

CSC 242: Homework 6

Assigned on Apr. 9

Not required for submission

If you want to earn the bonus points, you should hand in your solution to the bonus question on Apr. 28 in Class

Requirement

If you need to use your delay coupon, you can hand in your homework to my office. Include the **class ID number** assigned to you for this class, **your name**, and **how many days** late it is (if handed in late) in the headline. The ID number is used to simplify the grading procedure. If you do not know your ID number assigned to you, you can check the link.¹ If you are not on the list, please contact our TA Sean Esterkin (sesterki@u.rochester.edu). (*), (**), or (***) indicates the difficulty of each question.

Policy

We apply the late policy explained in syllabus to all homework. Any grading questions must be raised with the TA in two weeks after the homework is returned.

The homework must be completed individually. However, you are encouraged to discuss the general algorithms and ideas with classmates in order to help you answer the questions. You are also allowed to share examples that are not on the homework in order to demonstrate how to solve problems. If you work with one or more other people on the general discussion of the assignment questions, please record their names over every question they participated.

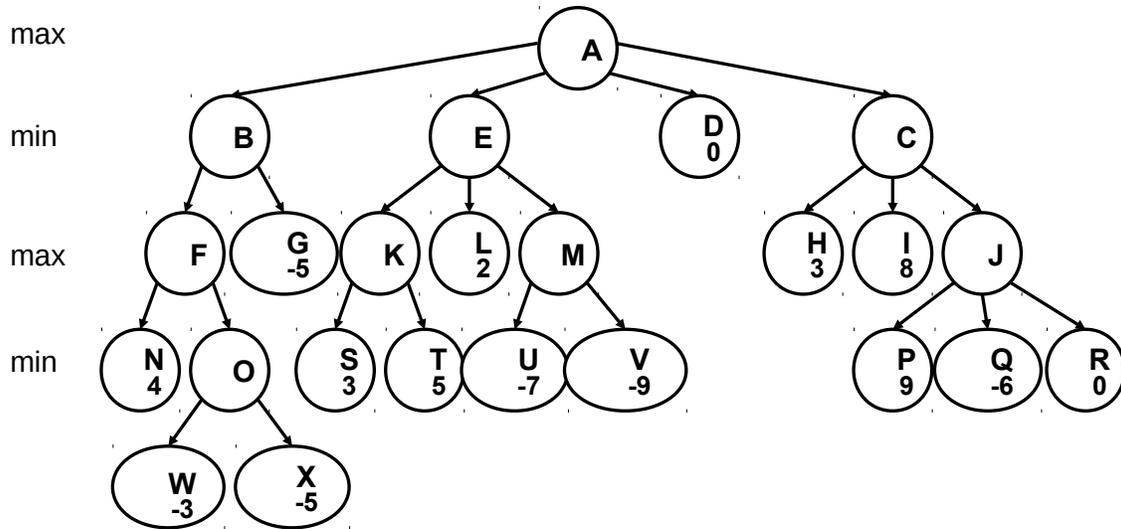
However, the following behaviors will receive heavy penalties (lose all points and apply the honest policy explained in syllabus)

- explicitly tell somebody else the answers;
- explicitly copy answers or code fragments from anyone or anywhere;
- allow your answers to be copied;
- get code from Web.

¹https://docs.google.com/spreadsheets/d/1hgxcxqFxBxZYb55_HHi2pWQt0rEyEppcStSZBKni8MU/edit?usp=sharing

1 (**) Alpha-Beta Pruning: 6 points

Please follow the alpha-beta pruning procedure to prune the following game tree. Please track the values for α and β on all nodes and write their values. And also indicate nodes which can be pruned.



2 (*) Nash Equilibrium for zero sum game: 2 points

Find the theoretical score for this zero sum game and the optimal strategy for player B. The number in the table is the score for Player A. The score for player B is the negative value of each number.

	B-I	B-II	B-III	B-IV
A-I	7	7	2	2
A-II	4	4	2	2
A-III	5	2	5	2
A-IV	5	2	5	2

3 (*) Nash Equilibrium for non-zero sum game: 2 points

Find all Nash Equilibrium points in the following matrix. The first number is the score for Player A while the second number is the score for Player B.

	B-1	B-II	B-III	B-IV
A-I	5, 3	9, 3	5, 8	9, 9
A-II	3, 1	2, 6	4, 1	5, 9
A-III	2, 3	6, 3	8, 4	6, 2
A-IV	3, 8	4, 5	3, 1	2, 3

4 (***) Nash Equilibrium for zero sum game with hidden information: 6 points

Recall the Morra example in our class. It is an unfair game for player E. Recall that the score matrix is

	O-I	O-II
E-I	2	-3
E-II	-3	4

Now to make this game be fair for both players, you are allowed to change the score matrix a little bit by introducing one variable t

	O-I	O-II
E-I	2	-t
E-II	-3	4

Please answer the following questions

- What is the value for t to obtain a fair game?
- When this is a fair game, what is the optimal mixed strategy for player-E?
- When this is a fair game, what is the optimal mixed strategy for player-O?

5 (***) (Bonus Question) Chinese Morra: 5 points

This question is motivated by the Chinese version of Morra. The rule is probably more complicated than any other versions of Morra. Two players (A and B) compete each other. Both of them need to show a pair of numbers simultaneously. We use the pair (n_A, N_A) to denote the numbers shown by A and the pair (n_B, N_B) for B. N_A (or N_B) is spoken out by A (or B), while n_A (or n_B) are displayed by the fingers of A (or B). The ranges are $N_A, N_B \in \{0, 1, 2, \dots, 10\}$ and $n_A, n_B \in \{0, 1, \dots, 5\}$. The goal of two players is to guess the sum of two numbers shown (using fingers) by both. If $N_A = n_A + n_B$ and $N_B \neq n_A + n_B$, then A wins B loses. If $N_B = n_A + n_B$ and $N_A \neq n_A + n_B$, then

B wins A loses. Please use the knowledge learned this class to find out the optimal mixed strategy of A (or B). Specifically, you are asked to provide the following things

- Provide the optimal mixed strategy, specifically showing the probability for each pair;
- Is this a fair game? Explain why.
- Think about the most frequent pair (or pairs). Provide some explanation it should be the most frequent pair.

To compute the optimal strategy, you need to solve the linear programming problem. You can use any exiting solver to solve it. To formulate the problem into a linear programming problem, you probably need to use the following equivalence: solving

$$\begin{aligned} \max_{x \in \mathbb{R}^n} \quad & \min(a_1^\top x, a_2^\top x, \dots, a_n^\top x) \\ \text{s.t.} \quad & \text{any constraint for } x. \end{aligned}$$

is equivalent to solving

$$\begin{aligned} \max_{x \in \mathbb{R}^n, t} \quad & t \\ \text{s.t.} \quad & a_i^\top x \geq t \quad \forall i = 1, 2, \dots, n \\ & \text{any constraint for } x. \end{aligned}$$