Defying Gravity and Gadget Numerosity: The Complexity of the Hanano Puzzle

Michael C. Chavrimootoo
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In this talk

- What is the Hanano Puzzle?
- Typical Game Complexities
- A Useful Tool: Nondeterministic Constraint Logic
- Defying Gravity: Visibility Representations
- Tool of the Trade: Gadget-Making

What is the Hanano Puzzle?

Creator: Qrostar, 2011

Windows-only video game

Combinatorial game with **gravity**

Played on a “**grid board**”

Played as a series of levels that must be “cleared”

Previous work: NP-hard (Liu and Yang, 2019)

Components of a level

Solving a level

As a decision problem

We want to determine the complexity of

$$\text{HANANO} = \{H \mid H \text{ is a solvable level of the Hanano Puzzle}\}$$

Known: HANANO is NP-hard (Liu and Yang, 2019)

Typical Game Complexities (Hearn and Demaine 2009)

Zero-player

- Bounded: P-complete
- Unbounded: PSPACE-complete

One-player

- Bounded: NP-complete
- Unbounded: PSPACE-complete

Two-player

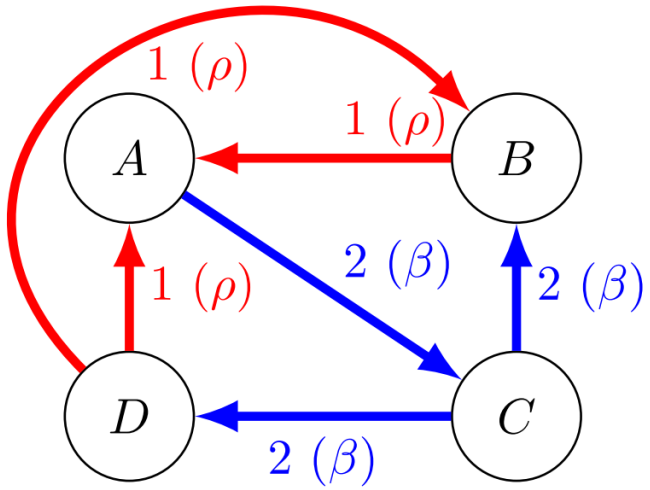
- Bounded: PSPACE-complete
- Unbounded: EXP-complete

Team

- Bounded: NEXP-complete
- Unbounded: RE-complete

See “Games, Puzzles, and Computation” by Hearn and Demaine (2009) for concrete examples.

Nondeterministic Constraint Logic (NCL) (Hearn and Demaine 2009)

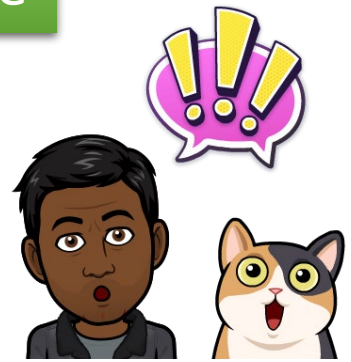


- Directed graph with only red (weight 1) and blue (weight 2) edges
- Inflow constraint: Sum of weight of incoming edges ≥ 2

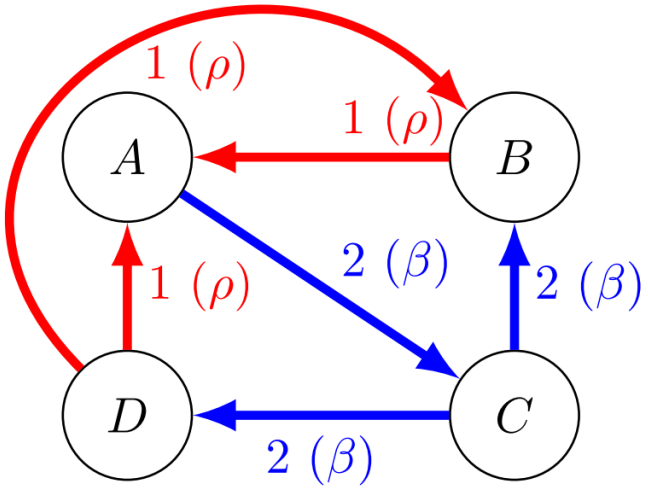
Question: Given an NCL graph and an edge in the graph, is there a sequence of flips such that the given edge is eventually flipped?

