VALIDITY & ENTAILMENT

A model (example)

\[ M = (D, I) \]
\[ \{ a, b \} \]
\[ A^I = a \]
\[ B^I = b \]
\[ P^I = \{ a, b \} \]
\[ Q^I = \{ a \} \]

for FO language
with:
ind. consts: A, B
pred._consts: P, Q
(1-place)
func.consts: none

Truth in a model (examples)

\[ = M \]
\[ \text{"} \psi \text{" is true in } M \]
\[ \text{"} M \text{ is a model of } \psi \text{"} \]

\[ = M (\forall x \ P(x)) \text{ for } M \text{ as above} \]
(proved on board & in handout)

\[ = M (\exists x \ Q(x)) \]

\[ = M (\forall x (Q(x) \Rightarrow P(x))) \]

Validity \( = \psi \)

= truth in all models
= satisfaction by all v.a. of rel. to all models
also called logical truth (independently of "how the world is")

\[ = \psi \text{ iff for all models } M, = M \psi \]

\[ = M \psi [U] \]

Examples

\[ P(c) \lor \neg P(c) \]
\[ (\forall x (x = x)) \]
\[ P(c) \equiv \neg P(c) \]

schemes \( \psi \lor \neg \psi \)
\( (\forall y (y = y)) \)
\( \psi \equiv \neg \neg \psi \)

prove: \[ = (\psi \lor \neg \psi) \]
Unsatisfiability (logical falsehood)

\( \varphi \) unsatisfiable
iff for all models \( M \), \( \not M \varphi \)
iff for all models \( M \), \( \not M \varphi \)
i.e., false in all models
i.e., negation true in all models \( \{ \} \models \varphi \)

\( \varphi \land \varphi ; \varphi \lor \varphi \), \( \varphi \lor \varphi \land \varphi \), etc.

Generalize: \( \Delta \) unsatisfiable iff
for all models \( M \), \( \not M \Delta \) (i.e.,
for some \( \varphi \in \Delta \), \( \not M \varphi \))

Contingent formulas

dependent on "how things are"

\( \varphi \) contingent iff for some model \( M \),
\( \not M \varphi \), and for some model \( M \), \( M \varphi \)

\( P(A) \), \( \text{Loves}(A, B) \), \( \forall x. P(x) \rightarrow Q(x) \), ...
prove contingent

Satisfiability

\( \varphi \) satisfiable iff for some model \( M \),
\( M \varphi \)
i.e., \( \varphi \) is either valid or contingent

Entailment \( \Delta \models \varphi \)

(logical consequence)

\( \Delta \models \varphi \) iff all models of \( \Delta \) are models of \( \varphi \)
i.e., iff for all models \( M \) s.t. \( \not M \Delta \),
we also have \( \not M \varphi \)
i.e., iff for all models \( M \) s.t. for all \( v \in U \),
\( M \Delta [U] \), we also have that for all \( v \in U \),
\( M \varphi [U] \).

\( \forall \varphi \models \varphi \), \( (\varphi, \varphi \rightarrow \psi) \models \psi \),
\( \forall x. P(x) \models P(A) \), \( P(A) \land \lnot P(A) \models \bot \)
true in no models