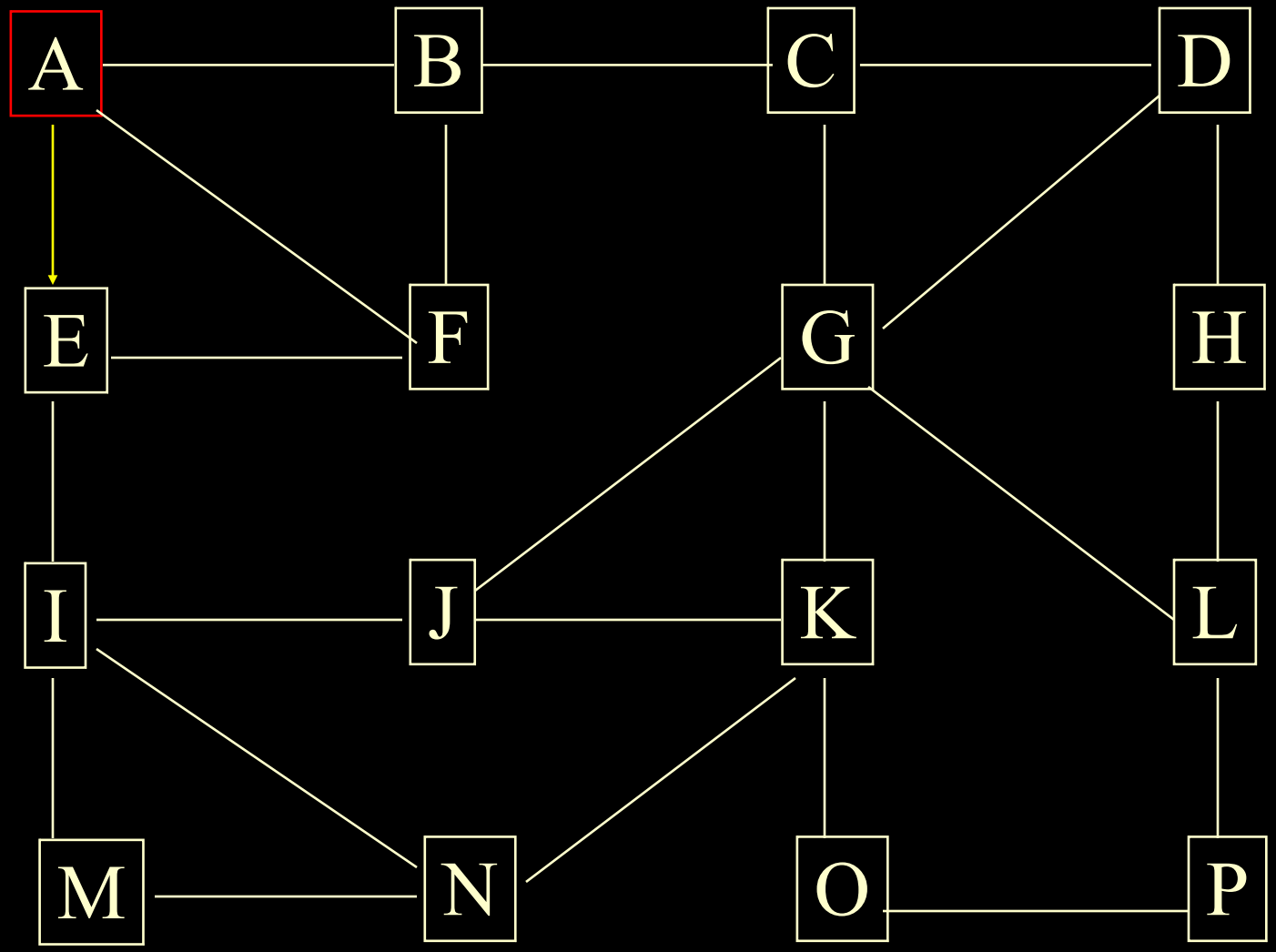

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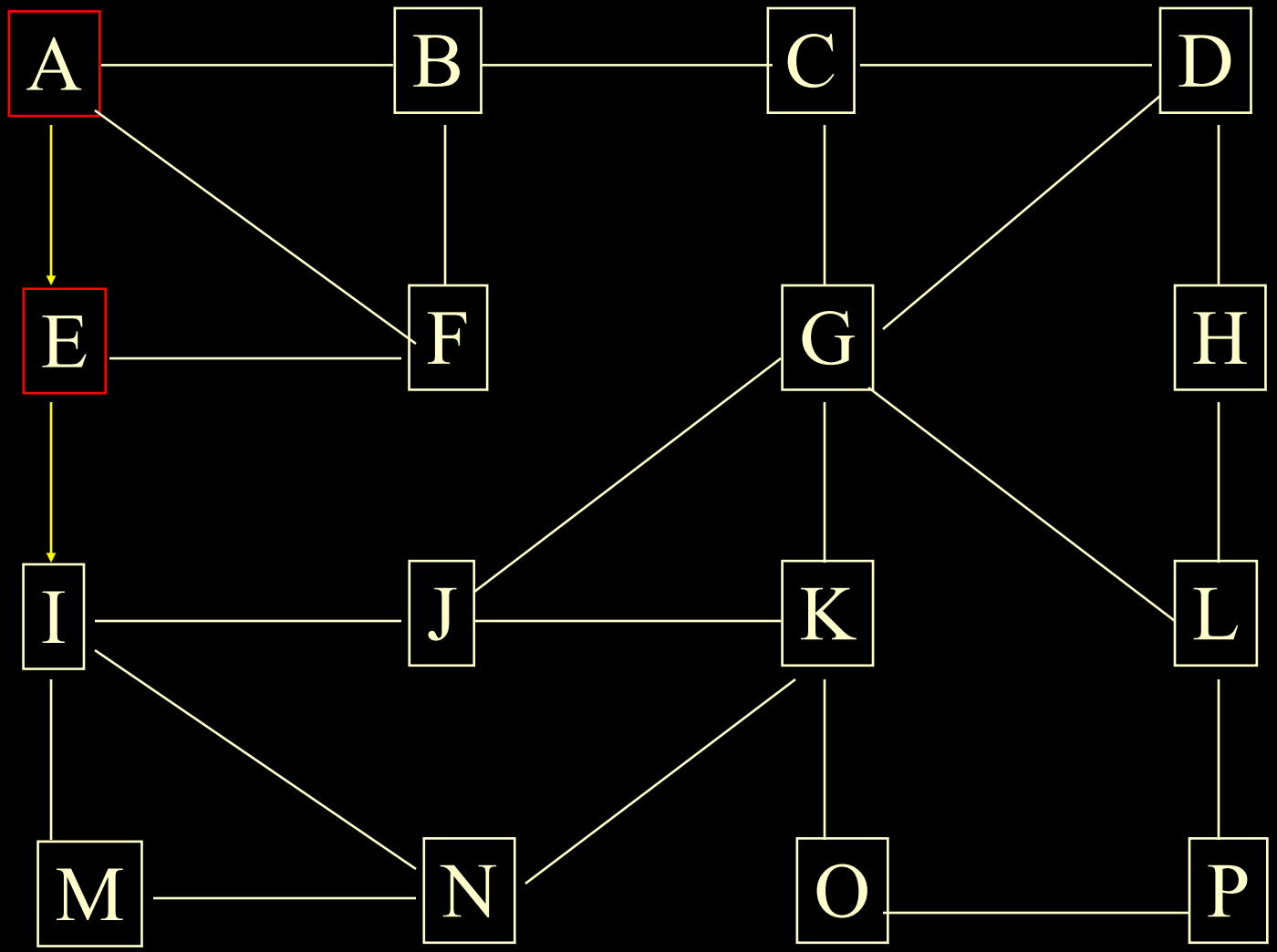

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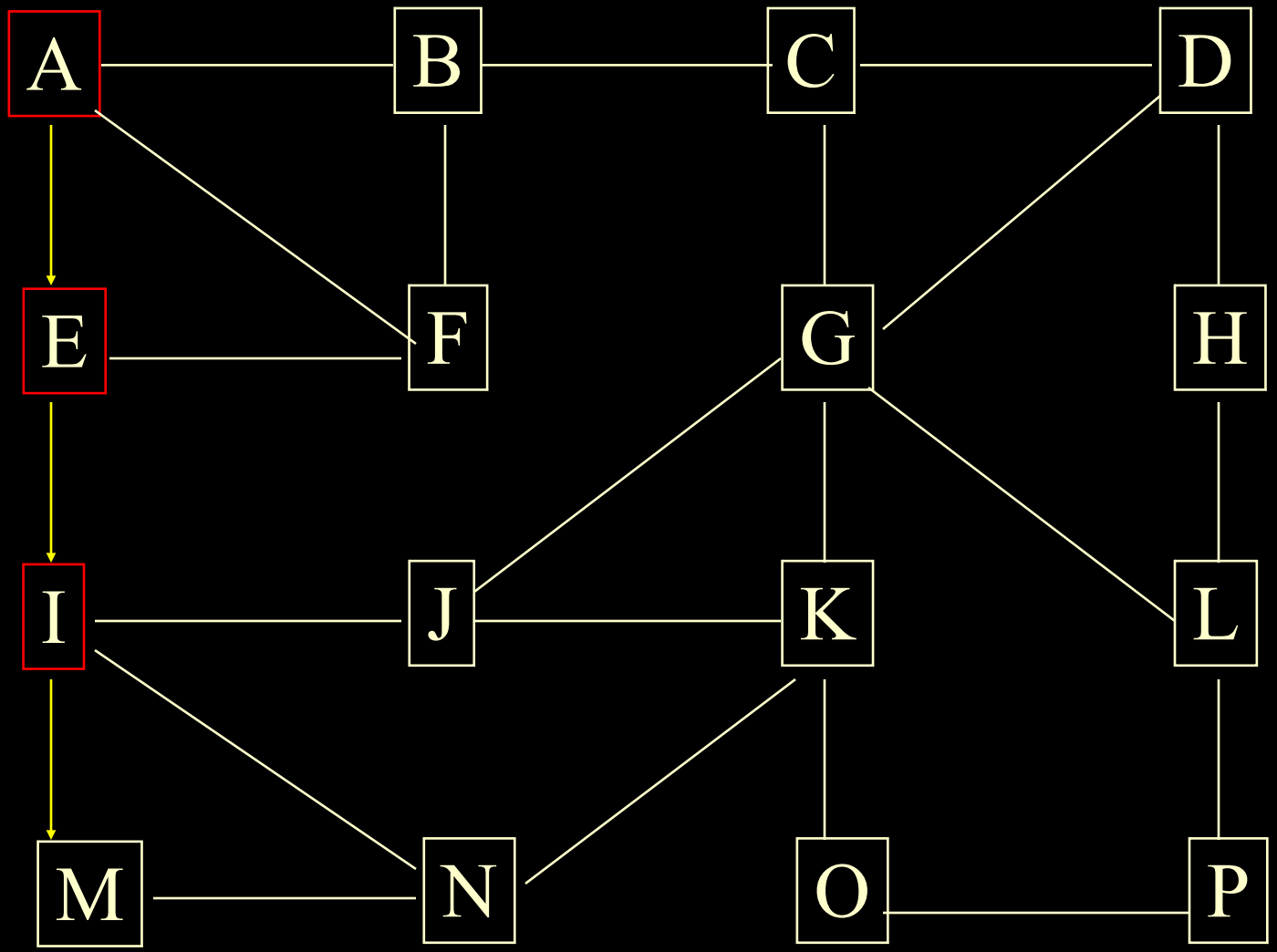

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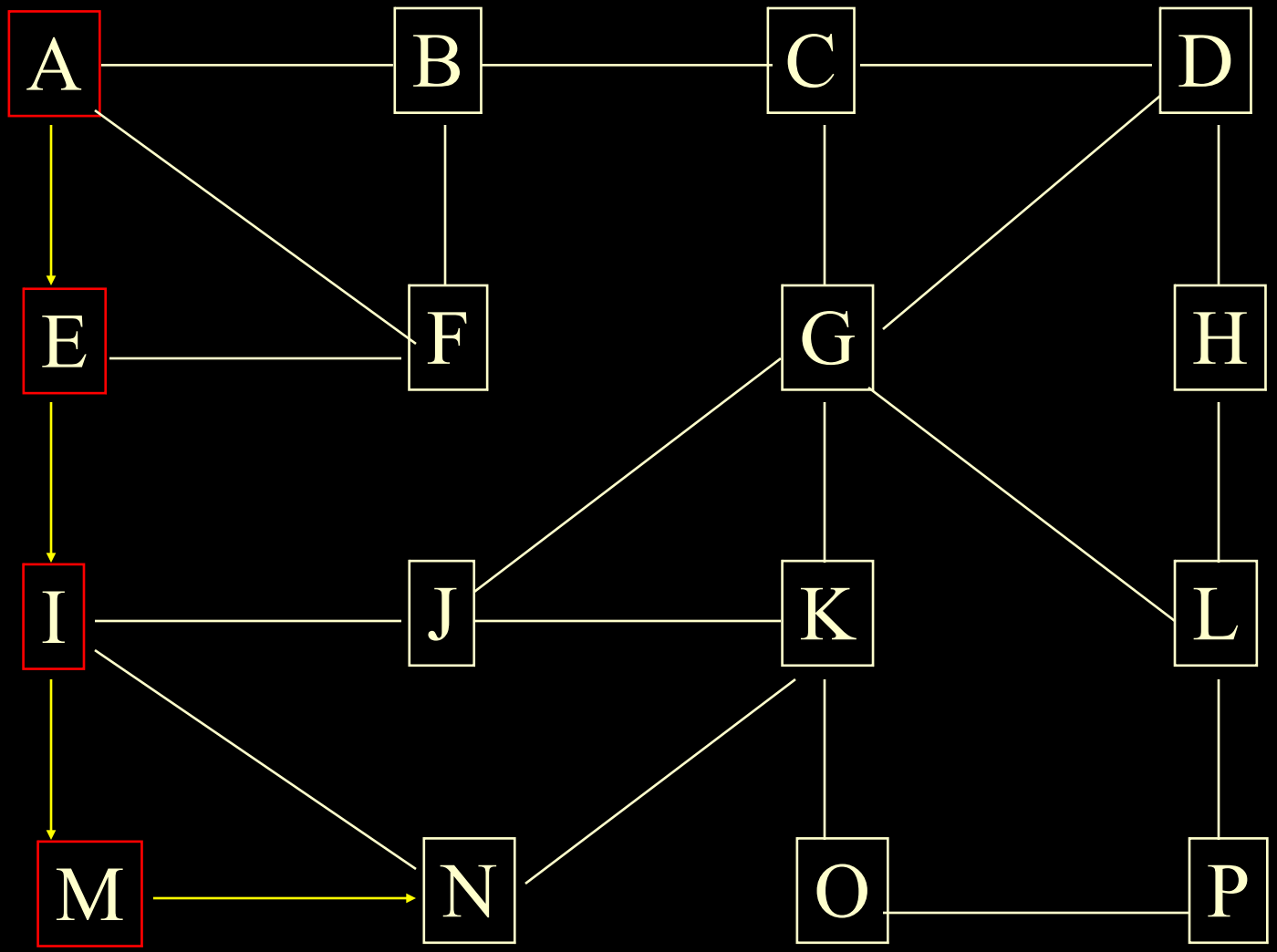

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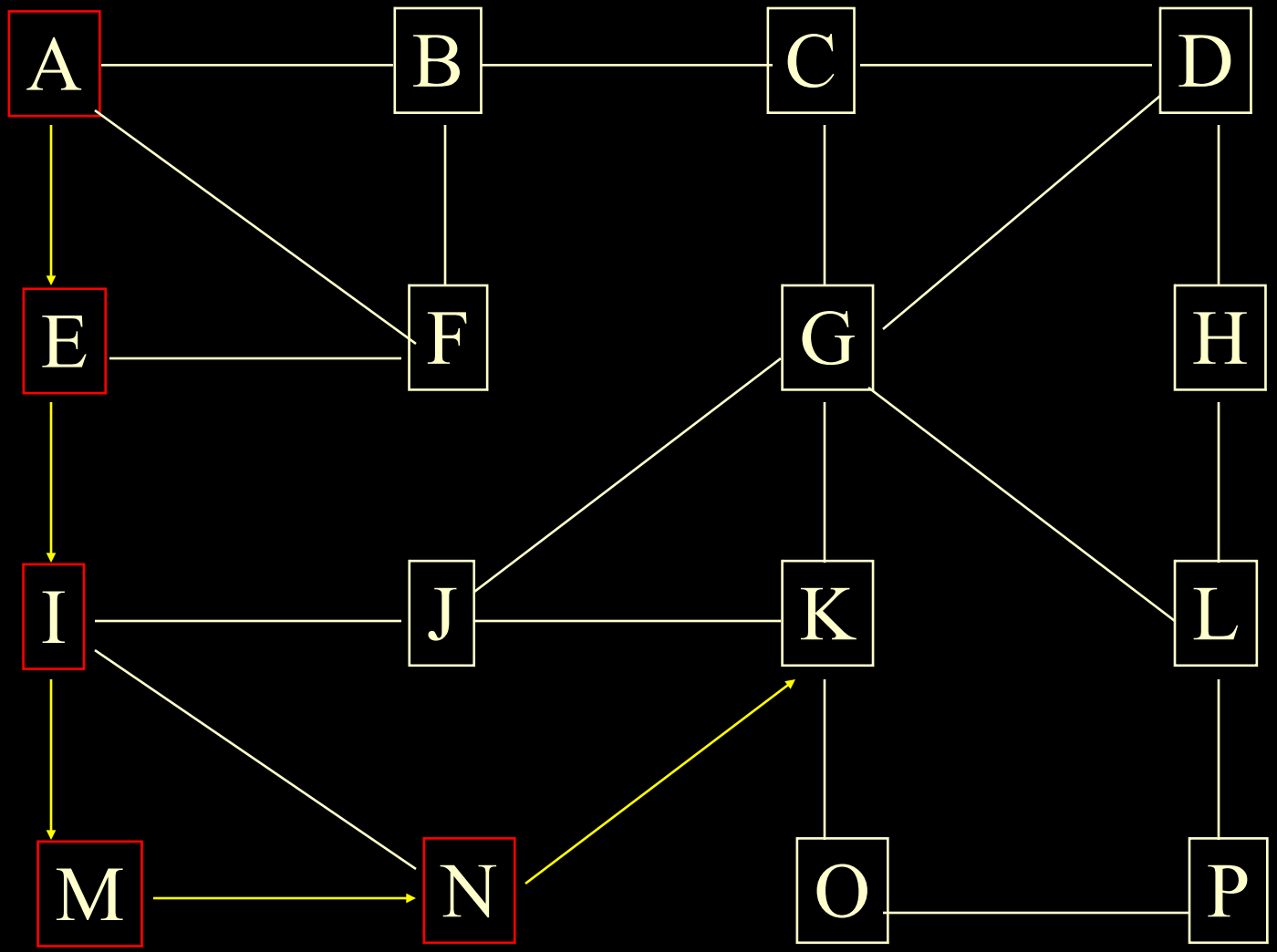