One move: flip an arbitrary edge

PSPACE-complete!



Additional restrictions



- Still PSPACE-complete if (1) all vertices are AND/OR vertices, (2) graph is planar.
- AND vertex: exactly 2 incident red edges and 1 incident blue edge
- OR vertex: exactly 3 incident blue edges
- **Only need 2 gadgets**

From now on, we will assume that the NCL graphs have these restrictions

How Will Gadgets Work?

Each vertex is represented by a gadget, and each gadget has 3 entry points (one for each incident edge).

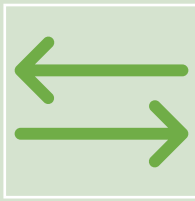
If an edge is into a vertex, then a blue block is placed at the corresponding entry point. The location of the block represents the direction of the edge.

For each block in a gadget, there is only one flower in that gadget that can bloom it.

If the target edge is (u, v), then gadget for v is modified to have one less flower.

Challenge

NCL is **fully reversible**, but Hanano is not. → Some edges may need to flip multiple times

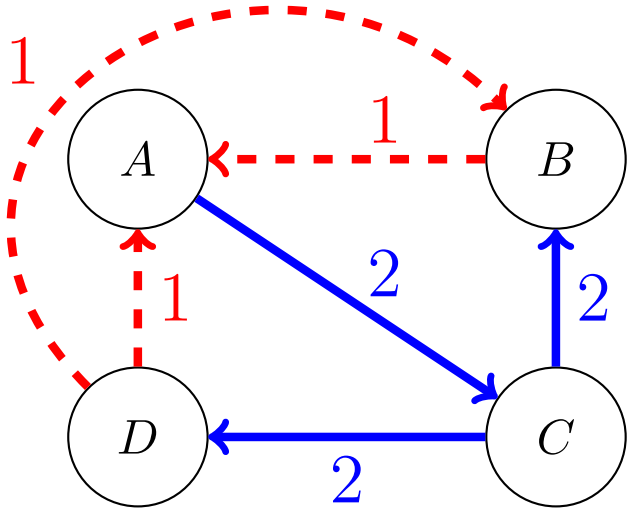


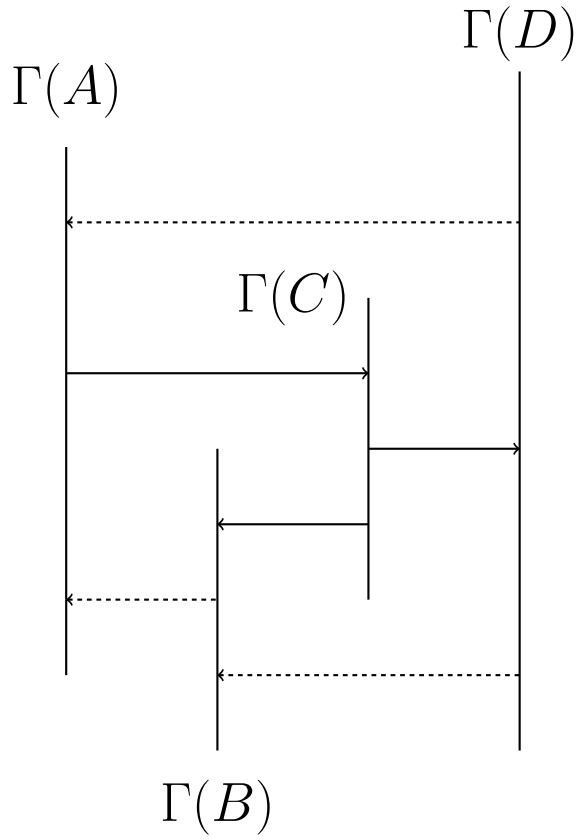
Our gadgets and their interactions need to be **fully reversible***

