Preface

Invitation

Secret 1 *Algorithms are at the heart of complexity theory.*

That is the dark secret of complexity theory. It is recognized by complexity theorists, but would be literally incredible to most others. In this book, we hope to make this secret credible. In fact, the real secret is even more dramatic.

Secret 2 *Simple algorithms are at the heart of complexity theory.*

A corollary of Secret 2 is that every practitioner of computer science or student of computer science already possesses the ability required to understand, enjoy, and employ complexity theory.

We realize that these secrets fly in the face of conventional wisdom. Most people view complexity theory as an arcane realm populated by pointy-hatted (if not indeed pointy-headed) sorcerers stirring cauldrons of recursion theory with wands of combinatorics, while chanting incantations involving complexity classes whose very names contain hundreds of characters and sear the tongues of mere mortals. This stereotype has sprung up in part due to the small amount of esoteric research that fits this bill, but the stereotype is more strongly attributable to the failure of complexity theorists to communicate in expository forums the central role that algorithms play in complexity theory.

Throughout this book—from the tree-pruning and interval-pruning algorithms that shape the first chapter to the query simulation procedures that dominate the last chapter—we will see that proofs in complexity theory usually employ algorithms as their central tools. In fact, to more clearly highlight the role of algorithmic techniques in complexity theory, *this book is organized by technique rather than by topic*. That is, in contrast to the organization of other books on complexity theory, each chapter of this book focuses on one technique—what it is, and what results and applications it has yielded.

The most thrilling times in complexity theory are when a new technique is introduced and sweeps like fire over the field. In addition to highlighting the centrality of algorithms in the proof arsenal of complexity theory, we feel that our technique-based approach more vividly conveys to the reader the flavor and excitement of such conflagrations. We invite the reader to come with us as we present nine techniques, usually simple and algorithmic, that burned away some of the field's ignorance and helped form the landscape of modern complexity theory.

Usage

We intend this book as a companion for students and professionals who seek an accessible, algorithmically oriented, research-centered, up-to-date guide to some of the most interesting techniques of complexity theory. The authors and their colleague Joel Seiferas have test-driven the book's approach in two different courses at the University of Rochester. We have used this technique-based approach in Rochester's one-semester basic complexity theory course, which is taken by all first-year computer science graduate students and also by those undergraduates specializing or specially interested in theoretical computer science, and in our second course on complexity theory, which is taken by all second-year graduate students as their theory "breadth" course.

We found in both these course settings that the technique-based approach allowed us to impart to students a significant amount of the feel and experience of complexity theory research and led to more student interest and involvement than occurred in earlier course incarnations using other texts. We expect that this will not only benefit the complexity theory students in the courses, but will also help all the course's students become prepared to do work that is theoretically aware, informed, and well-grounded.

At times, we stop the flow of a proof or discussion with a "Pause to Ponder." These are places at which we encourage the reader to pause for a moment and find his or her own solution to the issue raised. Even an unsuccessful attempt to craft a solution will usually make the proof/discussion that follows clearer and more valuable, as the reader will better understand the challenges posed by the hurdle that the proof/discussion overcomes.

With some exceptions due to result dependencies, the non-appendix chapters are generally ordered to put the easier chapters near the start of the book and the more demanding chapters near the end of the book.

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