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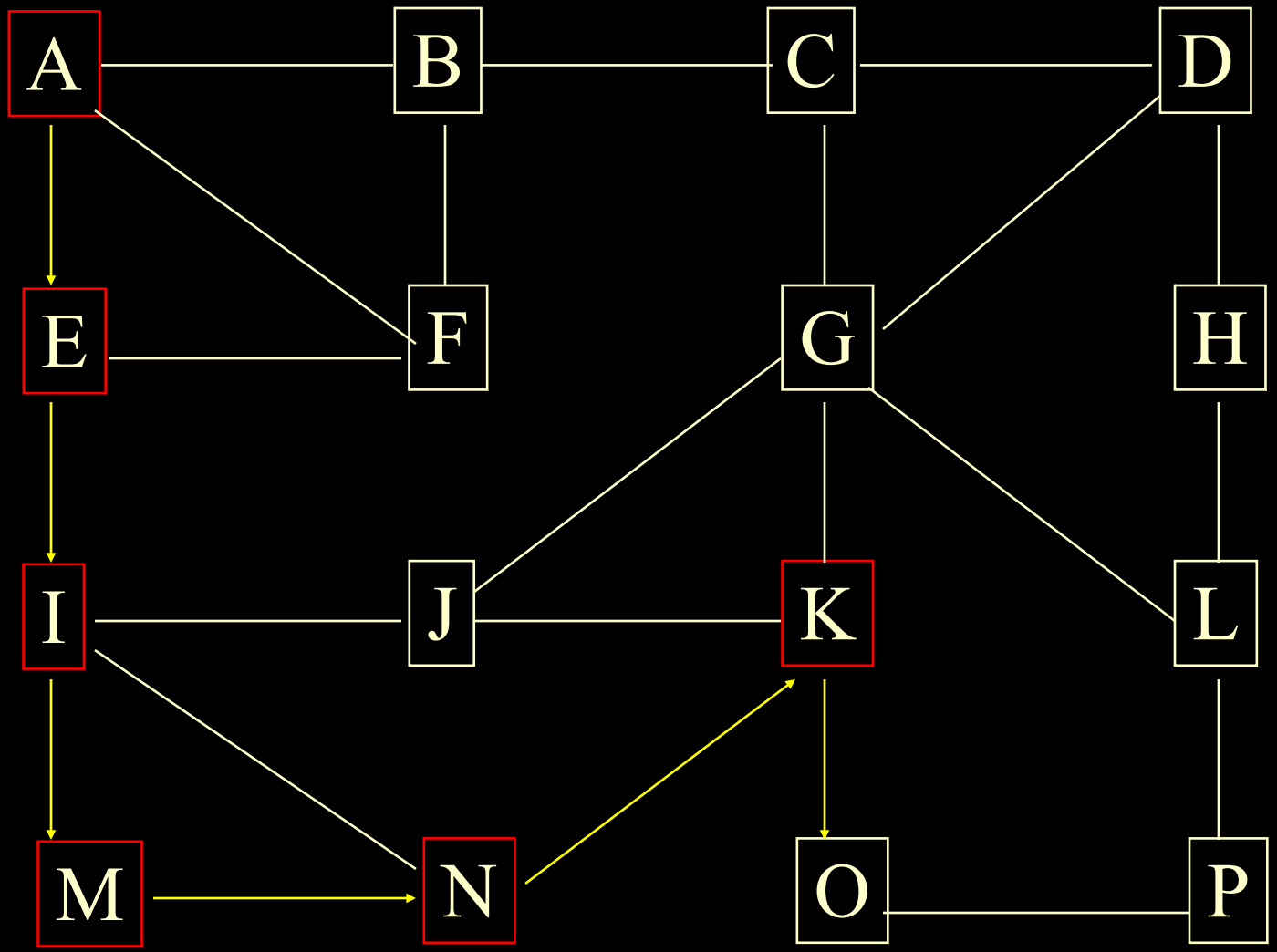

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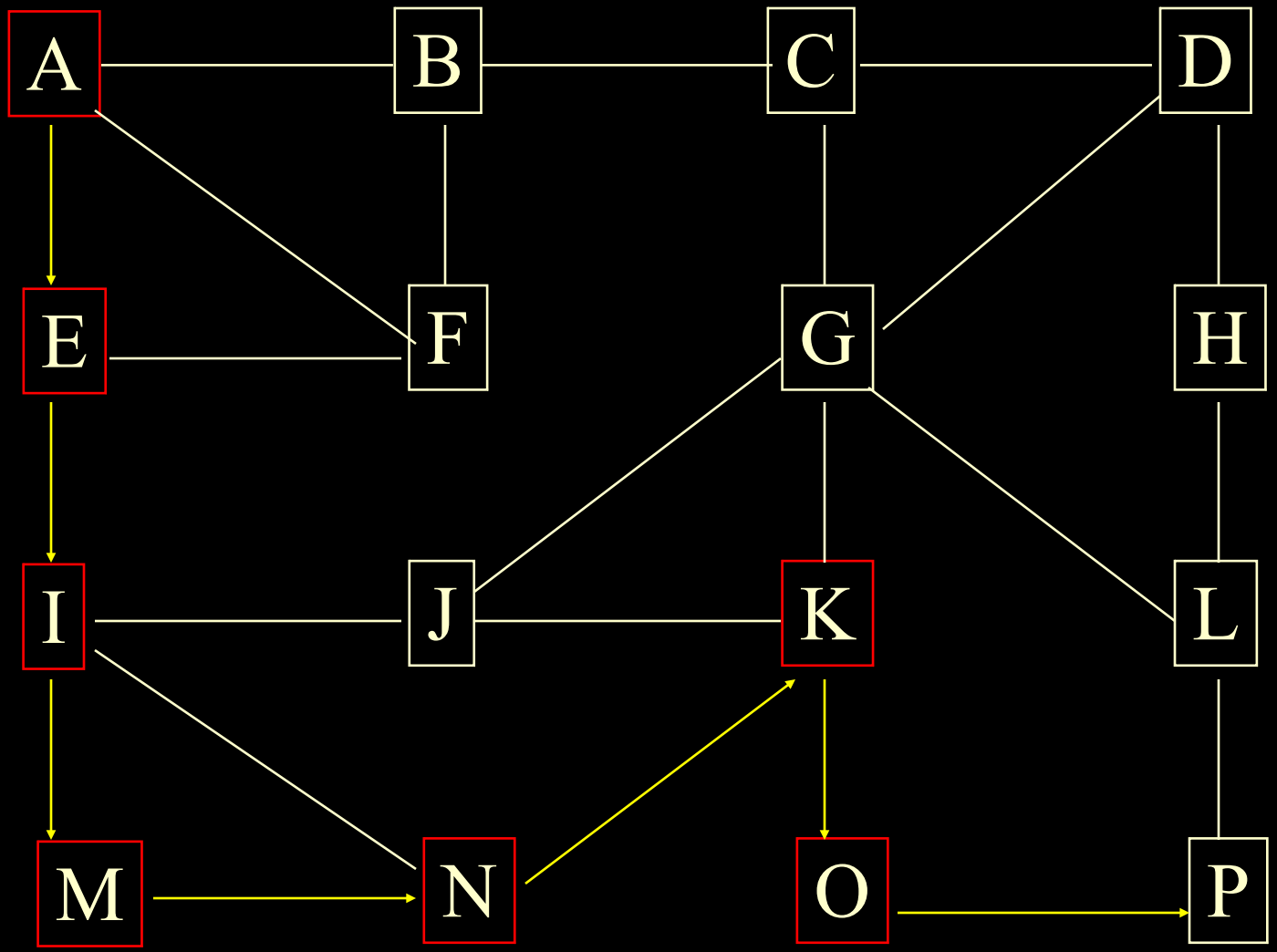

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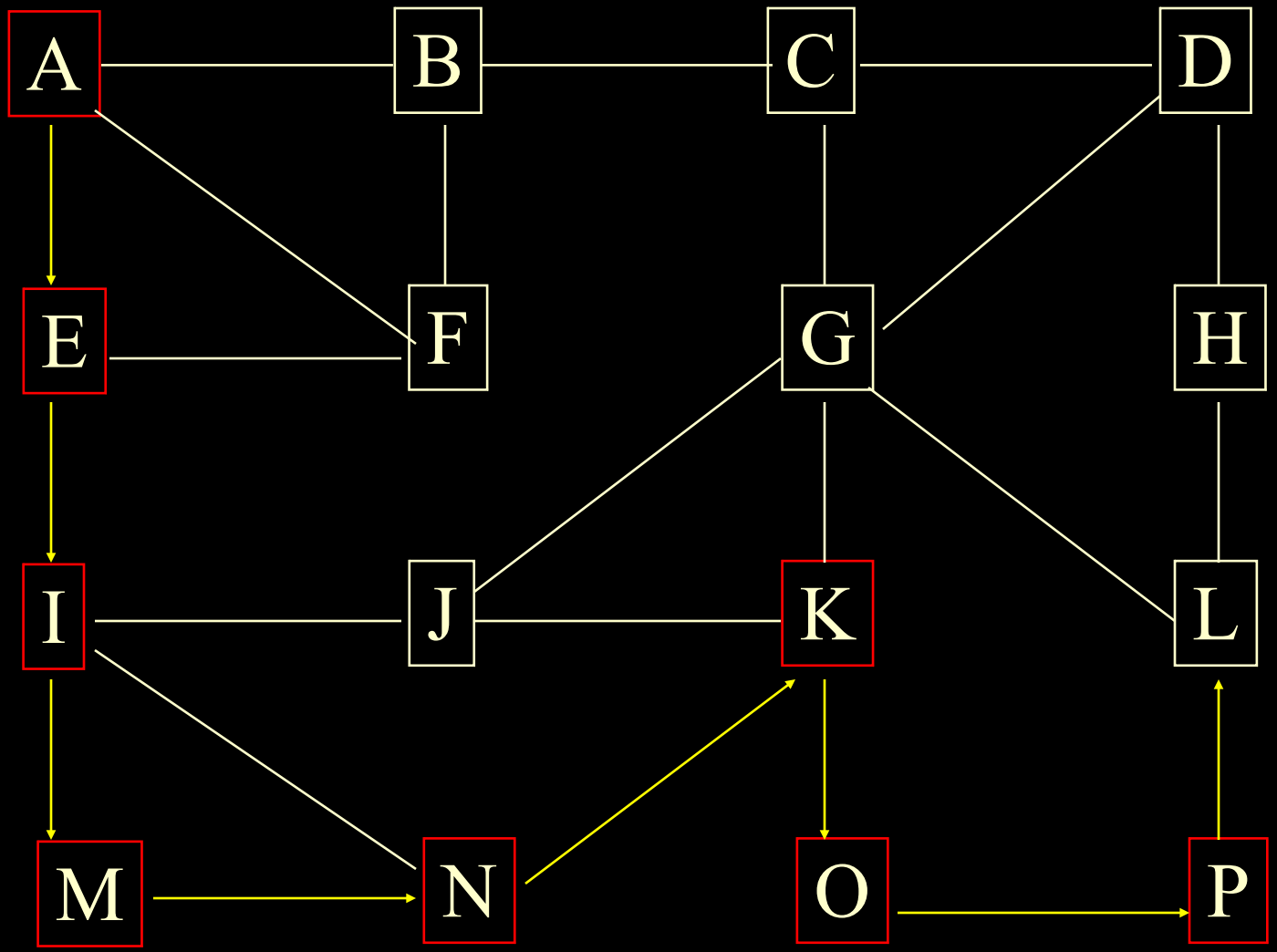

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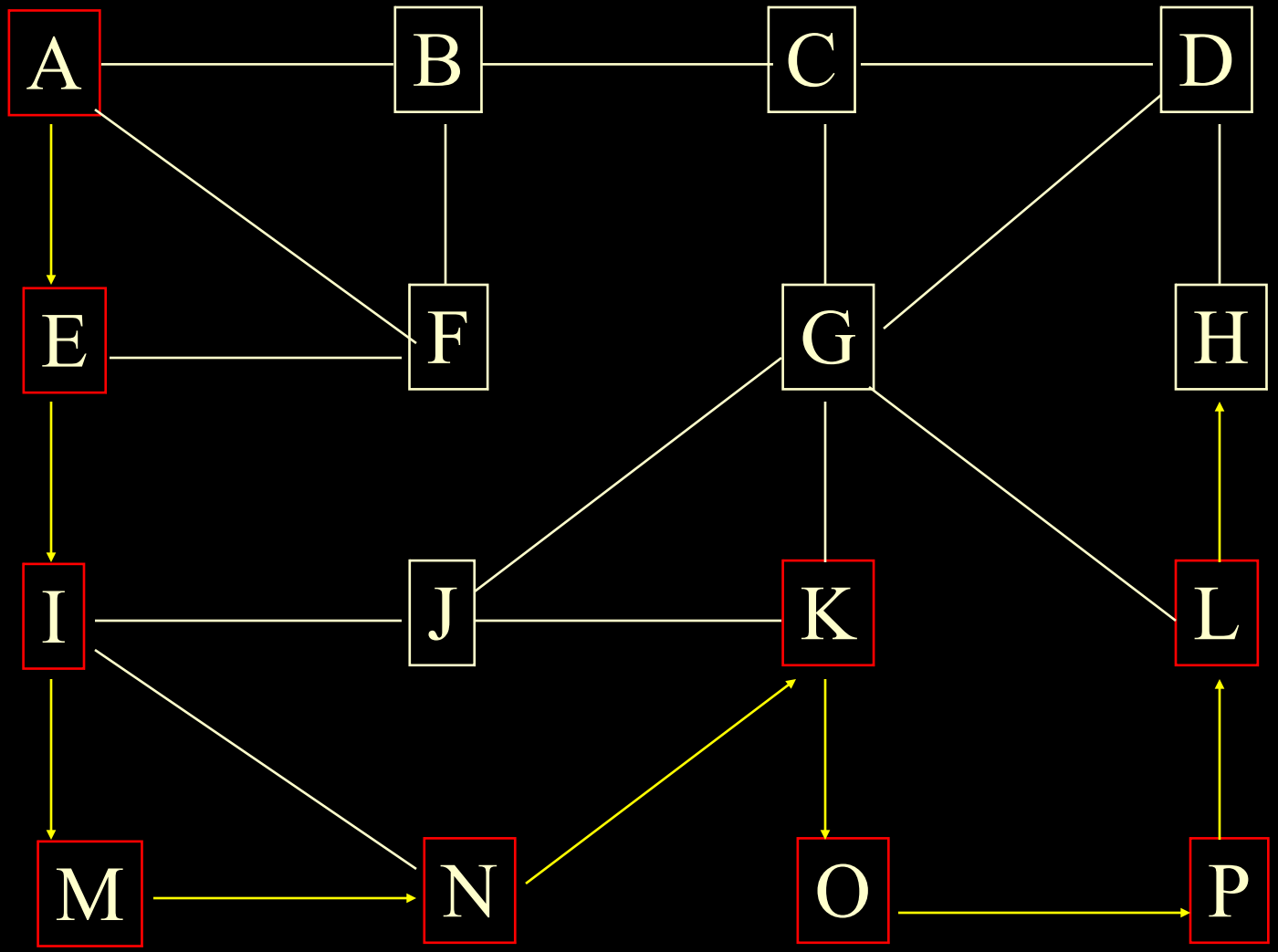

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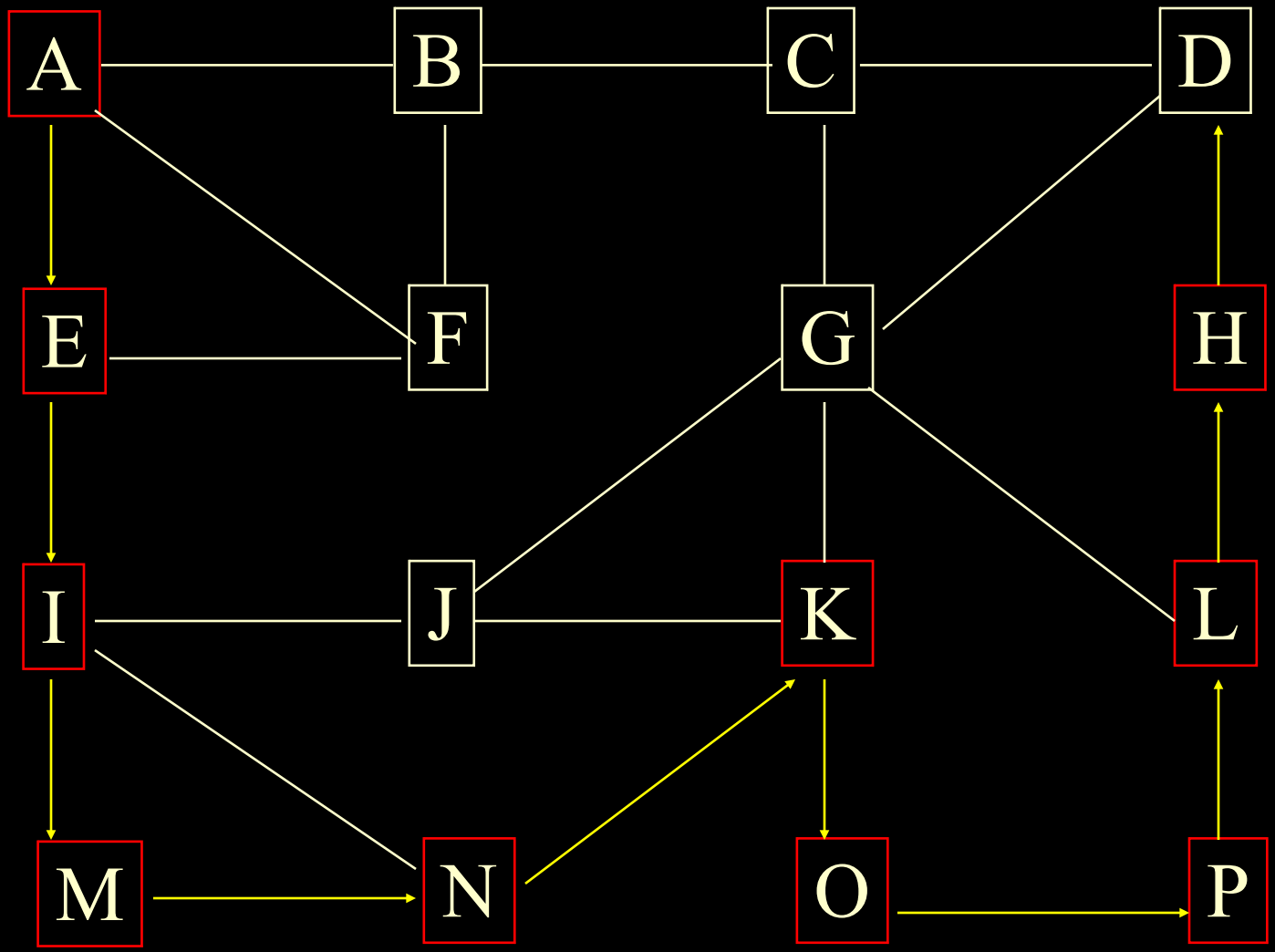