Contributions

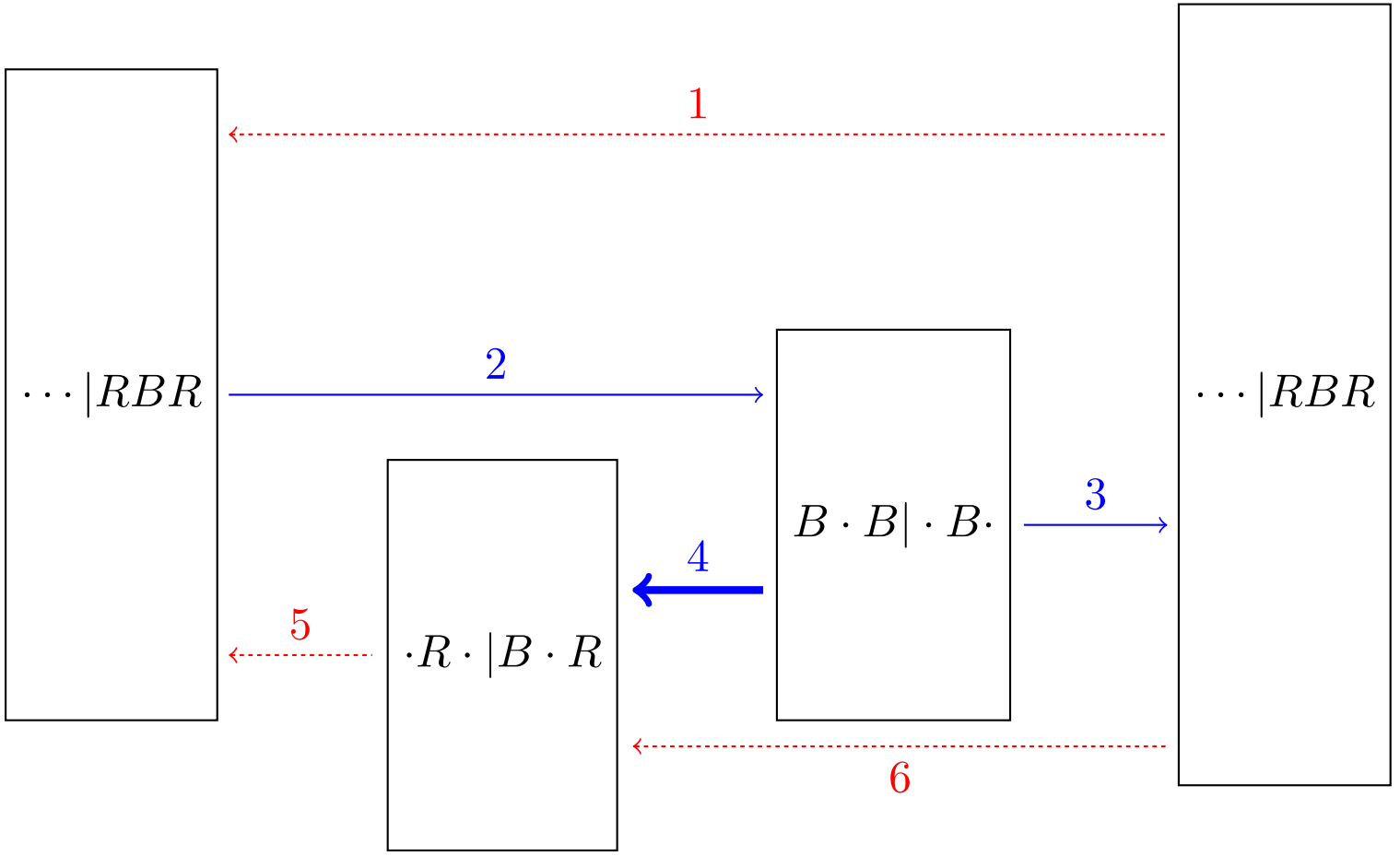
HANANO is PSPACE-complete	Show how to use structured NCL variant using only three gadgets (regardless of the game)
Even with one color and all blocks blooming upwards	Added structure creates an explosion in number of gadgets

