

RESOLUTION STRATEGIES

- Improve efficiency
- Preserve completeness

Clause elimination

- "pure" literal elimination



- tautology elimination

$P(A) \vee \neg P(A) \vee P(B), P(x) \vee \neg P(x) \vee Q(A, x)$

cannot help in a refutation

eg., we would have to have clauses with literals resolvable against $P(A), \neg P(A) \dots$ but then these clauses can be resolved against each other.

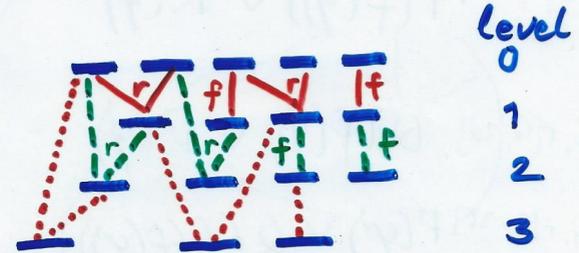
- subsumption elimination

$P(A)$ subsumes $P(A) \vee Q(B)$
 $P(x)$ subsumes $P(y) \vee Q(y)$
 $P(f(x), B) \vee Q(y)$ subsumes $P(f(B), B) \vee Q(A) \vee R(w)$

Proof strategies

- Breadth-first

complete but inefficient

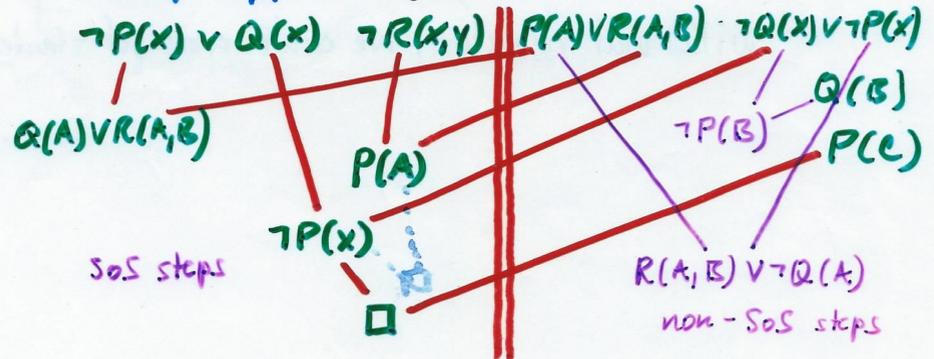


- Depth-first
- efficient but incomplete

- Unit resolution (one or both parents length 1)
- efficient but incomplete $\psi \vee \psi, \neg\psi \vee \neg\psi, \neg\psi \vee \psi, \psi \vee \neg\psi$

- Unit preference (use unit resolution if possible)
- efficient & complete

- Set of support
- denial clauses "guide" proof



- Connection-graph method: (grads)

- Ordered resolution

Resolve initial literals only

Complete for Horn clauses (prolog)
 Colmerauer
 ≤ 1 positive literal

$P(A) :- , \quad :- Q(x), \quad Q(x) :- P(x)$
 $P(A) \Leftarrow \text{True}, \quad \text{False} \in Q(x), \quad Q(x) \Leftarrow P(x)$
 $P(A), \quad \neg Q(x) \quad P(x) \Rightarrow Q(x)$

$\text{Quacks}(x) :- \text{Happy}(x), \text{Duck}(x)$
 $\quad \quad \quad :- \text{Quacks}(\text{Daffy})$ *goal denial*
 $\quad \quad \quad :- \text{Happy}(\text{Daffy}), \text{Duck}(\text{Daffy})$

If we also have $\text{Happy}(\text{Daffy}) :-$ & $\text{Duck}(\text{Daffy}) :-$
 we'll have a refutation & thus a proof of $\text{Quacks}(\text{Daffy})$;

The latex notes contain James Allen's less trivial example,
 Where we show that some bird quacks: $:- \text{Bird}(x), \text{Quacks}(x)$

- Model Generation (elimination)

Efficient! Think of "model" as "all true ground atoms"

Use prolog on Horn subset

If refuted \rightarrow done

If fail \rightarrow expand Horn set using a +ve literal from a non-Horn clause whose antecedent (if any) we can prove.

If we refute all possibilities \rightarrow done

O.W. expand Horn set again, ...

E.g. (BTW, SAT methods (later) can solve this too)

$\neg P, \neg Q, \neg R \vee S; \neg S \vee P \vee Q, R \vee P \vee Q$
 (1) (2) (3) (5) (6)

can't refute

Add R from (6)

can't refute \therefore retain R in model

Add P from (5)

can refute \therefore retract P

Add Q ... can refute ...

can prove antecedent, which is empty (prove antecedent = eliminate -ve literals)

can prove S (eliminate $\neg S$)

Retract R, Q; back to (6), try options P, Q; fail; DONE!