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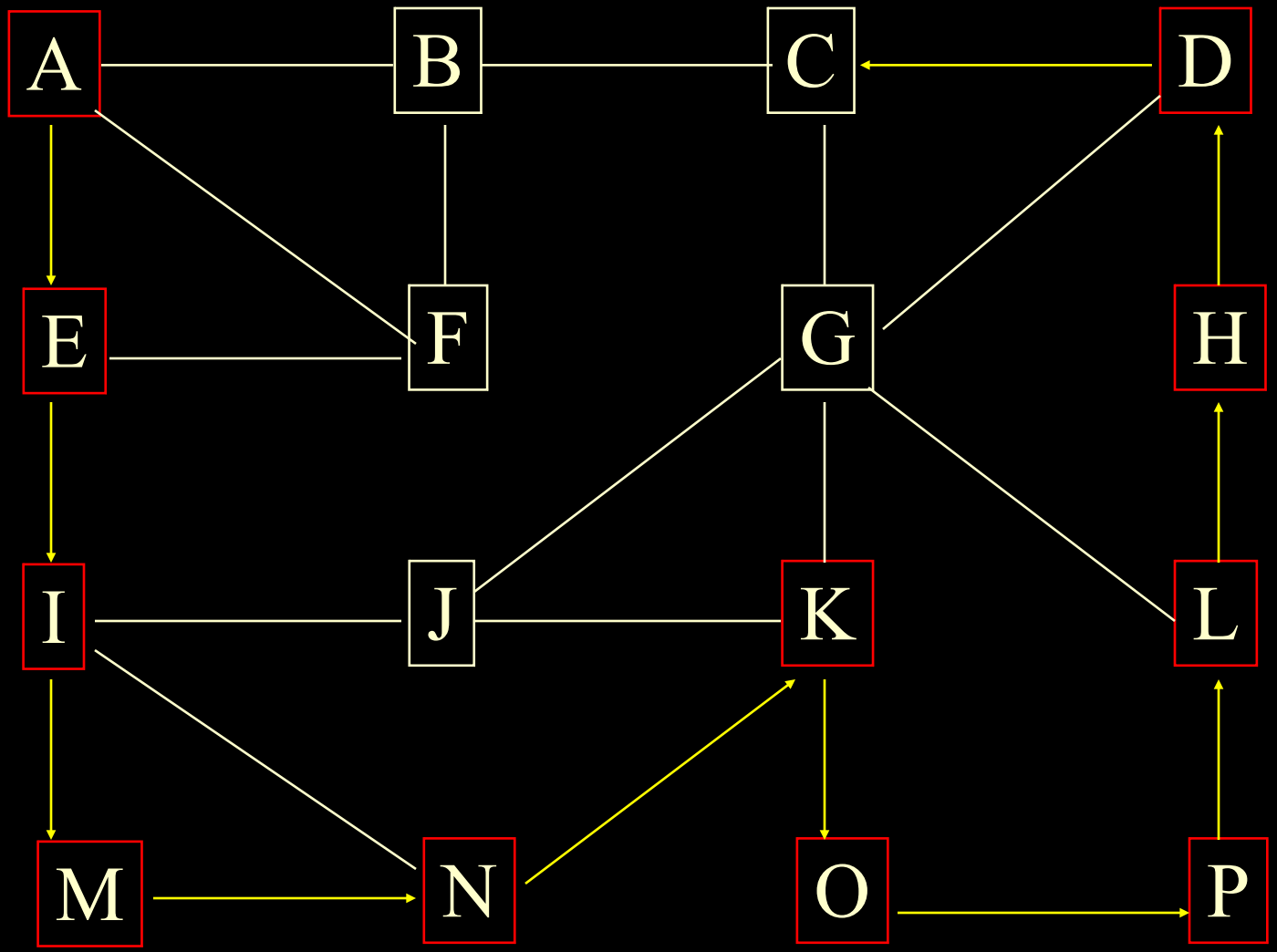

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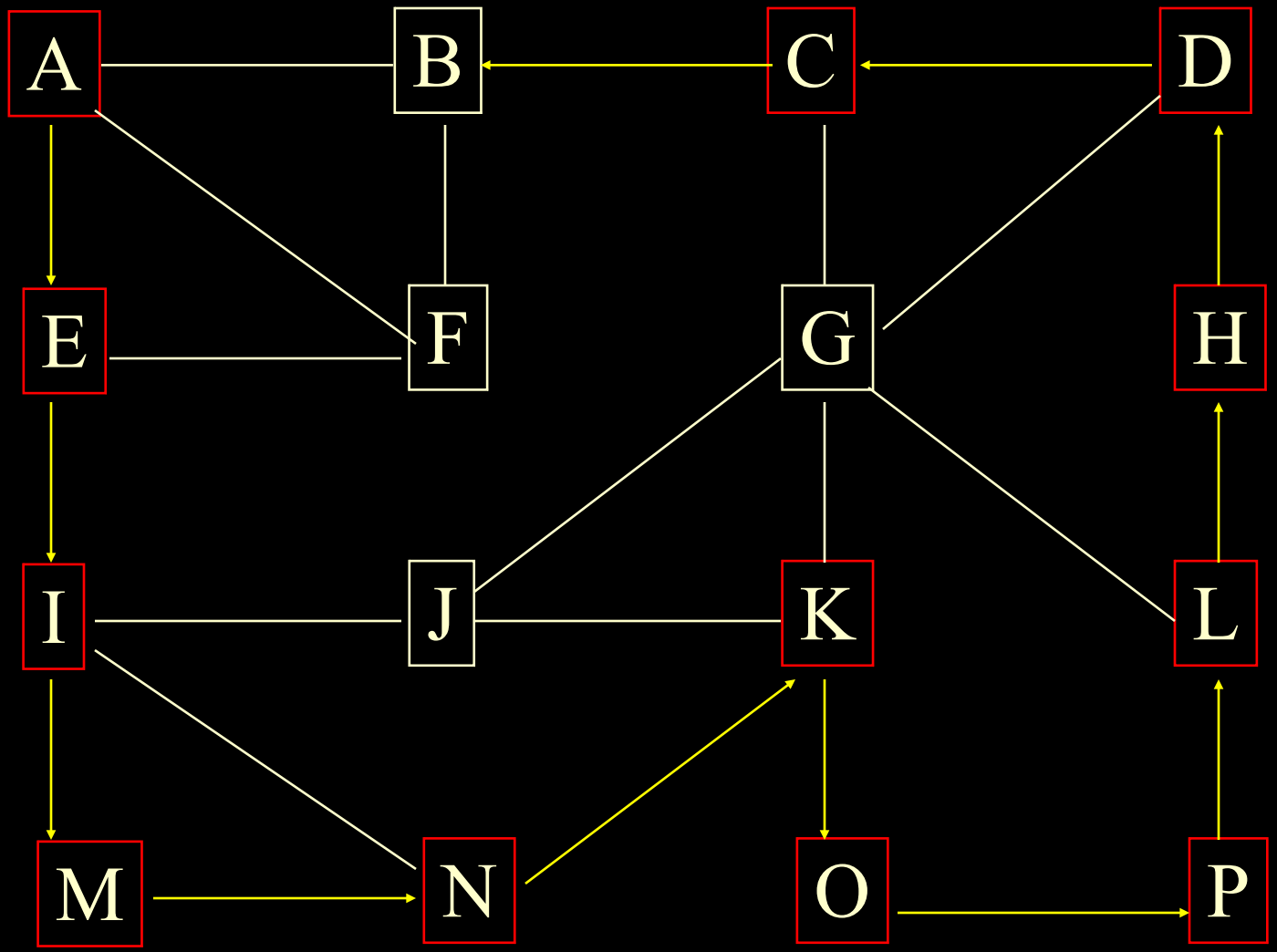

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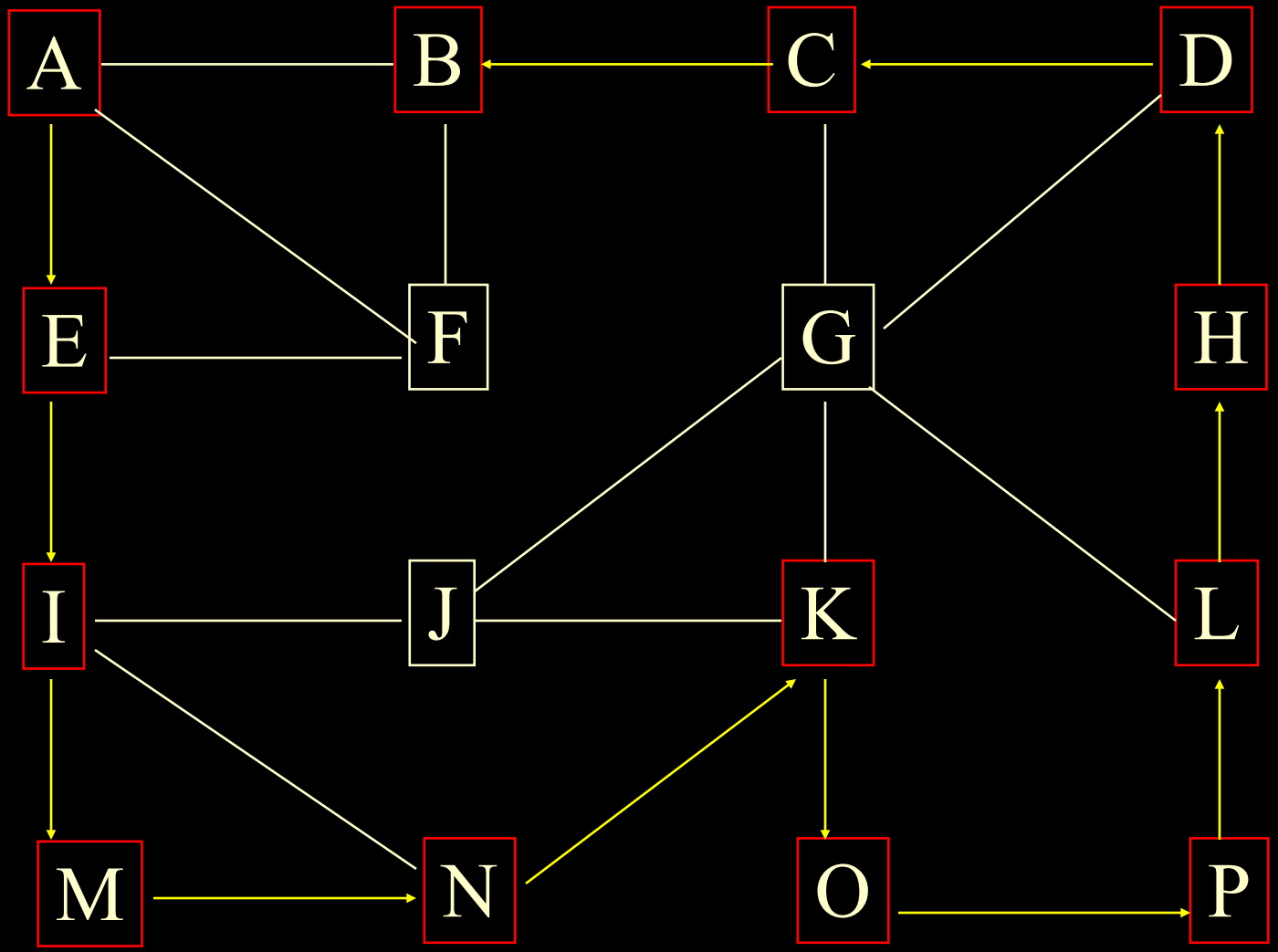

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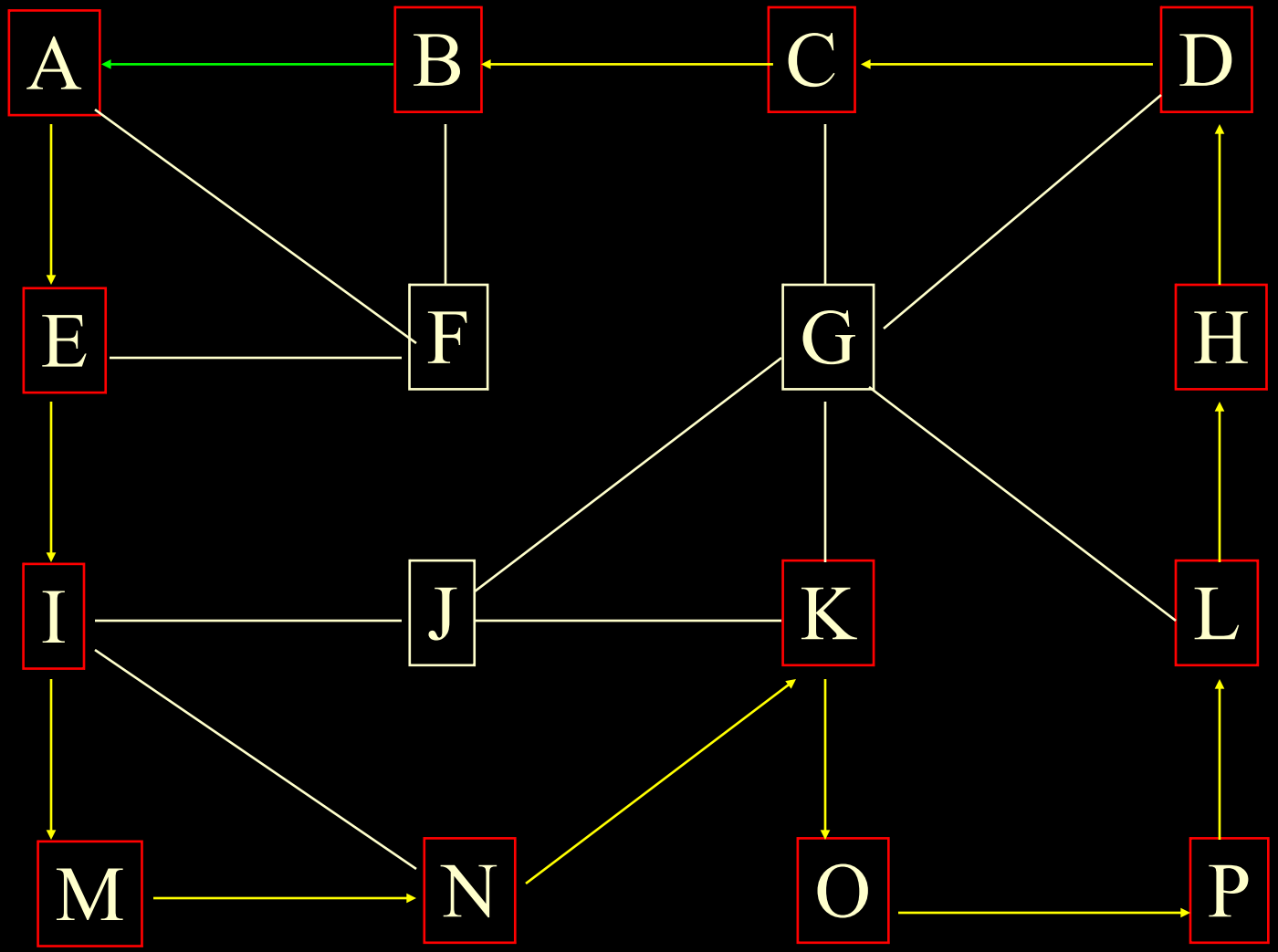