Defying Gravity: Visibility Representation





Sketch of a Reduction

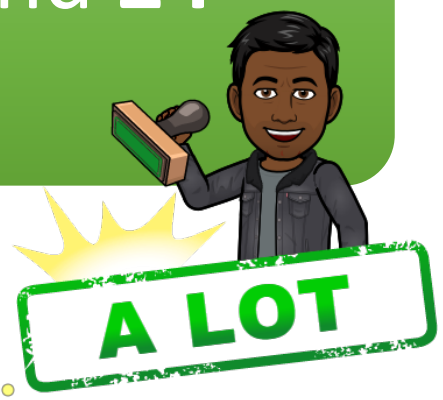


So how many gadgets do we need?

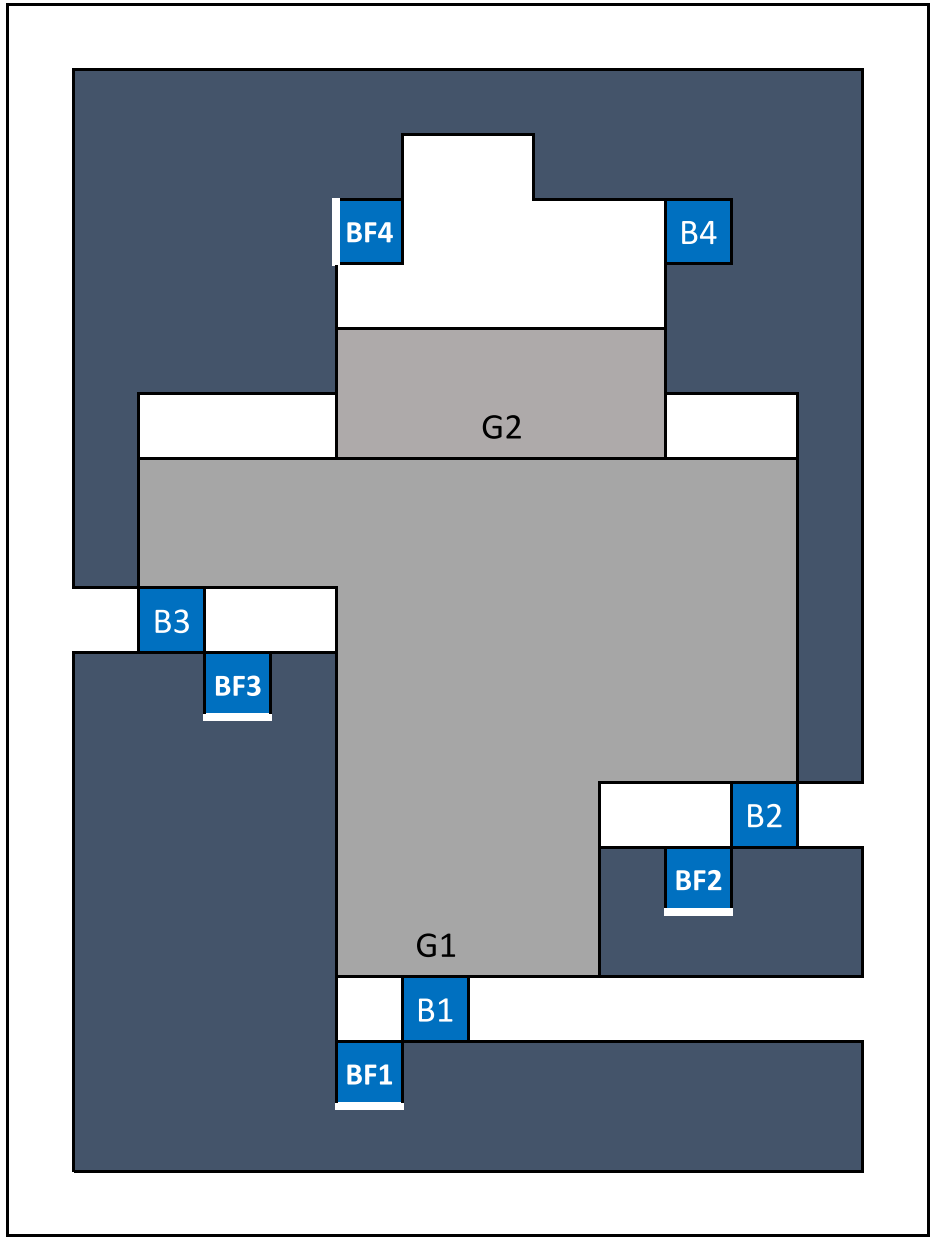
Each gadget will have exactly three entry points.

