

# **NSF ITRG Project Proposal**

Project on Counting Problems  
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Recognizing that many of the most creative endeavors in history have been accomplished by very young people, the Federal Government has decided to offer ITRG graduate awards. These awards are intended to support highly innovative research in information technology primarily conceived by students under the age of 25 that is high risk, but has high potential payoff. Grants of up to \$100K (and possibly more) are available to support investigations into basic information technology or its applications that have the potential to produce revolutionary contributions in areas of national public interest, including (but not limited to) industrial productivity, energy production, transportation, security, defense, law enforcement, housing, urban development, workforce utilization, medicine, health care, education, communication, environment.

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# Chapter 1

## Project Summary

### 1.1 Overview

This project is made with the hope of cracking some well known counting problems. The way to solve this is through solving an easier problem i.e. counting popular matchings for some specific instances. Though the starting problem is not famous enough it has strong relation with some classic counting problems such as counting Max Flows, Counting Stable Matchings, Counting Perfect Matchings, Counting Forests (as a subgraph) of a given graph. So the project will start by solving the starting problem i.e. the popular matching problem. Then it will try to create the relations between this and the classic problems which can lead to both positive and negative results for the classic problems(which is good enough in this area). Above all this project is dealing with some unconventional way to crack some classic problems which is very likely to work and not thought by anybody before.

### 1.2 Broader Impact

All these problems are really wellknown classic and many people are still working on these. Solving these will be a great contribution in these area.

# Chapter 2

## Project Description

A *popular matching problem* instance  $I$  comprises a set  $\mathcal{A}$  of *agents* and a set  $\mathcal{H}$  of *houses*. Each agent  $a$  in  $\mathcal{A}$  ranks the houses in  $\mathcal{H}$  (lower ranks specify higher preference). The ordered list of houses ranked by  $a \in \mathcal{A}$  is called  $a$ 's *preference list*. The preference list of  $a \in \mathcal{A}$  defines a set of edges  $E_a$  from  $a$  to houses in  $\mathcal{H}$ . Define  $E = \cup_{a \in \mathcal{A}} E_a$ . The problem instance  $I$  is then represented by a bipartite graph  $G = (\mathcal{A} \cup \mathcal{H}, E)$ . A *matching* of  $I$  is a matching of the bipartite graph  $G$ . We use  $M(a)$  to denote the house assigned to agent  $a$  in  $M$  and  $M(h)$  to denote the agent that is assigned house  $h$  in  $M$ . An agent *prefers* a matching  $M$  to a matching  $M'$  if (i)  $a$  is matched in  $M$  and unmatched in  $M'$ , or (ii)  $a$  is matched in both  $M$  and  $M'$  but  $a$  prefers the house  $M(a)$  to  $M'(a)$ . Let  $\phi(M, M')$  denote the number of agents that prefer  $M$  to  $M'$ . We say  $M$  is *more popular than*  $M'$  if  $\phi(M, M') > \phi(M', M)$ , and denote it by  $M \succ M'$ . A matching  $M$  is called *popular* if there exists no matching  $M'$  such that  $M' \succ M$ .

The popular matching problem was introduced in [1] as a variation of the stable marriage problem [2]. The idea of popular matching has been studied extensively in various settings in recent times [3, 4, 5, 6, 7, 8, 9], mostly in the context where only one side has preference of the other side but the other side has no preference at all. We will also focus on this setting. Much of the earlier work focuses on finding efficient algorithms to output a popular matching, if one exists.

The problem of counting of many well-studied combinatorial objects fall into the complexity class  $\#P$ . The counting of matchings in graphs has an important history. In their seminal work, Jerrum and Sinclair [10] introduced a Markov Chain Monte Carlo method to approximately count the number of perfect matchings in *dense* bipartite graphs (improved to general bipartite graphs in [11]). Subsequently, approximation schemes were proposed for counting other combinatorial objects such as hamilton cycles [12], independent sets [13], network flows [14] etc (some in limited settings).

An area of interest that has recently gathered a certain amount of attention is the problem of counting stable matchings in graphs. The Gale-Shapely algorithm [2] gives a simple and efficient algorithm to output the stable matching, but counting them was proved to be  $\#P$ -hard in [15]. Bhatnagar, Greenberg and Randall [16] showed that the random walks on the *stable marriage lattice* are slowly mixing, even in very restricted versions of the problem. [17] give further evidence towards the conjecture that there may not exist an FPRAS at all for this problem.

Our motivation for this study is largely due to the similarity of structures between stable matchings and popular matchings. The interest is further fueled by the existence of a linear time algorithm to exactly count the number of popular matchings in the standard setting [5]. We earlier proved some generalizations of the standard version - preferences with ties and houses with capacities - where the counting problem is  $\#P$ -hard. We gave an FPRAS in HAT instance. While the FPRAS for the case of ties is achieved via a reduction to a well known algorithm, the  $\#P$ -hardness for the capacitated case are involving, making it the more interesting setting of the problem. But the FPRAS for CHA instance is not yet known. So our first goal is to solve this problem using standard MCMC technique. Though it is a small problem we are specially interested to solve it because we have seen some strong relations between CHA problem and MAX FLOW problem, generalized result of which is still unknown. So our goal is to see the connection between this two problems explicitly and solve some specialized versions of the core counting problems. Also we have given an alternate proof of counting all matchings in a general graph through counting popular matchings of a particular kind of CHA instance. In which we have done random walks on the set of FORESTS as subgraph of a given graph. So we want to establish the relation between this two structure explicitly. Again this motivates us because the sampling forests is still an open question. And also the strong relations between popular matchings and stable matchings always at the heart of our motivation, which we hope to figure out at the end of this project and be able to produce positive or negative result both of which is same appealing in this field.

# Chapter 3

## Resources

For these project we need 5-6 experienced people in the field of counting algorithms and also they should have very well understanding in Graph Theory and MCMC techniques.

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