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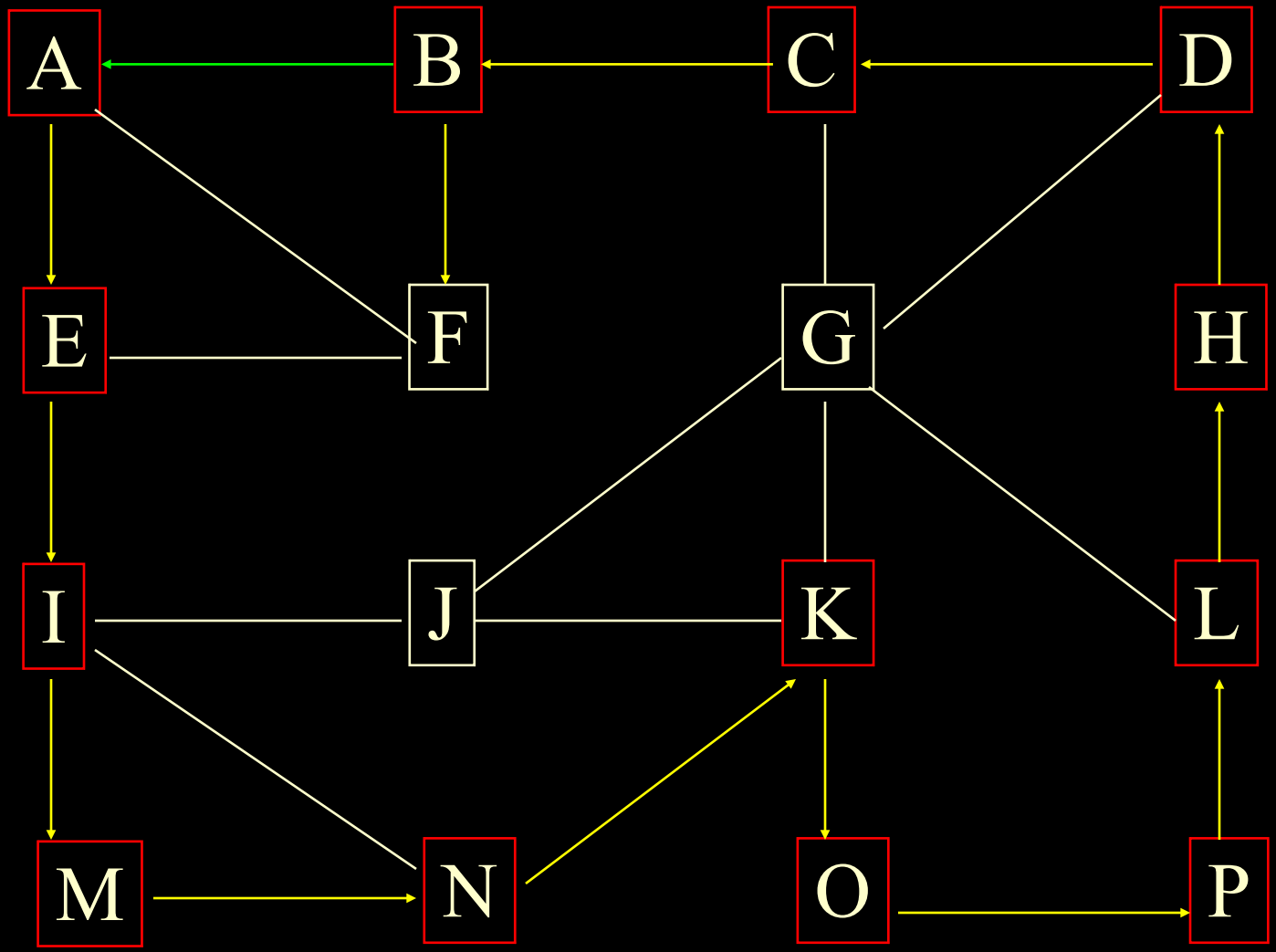

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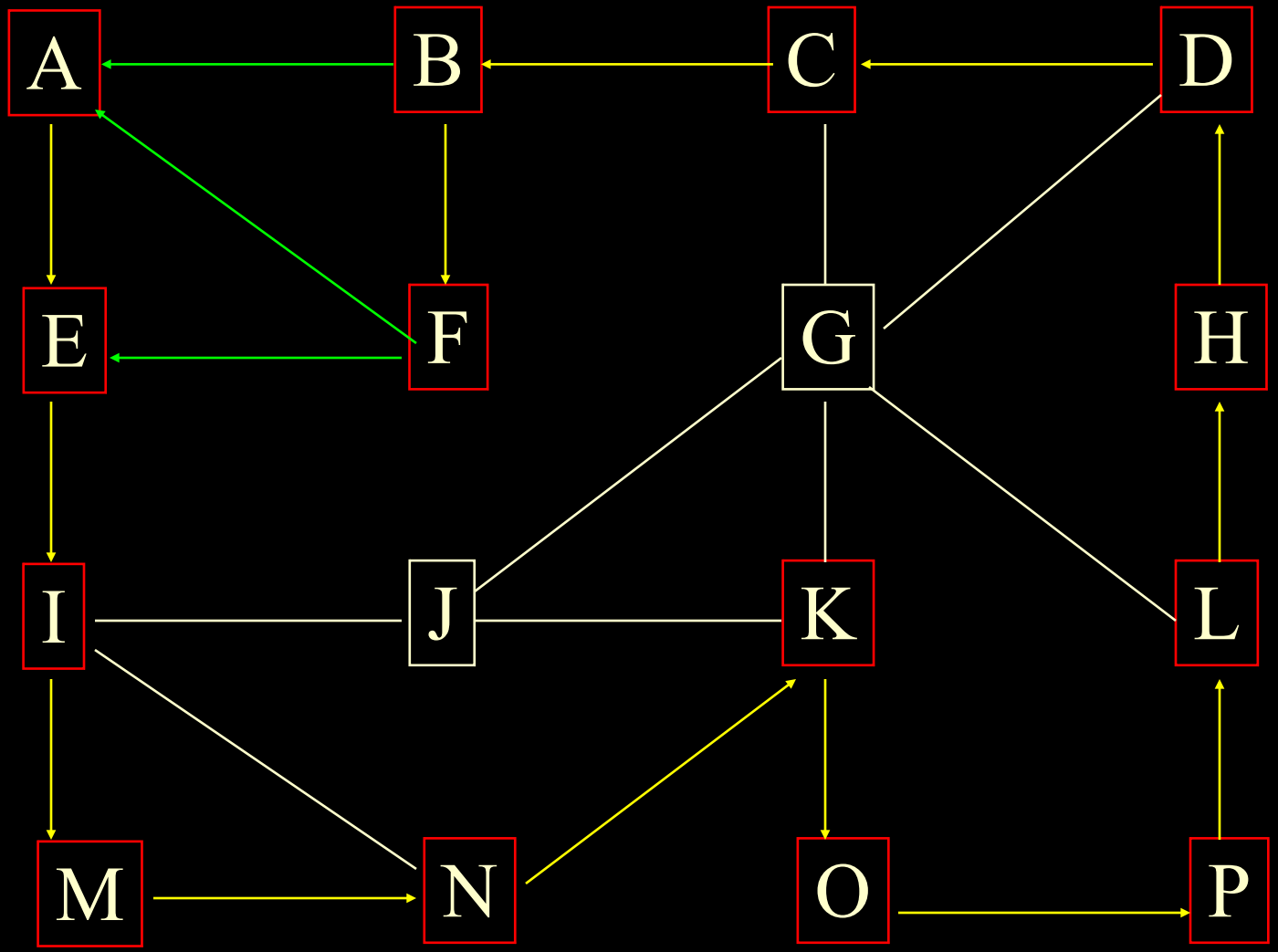

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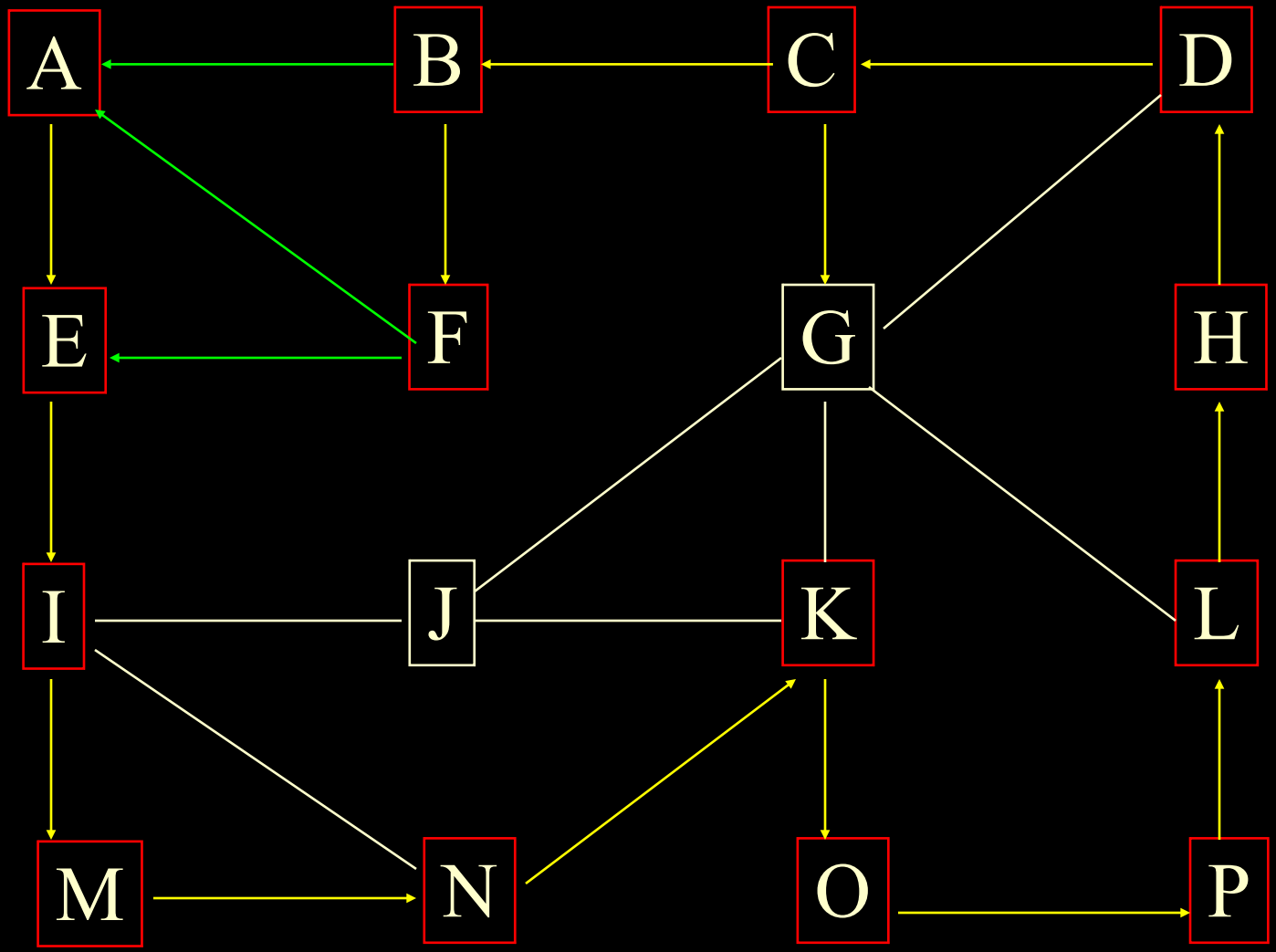

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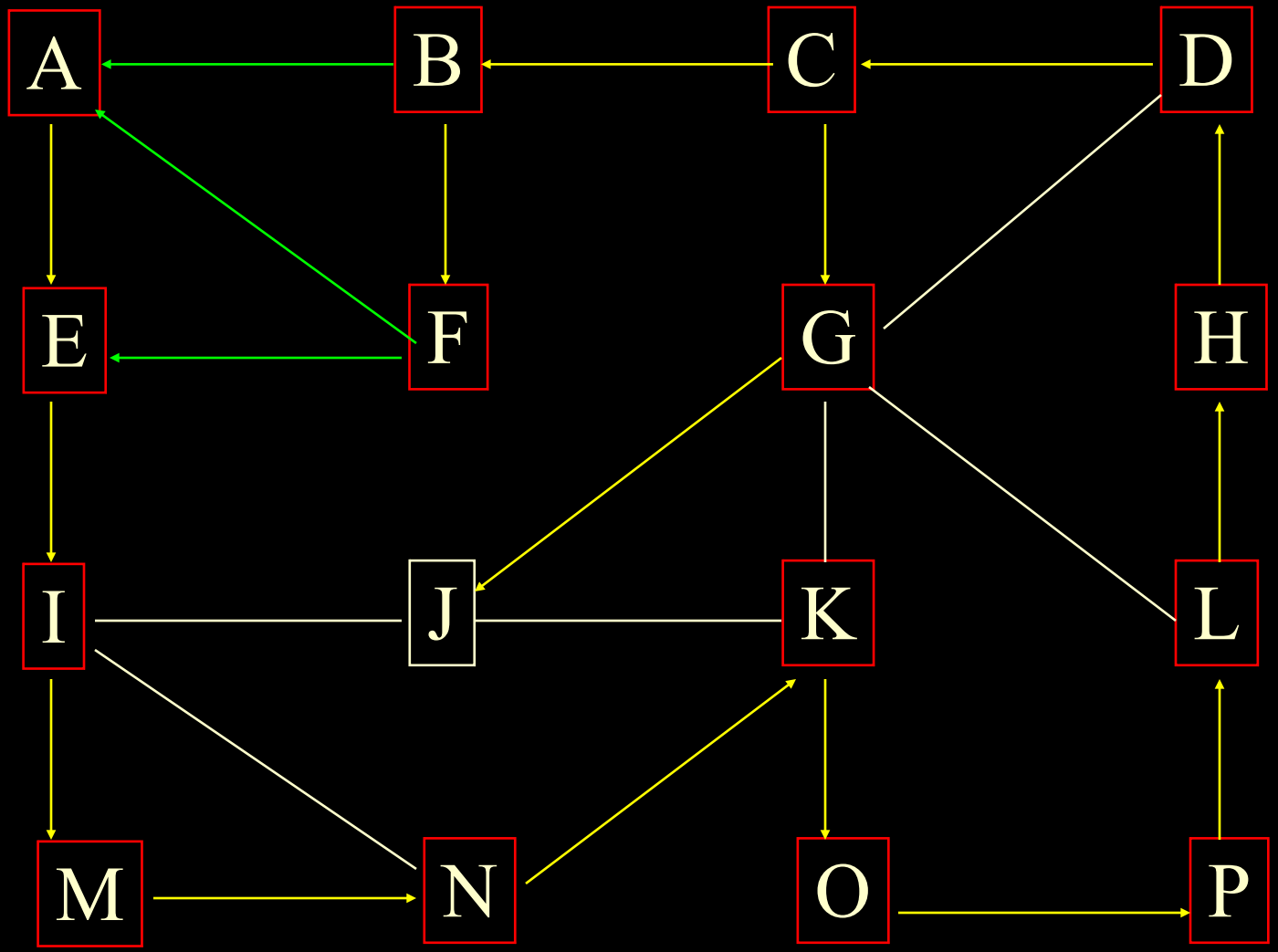

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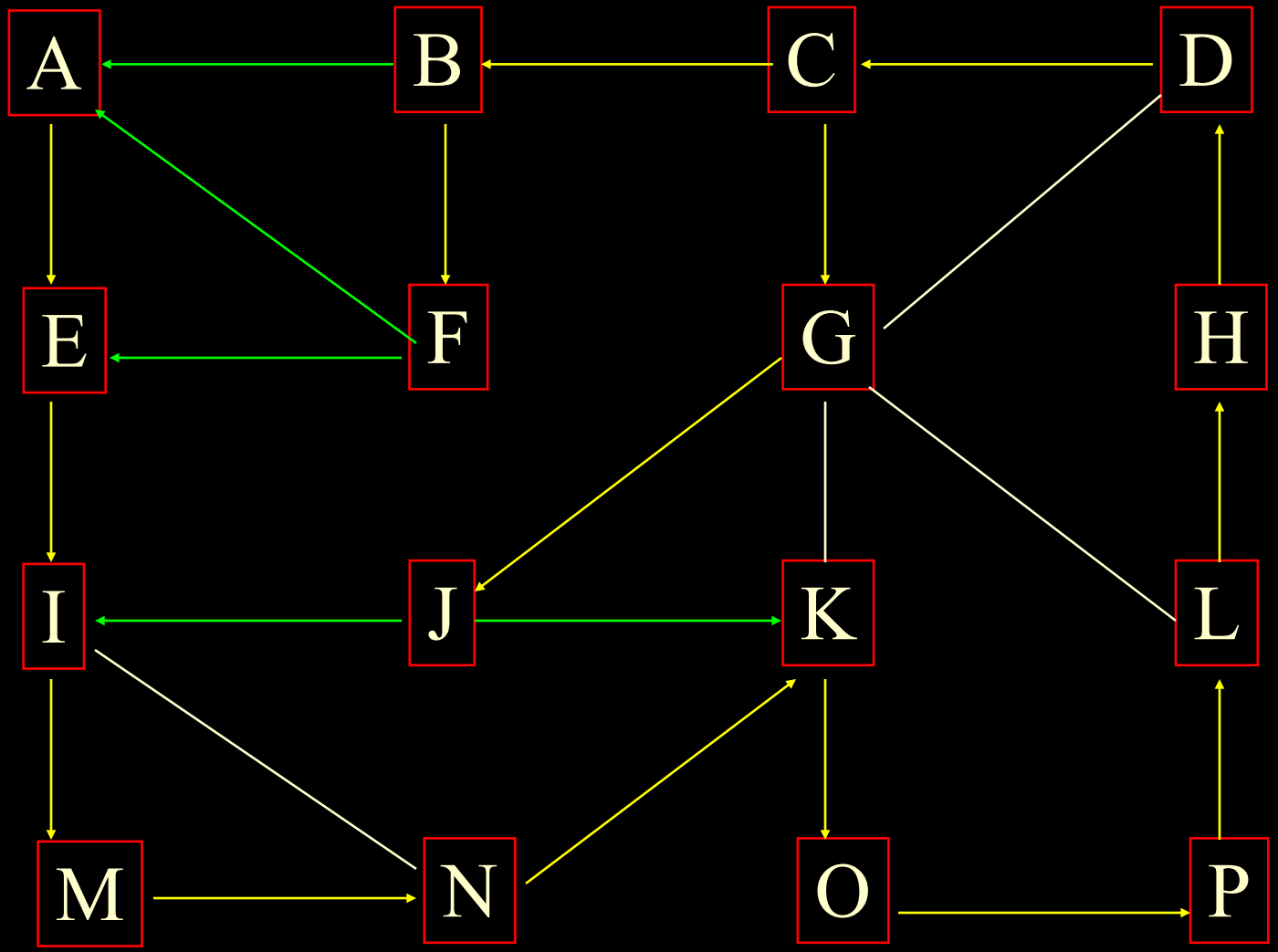