They can each lie either on the left or the right of the gadget.

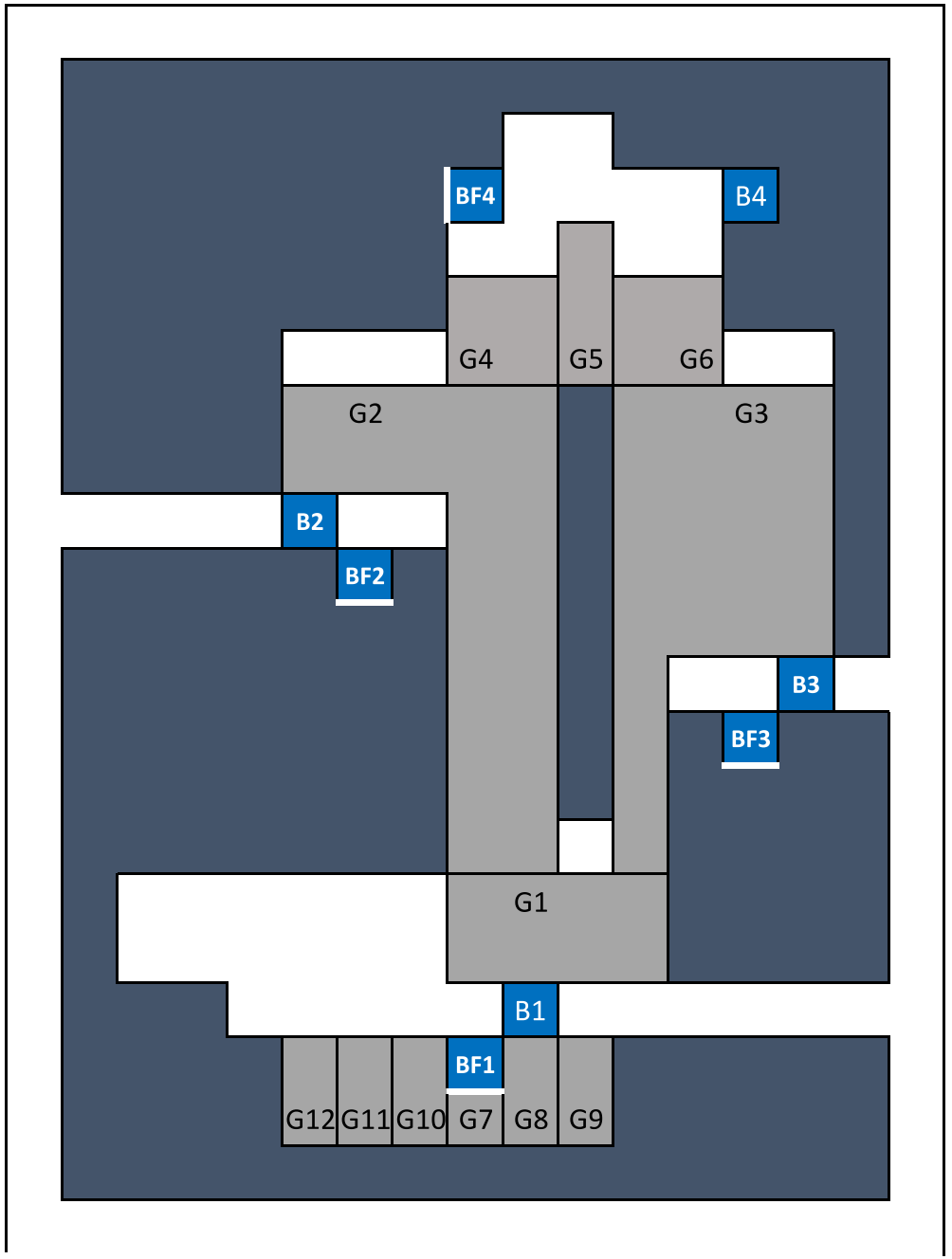
We need **8 OR gadgets** and **24 AND gadgets**.



