Problem 1

Resolution, paramodulation and answer extraction ...

Given:

1. Everyone votes for some candidate.
2. C and D are (distinct) candidates.
3. Every candidate scolds every other candidate.
4. Whoever scolds someone doesn’t vote for him/her.

Question: Who votes for him/herself?

Express the given premises and the appropriate conclusion in ordinary first-order logic, and convert the premises and the denial of the conclusion to clause form. Then use resolution, paramodulation and answer extraction to obtain a Skolem-free answer to the question.

Besides equality, use only the predicates \( V(x, y) \) (\( x \) votes for \( y \)), \( P(x) \) (\( x \) is a [presidential] candidate), and \( S(x, y) \) (\( x \) scolds \( y \)). (We restrict the domain of discourse to people, so there is no need to use a “person” predicate.)

Note A. There is some redundant information here, which could lead you up the garden path. (In real theorem proving, there’s usually a lot more!) So it’s a good idea to sketch out an intuitive proof and try to follow it in the resolution proof.

Note B. A Skolemized answer can be obtained trivially, but the “real” proof requires a number of steps.

Problem 2: Resolution, etc.

This one’s a bit more challenging...

We are given the following premises (with the numbering telling you how many clauses each sentence should eventually lead to):

1. Every animal lover has a dog or a cat.
3., 4. Whoever (or whatever) has a dog or a cat is an animal lover.
5. If one is married to another, then whatever one has, the other has also.
6. Nothing is married to itself.
7. Edith is married to Ike or Jake.
8., 9. Edith is an animal lover, while Ike is not.

Using resolution, factoring and paramodulation, and Cordell Green’s method of answer extraction, derive an answer to the question

Who (or what), other than Edith, is an animal lover?

Note that the qualification “other than Edith” requires (in)equality. Start by representing the premises and the denial of the appropriate conclusion in FOL, then convert these to clause form, find a refutation, and carry an answer literal through the refutation. Use $A(x)$ for “$x$ is an animal lover”, $H(x,y)$ for “$x$ has $y$”, $D(x)$ for “$x$ is a dog”, $C(x)$ for “$x$ is a cat”, $M(x,y)$ for “$x$ is married to $y$”, $E$ for “Edith”, $I$ for “Ike”, and $J$ for “Jake”. (Advice: Figure out how you would informally prove this, note the order in which you’re using the given facts and denial of the conclusion, then structure your proof analogously.)

**Problem 3 – for grads** (Natural deduction in the Kalish & Montague system)

Given that

\[ \forall x (\neg P(x) \Rightarrow Q(x)) \],

use the Kalish & Montague natural deduction system (as adapted by Pelletier) to prove

\[ \exists x (\neg Q(x) \Rightarrow P(x)). \]

Be sure to use only rules stated in the hand-out on Pelletier’s system. For instance, you can’t freely distribute negation, or express $\Rightarrow$ in terms of $\lor$, etc.!

**Problem 4: Soundness and completeness**

(a) Formally prove that the following special case of resolution is sound:

\[
\frac{P(A), \neg P(x) \lor Q(x)}{Q(A)}.
\]

Do not use syntactic manipulations but rather use the definition of soundness, truth in a model (|=M), and satisfaction (|=M...[U]).

(b) Formally prove that the following special case of paramodulation is sound:

\[
\frac{x = A, P(A)}{P(x)}.
\]

(Same guidelines as in (a))
(c) **(for grads)** The B&L text (section 4.3.2) explains what is meant by the *Herbrand universe* of a set of clauses. The text mentions that the Herbrand universe is finite if there are no function symbols and only finitely many constants in the given set of clauses (the latter part will certainly be true if the set of clauses is finite!). Now suppose we consider an arbitrary, finite set of FOL formulas (i.e., not yet in clause form). Suppose this contains no function symbols. In this case, we’re not guaranteed that the Herbrand universe will be finite; (i) Why not? (ii) What assumption about the way quantifiers occur in the given set of FOL formulas will assure a finite Herbrand universe (and hence decidability of satisfiability of the formulas)? (This problem is easy, just requiring *understanding* the concepts at issue.)

**Problem 5: Parsing by prolog**

Let

Everyone(W1,I0,I1), Loves(W2,I1,I2), Mary(W3,I2,I3)

denote that W1 is an occurrence of the word ‘everyone’ at indices I0 to I1; that W2 is an occurrence of the word ‘loves’ at indices I1 to I2; and that W3 is an occurrence of the word ‘Mary’ at indices I2 to I3. (Imagine the indices I1, I2, I3 as lying between the words.) We are abiding by the CSC 244/444 convention here that individual constants are capitalized and variables are lower-case (contrary to common prolog practice).

We also know the following facts. If x is an occurrence of ‘everyone’ from y to z, then x is an NP (noun phrase) from y to z; if x is an occurrence of ‘loves’ from y to z, then x is a V_np (a verb expecting to be followed by an NP, i.e., a transitive verb) from y to z; if x is an occurrence of ‘Mary’ from y to z, then x is an NP from y to z; if x is a V_np from u to v and y is an NP from v to w, then there exists a z that is a VP (verb phrase) from u to w; and if x is an NP from u to v and y is a VP from v to w then there exists a z that is an S (sentence) from u to w.

Prove that there exists a sentence from I0 to I3, by representing the above premises, and the denial of the conclusion, in FOL and then as Prolog clauses (using standard Prolog notation), and then deriving a contradiction (following the Prolog proof strategy — which is *ordered*, among other constraints).

(Note incidentally that we could have added feature information such as that if x is an occurrence of ‘everyone’ from y to z, then x is singular, and similarly for ‘loves’, and then we could have used extra premises and conclusions in the rules to ensure “propagation of

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1 Individual constants are interpreted as themselves, any n-ary function constant f is interpreted as the function that forms the term f(α1, ..., αn) when applied to ground terms α1, ..., αn, and together these terms form the Herbrand universe. Predicates are interpreted as subsets of the “Herbrand base” — which is just the set of ground atoms (predications over ground terms) that can be formed, given the vocabulary of individual, predicate, and function constants. The significance of this ingenious construction lies in the fact that proving completeness of a proof system for FOL need only consider models whose domain consists of the Herbrand universe — an enormous conceptual simplification.
features” – e.g., propagation of the “singular” property from the verb to the verb phrase, and “agreement” – e.g., both the NP and the VP in the sentence rule should be singular, or both should be plural.)