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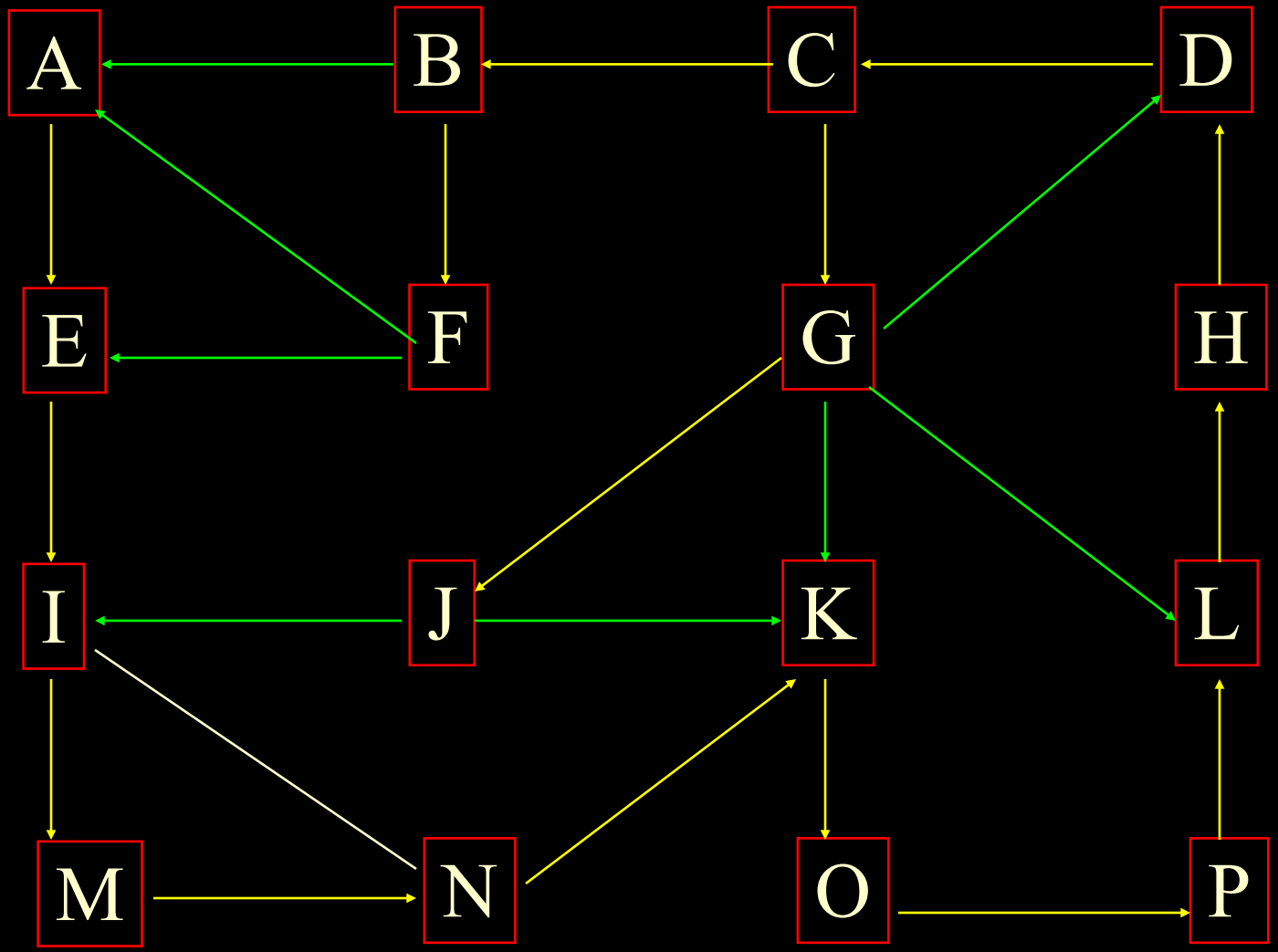

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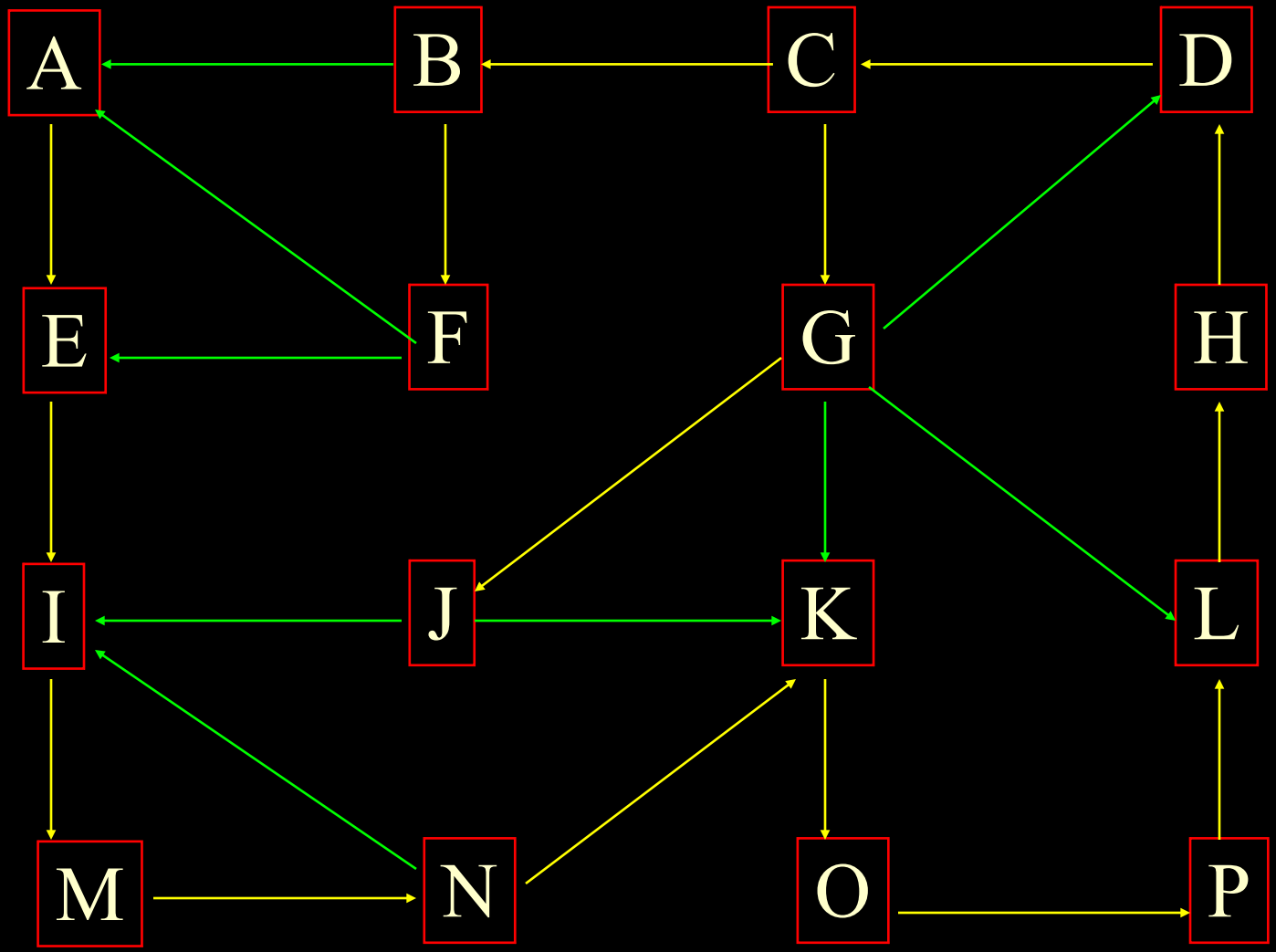
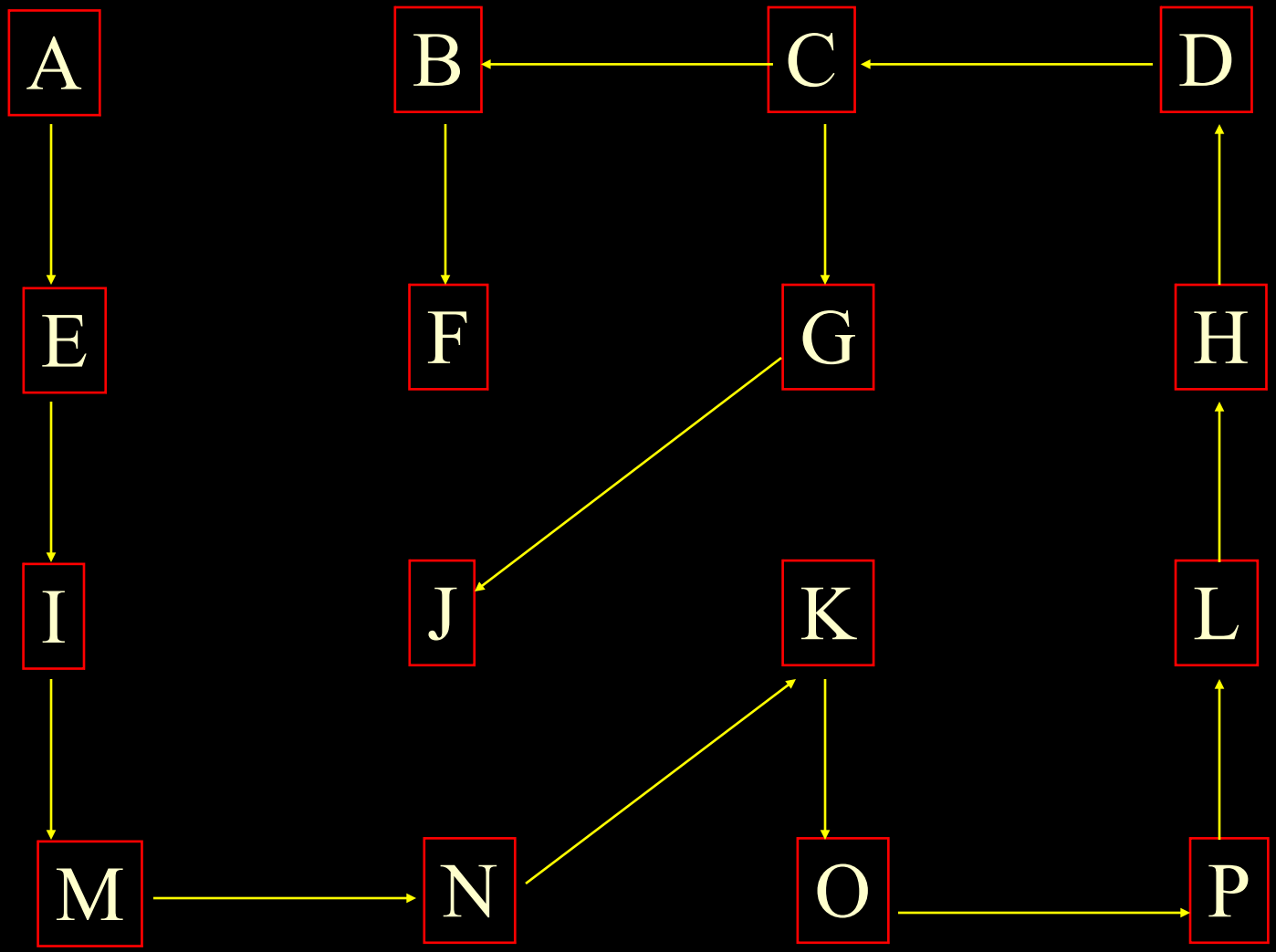


DFS Tree



DFS Properties

Starting at s

- The traversal visits al the vertices in the connected component of s
- The discovery edges form a spanning tree of the connected component of s

DFS Runtime

DFS is called on each vertex exactly once

Every edge is examined exactly twice (once from each of its vertices)

So, for n_s vertices and m_s edges in the connected component of the vertex s, the DFS runs in $O(n_s+m_s)$ if:

- The graph data structure methods take constant time
- Marking takes constant time
- There is a systematic way to examine edges (avoiding redundancy)

Marking Verticies

- Extend vertex structure to support variable for marking
- Use a hash table mechanism to log marked vertices

Breadth-First Search

Starting vertex has level 0 (anchor vertex)

Visit (mark) all vertices that are only one edge away

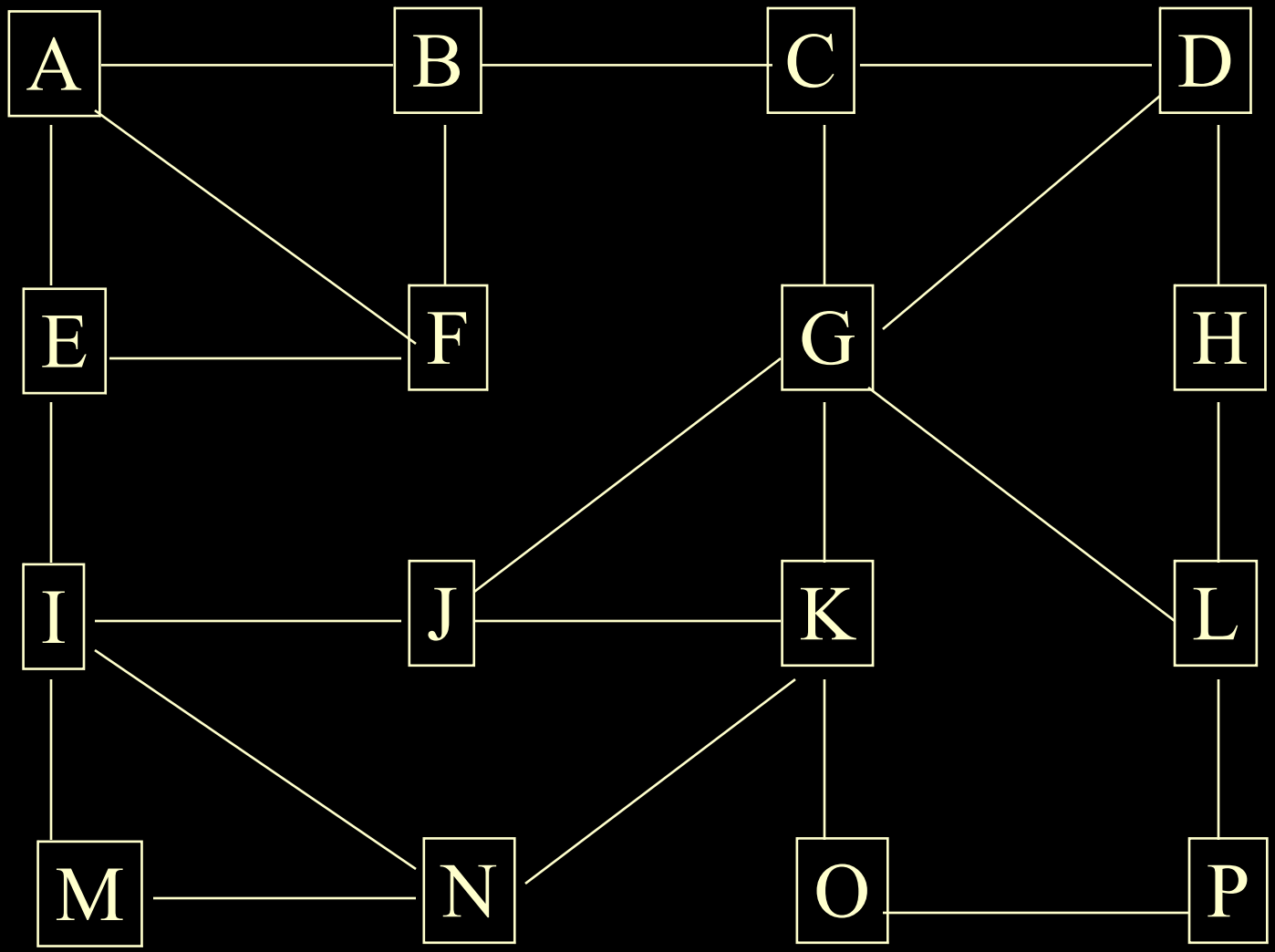
mark each vertex with its “level”

One edge away from level 0 is level 1

One edge away from level 1 is level 2

Etc. . . .

Example



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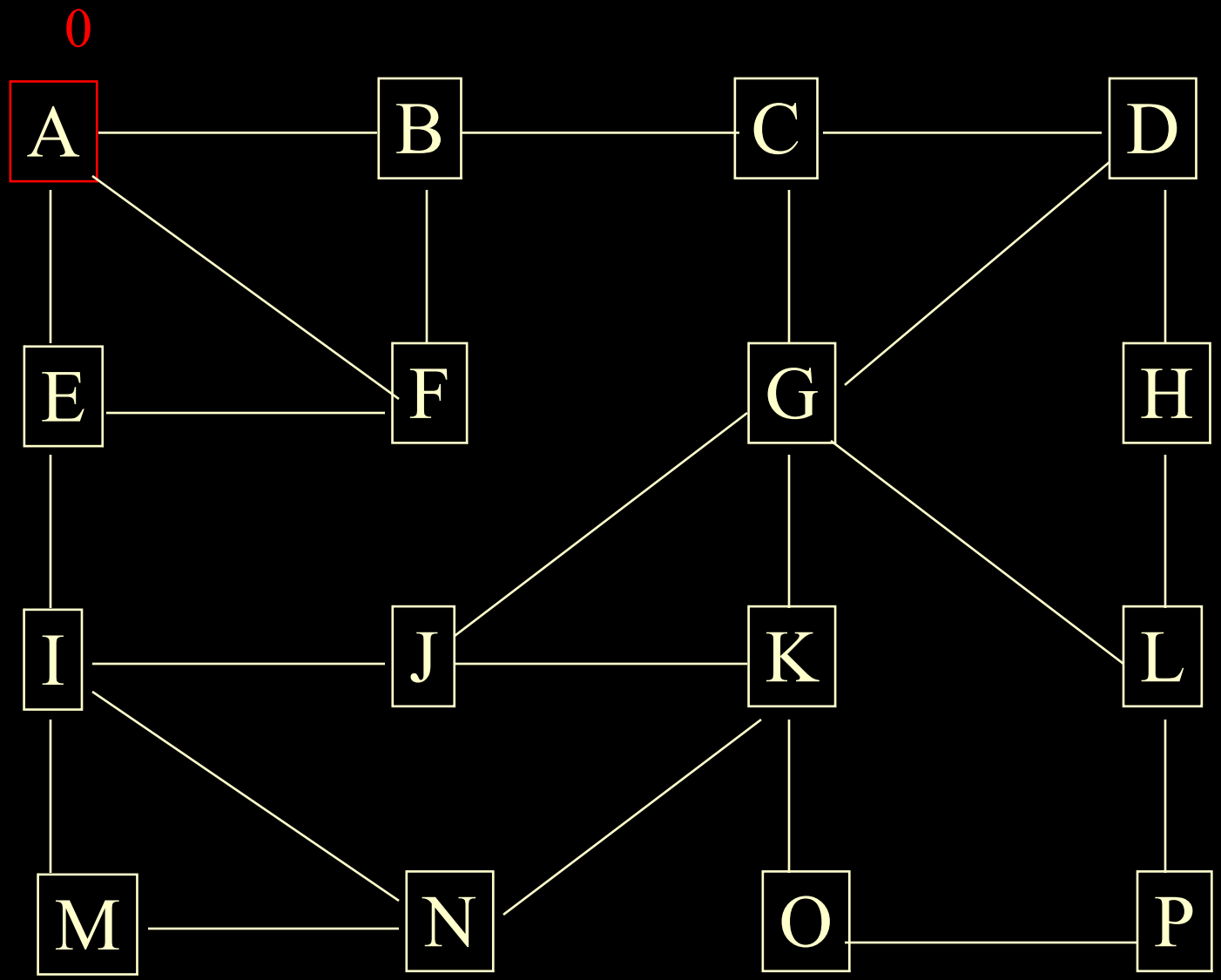