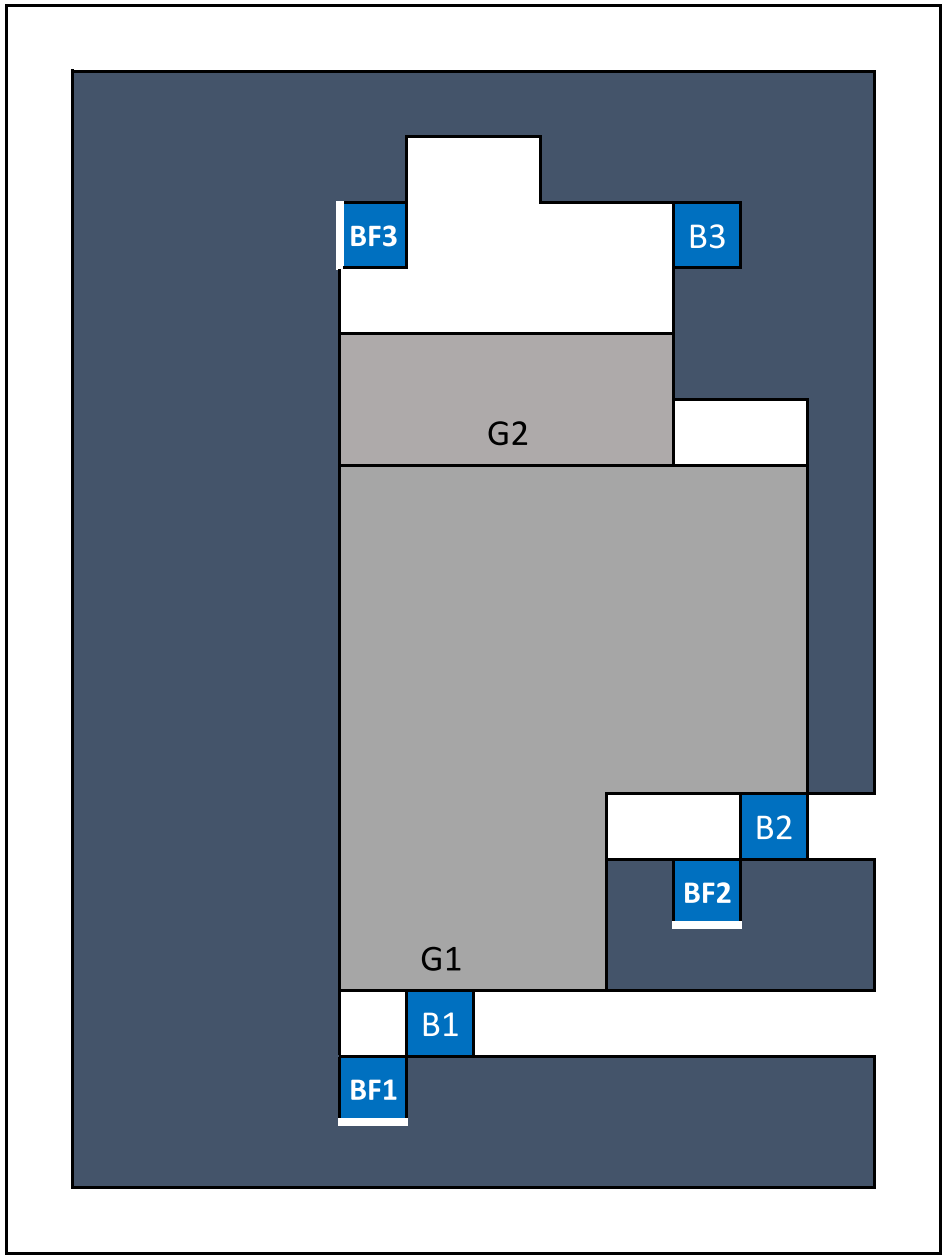
OR Gadget ($B \cdot\cdot| \cdot BB$)