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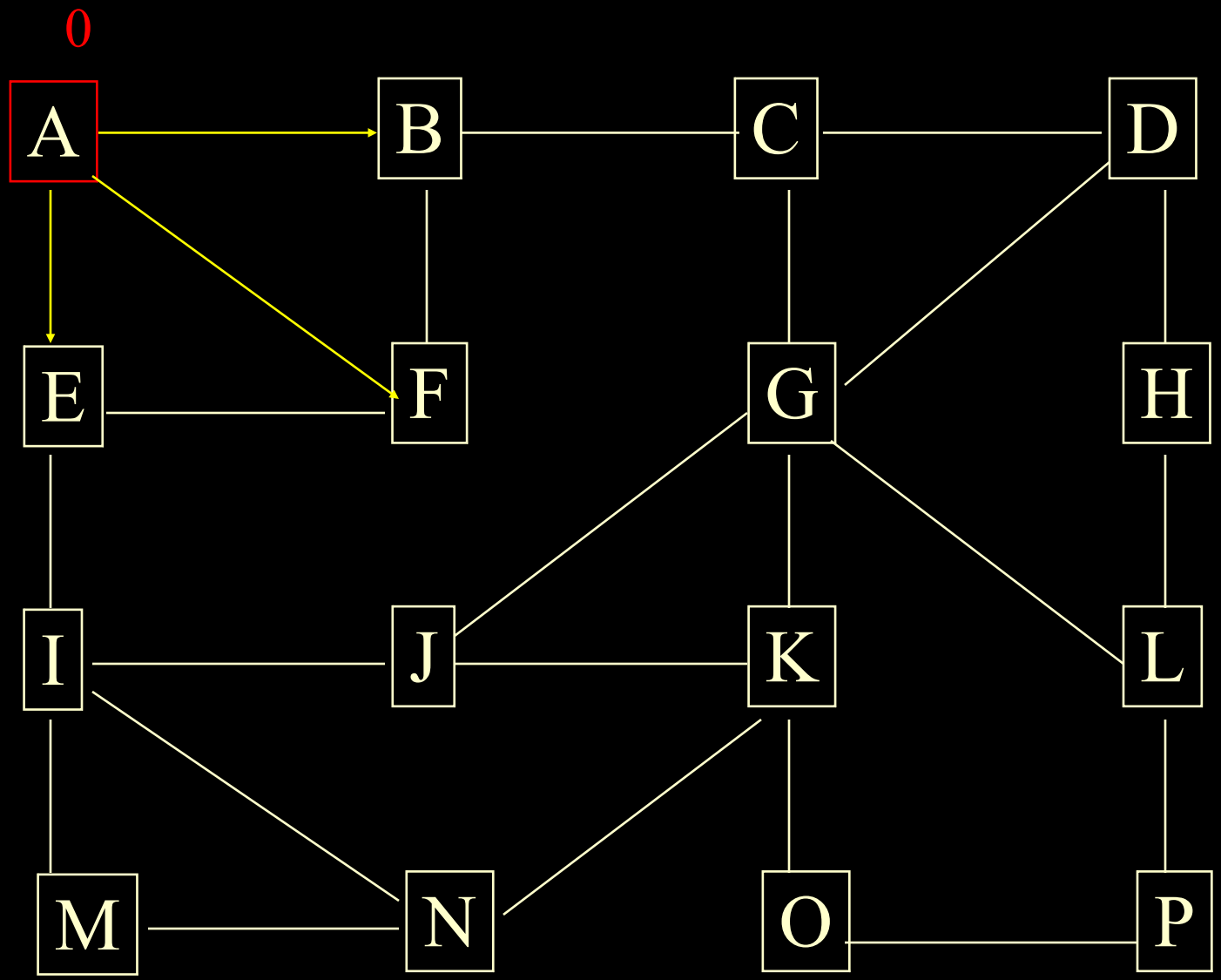

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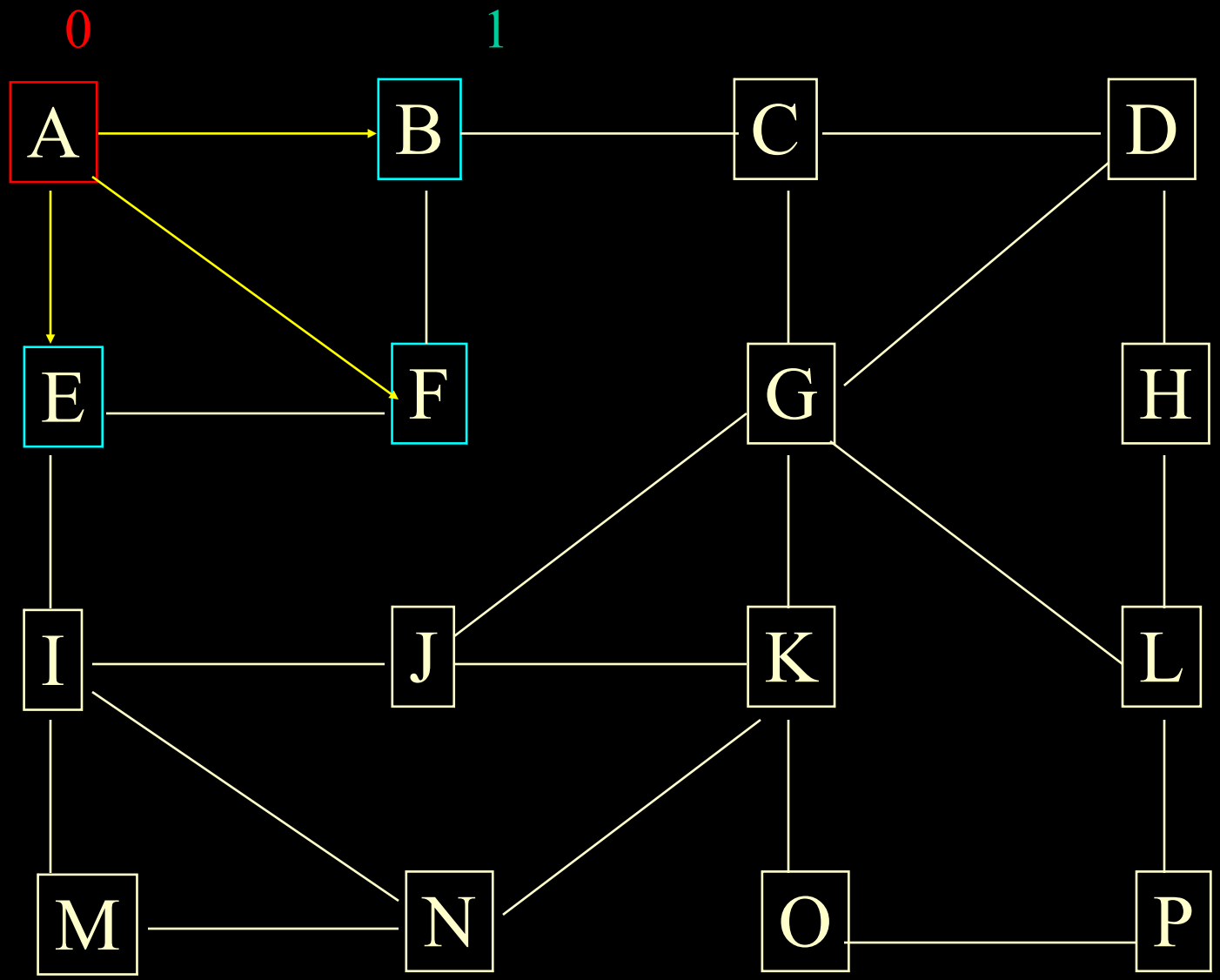

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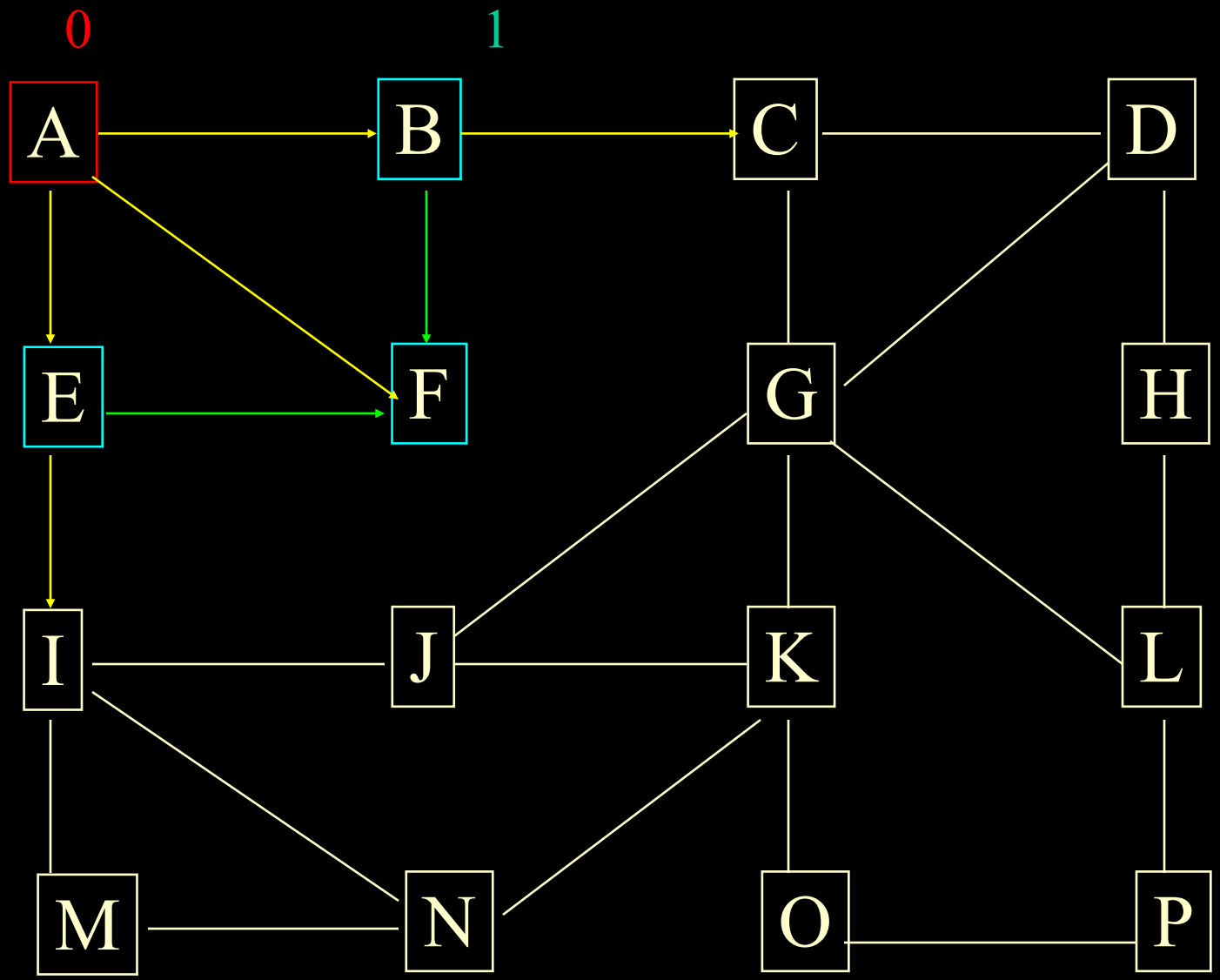

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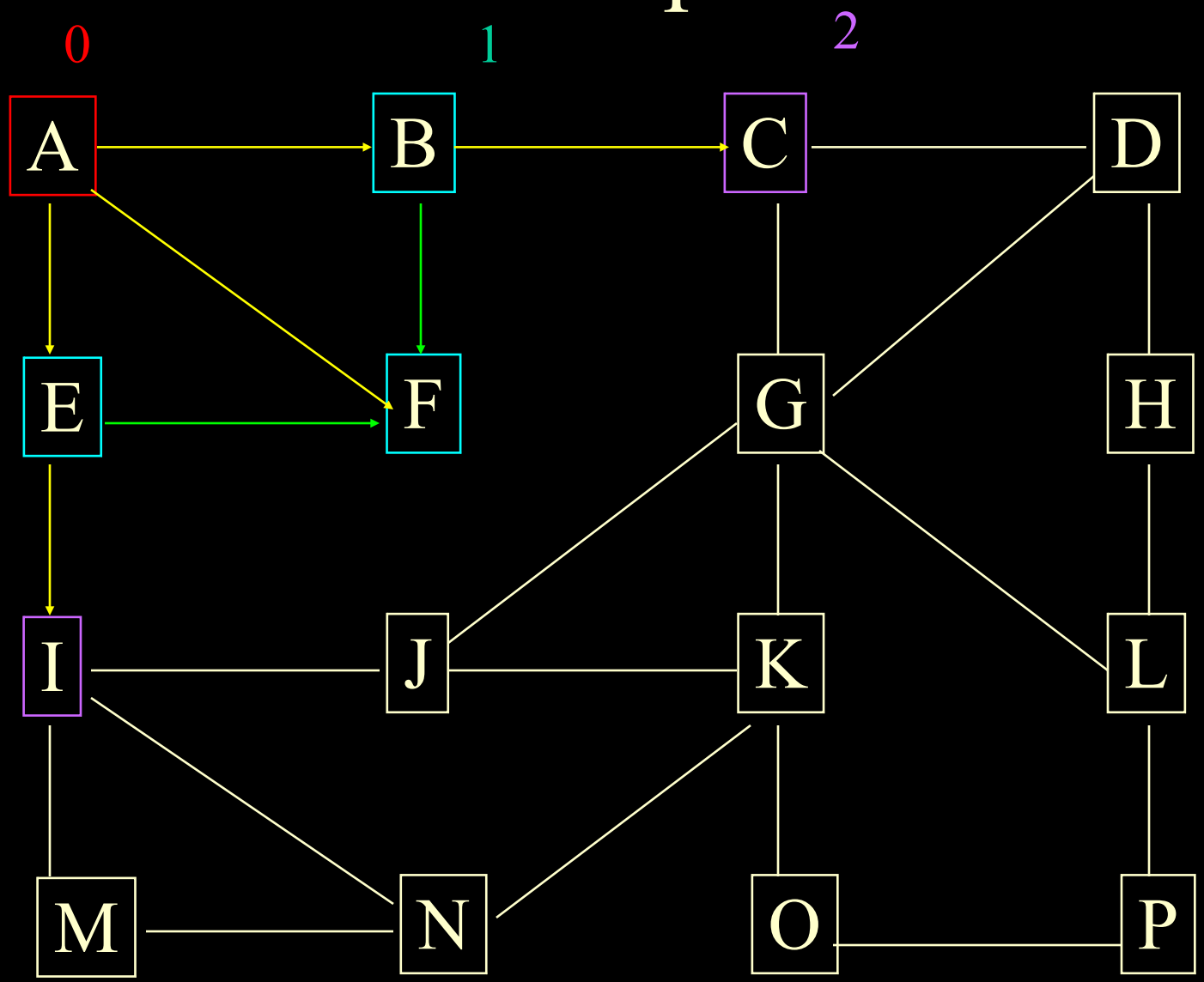

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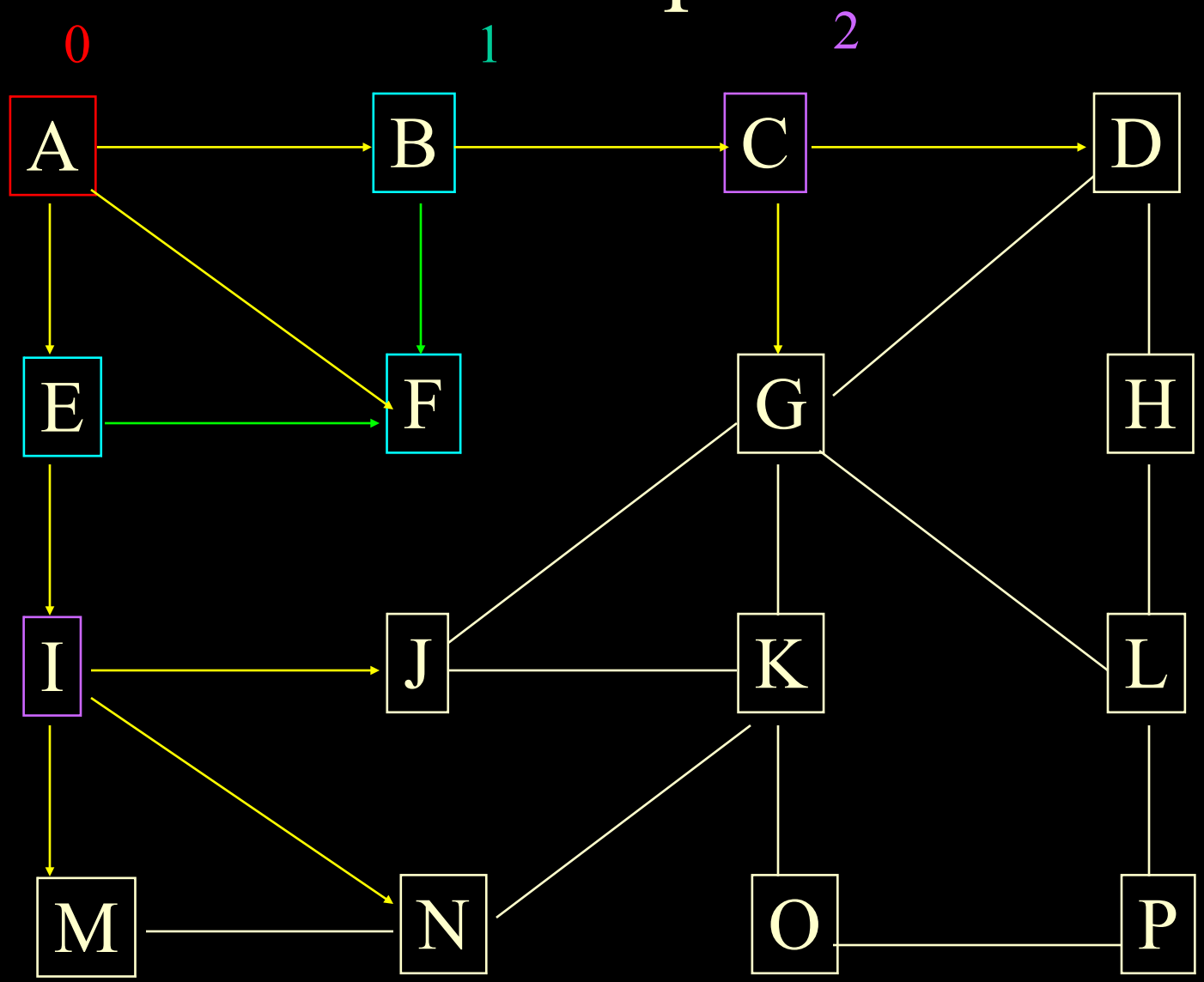

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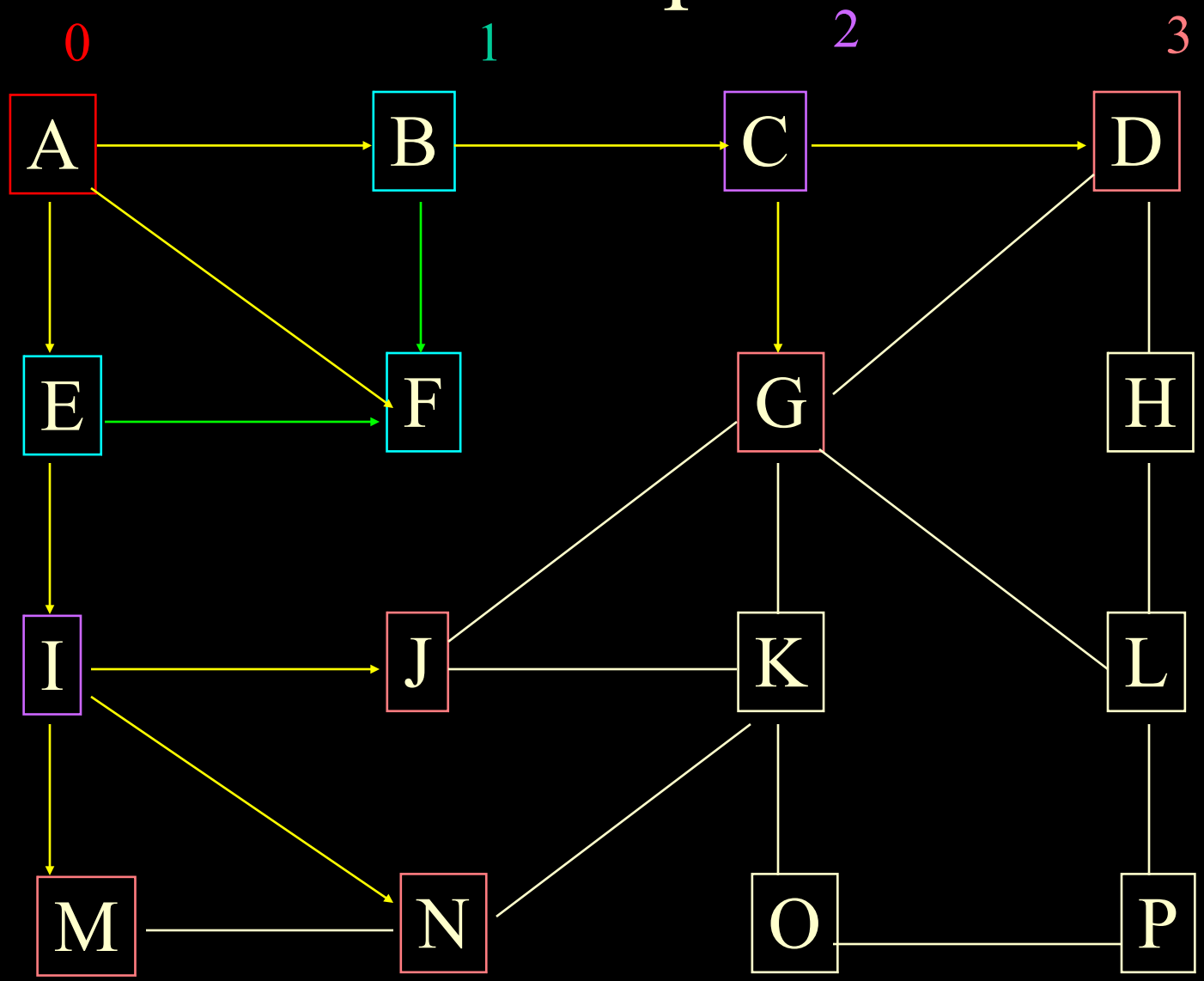

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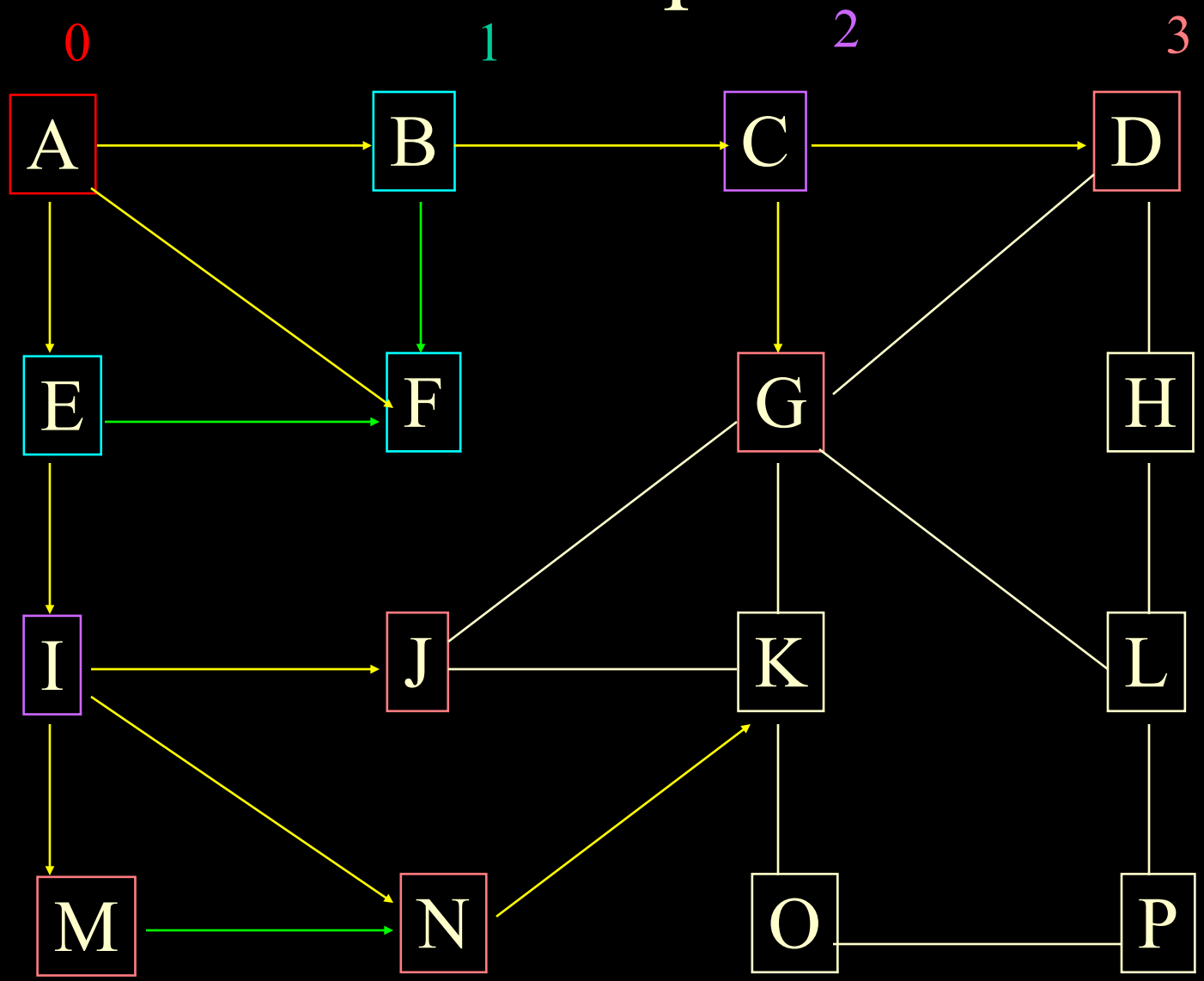

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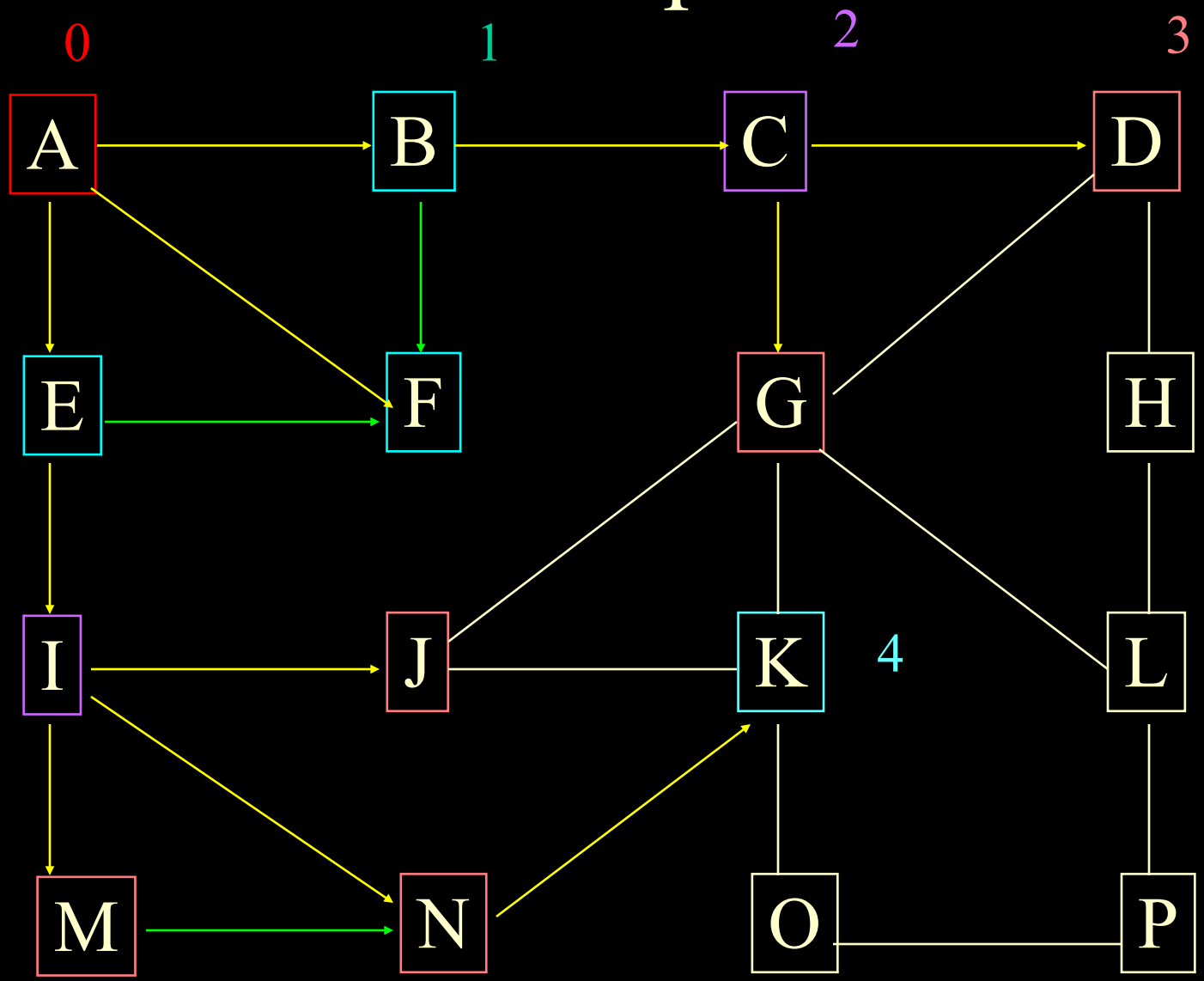

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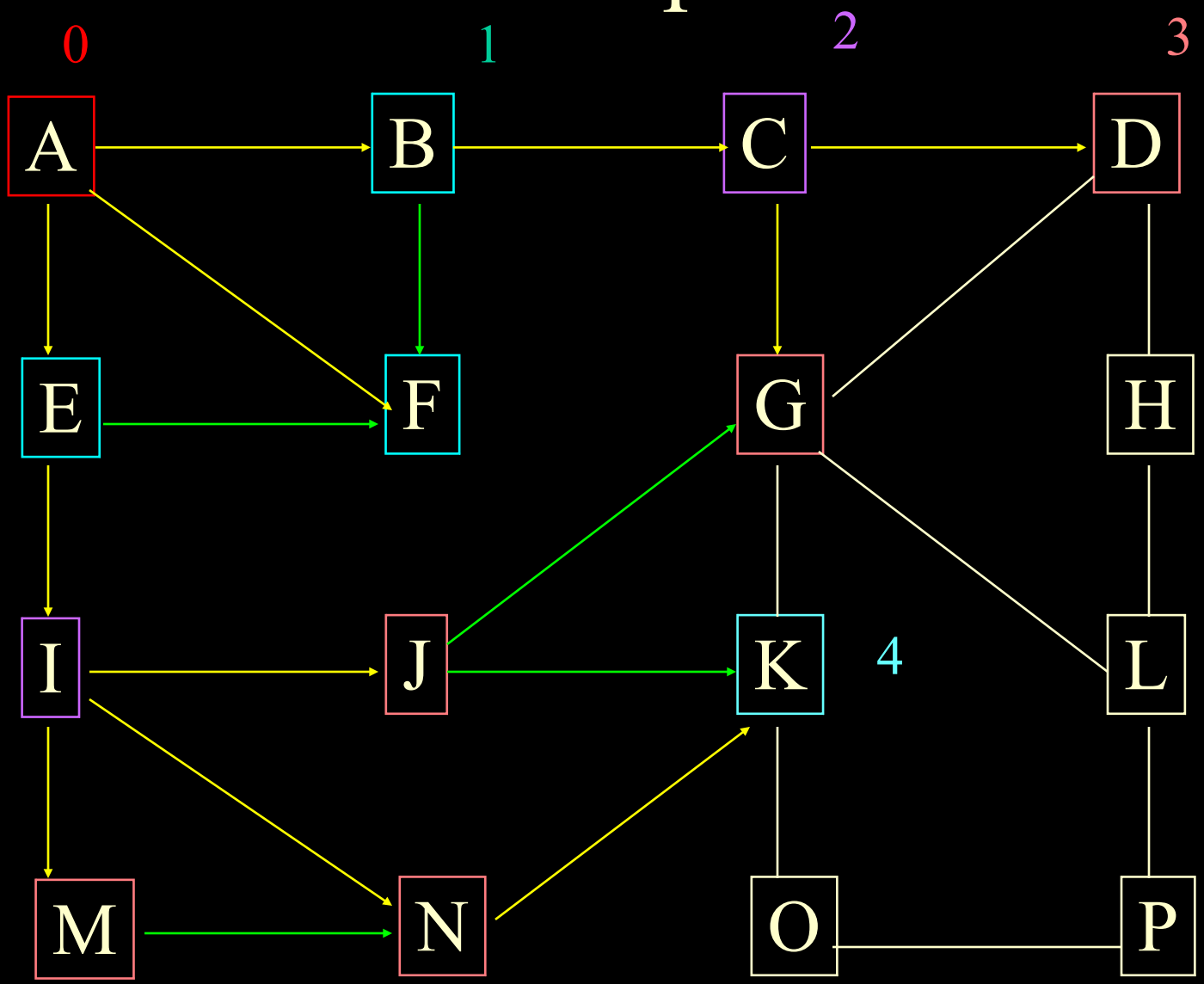