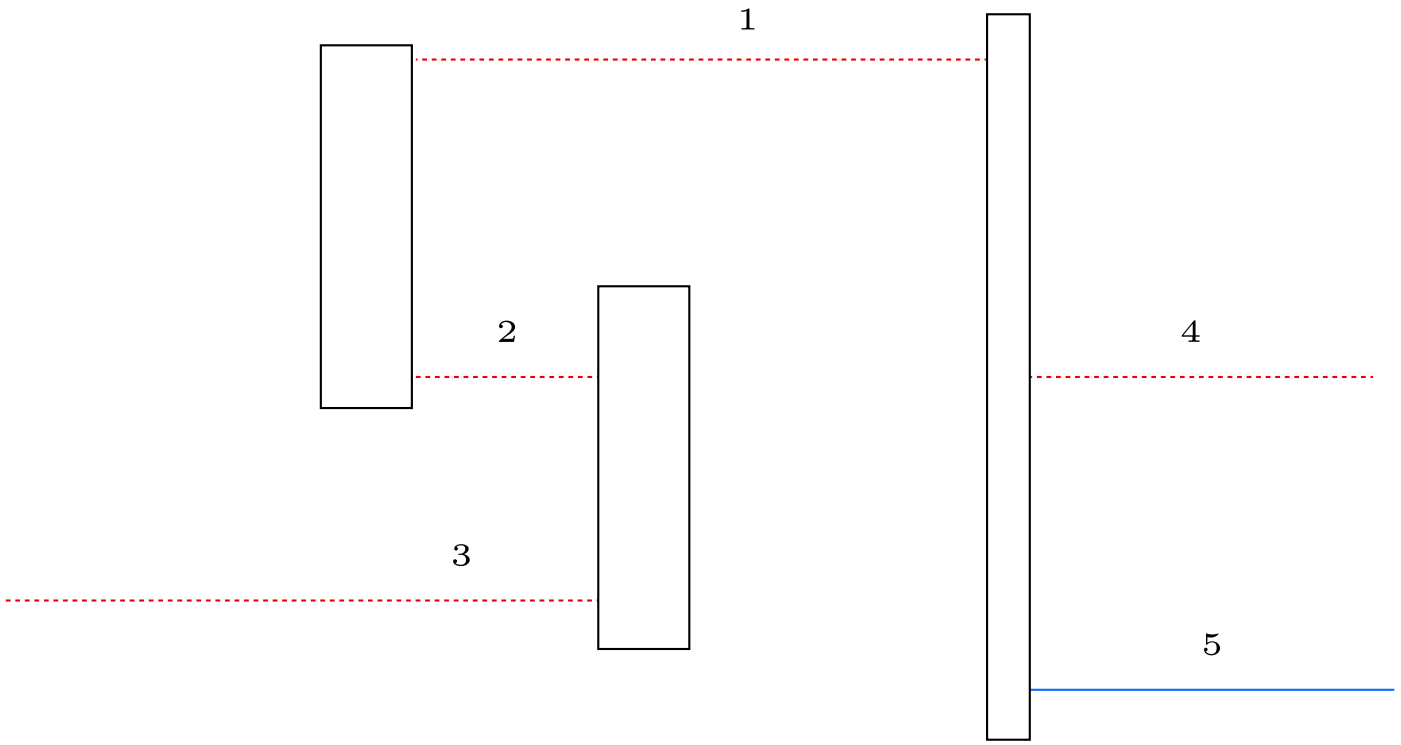
AND Gadget ($R \cdot\cdot|\cdot RB$)



Red Bend Gadget (not needed in Hanano)



Defying Gadget Numerosity: Schemas



Schema of $\cdot R \cdot | R \cdot B$

Some Future Directions

Apply	Reduce	Lower
Apply the technique to other games	Reduce number of gadgets	Lower number of faces on gray blocks


Thank You
Location: Le Morne, Mauritius