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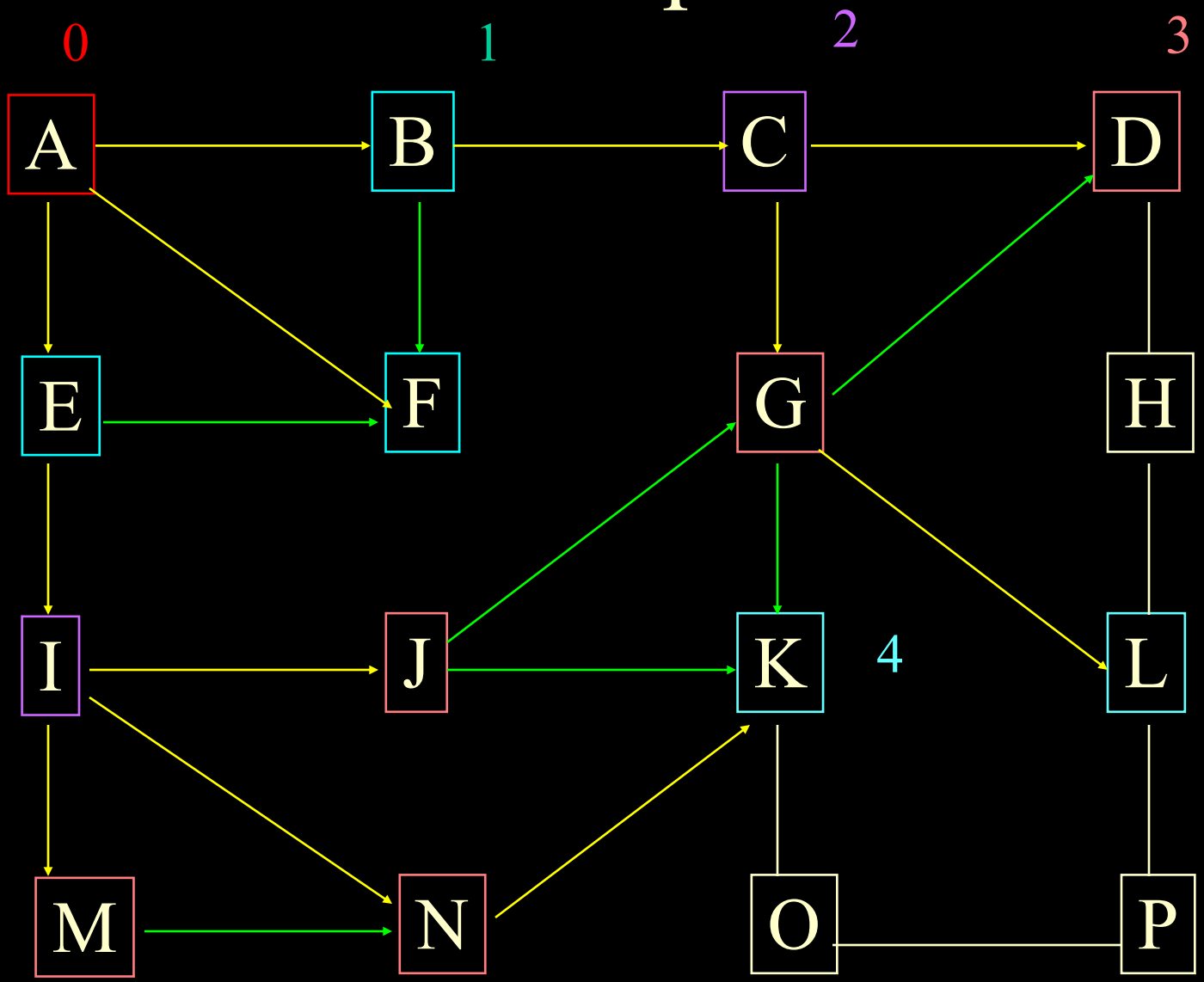

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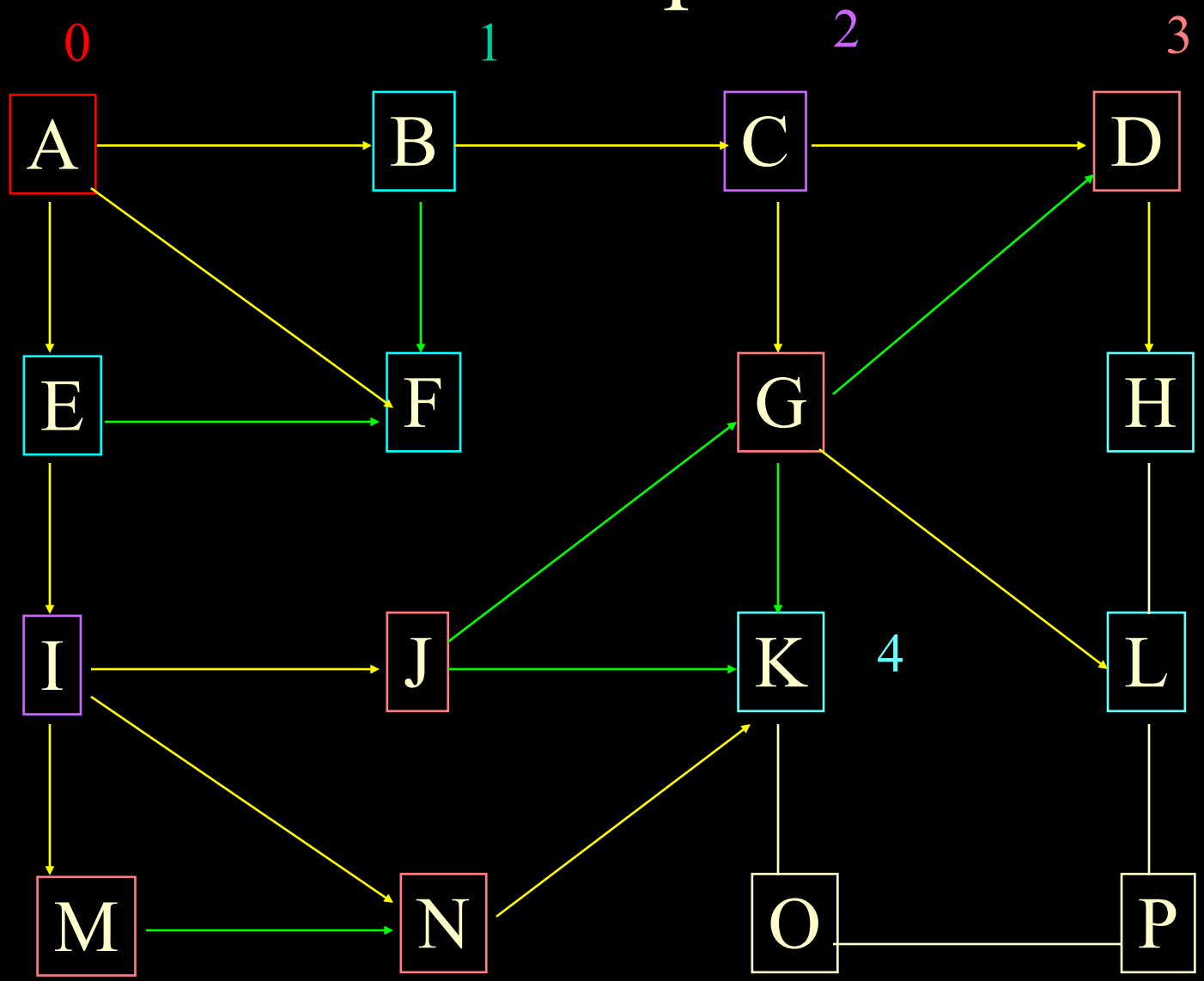

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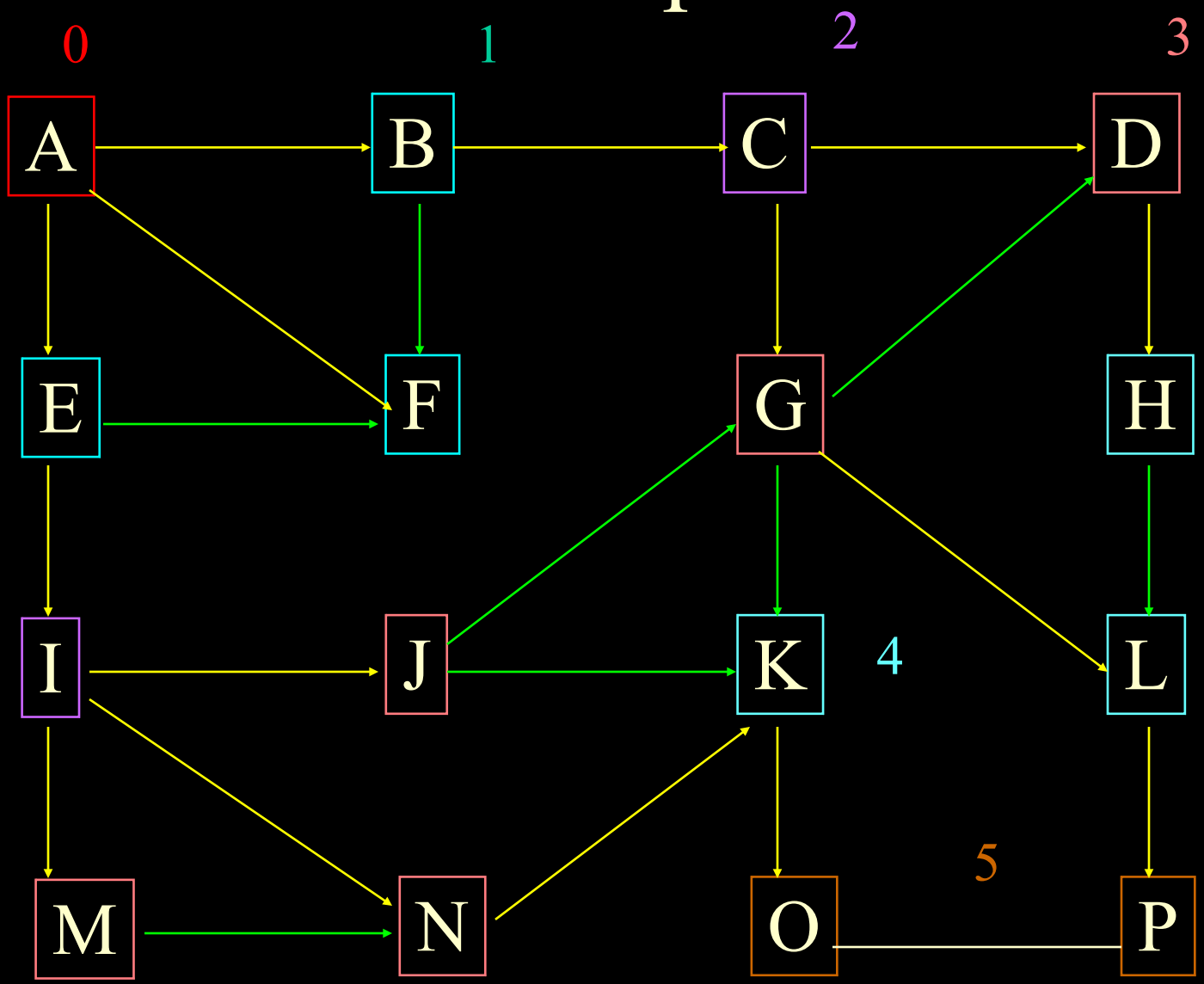

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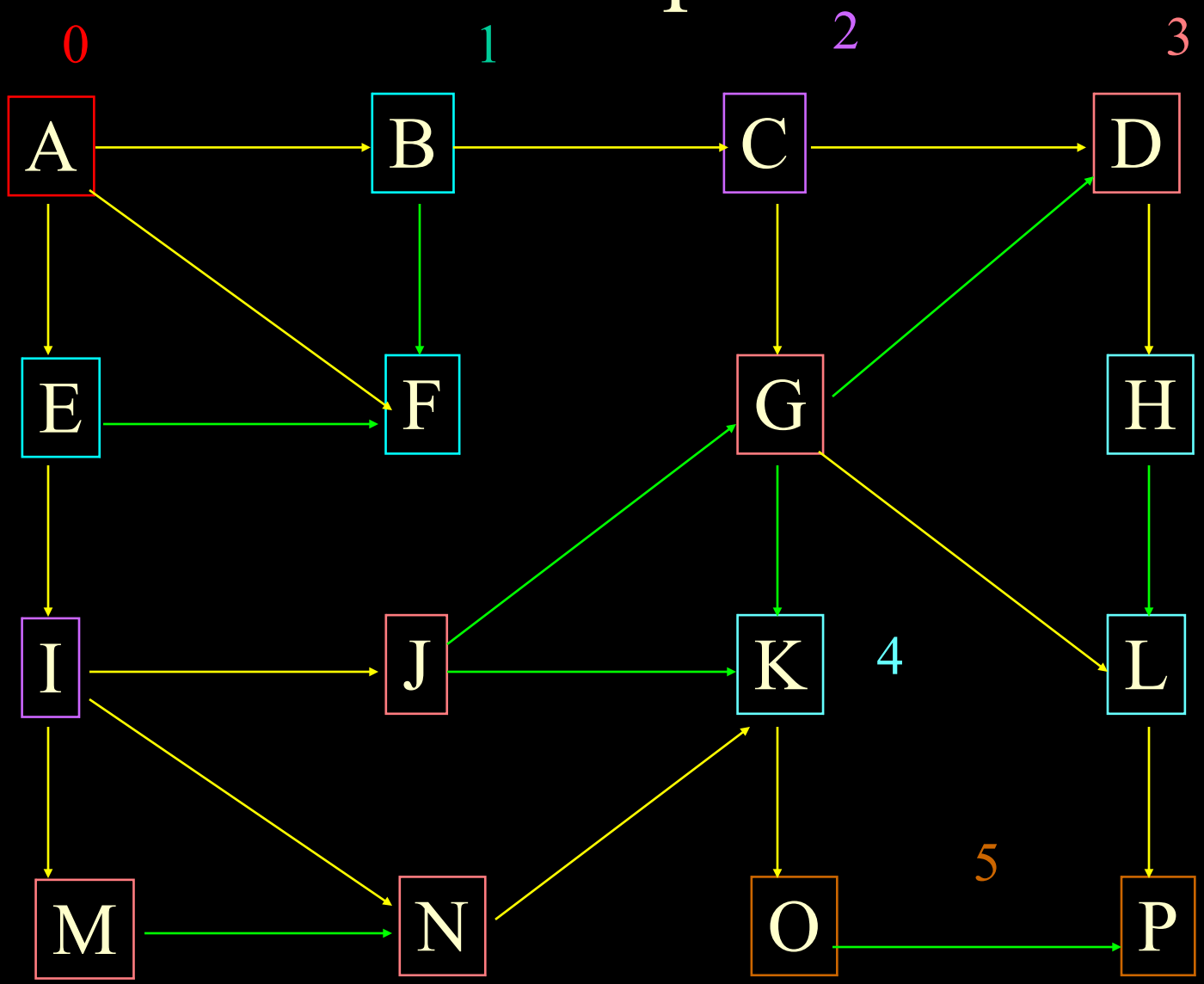
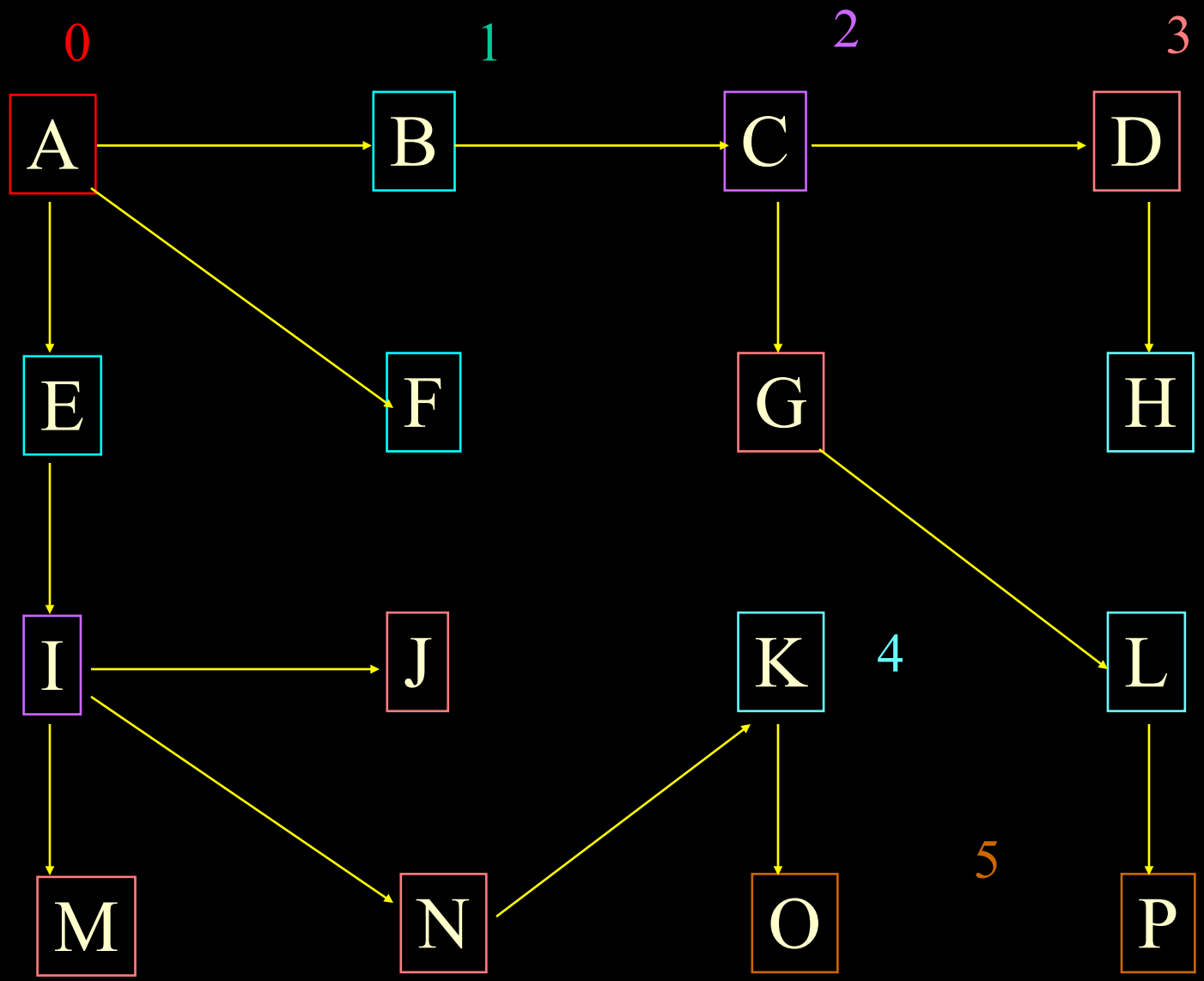


BFS Tree



BFS Pseudocode (1 of 2)

```
BSF(Vertex s)
    initialize container L0 to contain vertex s
    i ← 0
    while Li is not empty do
        create container Li+1 to initially be empty
        for each vertex v in Li do
            // next slide
        i ← i+1
```

BFS Pseudocode (2 of 2)

```
// for each vertex v in L_i do
    if edge e incident on v do
        let w be the other endpoint of e
        if w is unexplored then
                label e as a discovery edge
                insert w into L_{i+1}
        else
                label e as a cross edge
//i ← i+1
```

BSF Properties

The traversal visits all vertices in the connected component of s

The discover edges form a spanning tree of the cc

For each vertex v at level I, the path of the BSF tree T between s and v has I edges and any other path of G between s and v has at least I edges

If (u,v) is an edge that is not in the BSF tree, then the level number of u and v differ by at most one

Run Time

A BSF traversal takes O(n+m) time

Also, there exist O(n+m) time algorithms base on BFS which test for

- Connectivity of graph
- Spanning tree of G
- Connected component
- Minimum number of edges path between s and v