Nonmonotonic Inheritance Networks

Show "normal" relationships among monadic predicates (sets).

\[ \begin{align*}
\text{e.g.,} & \quad \text{Fly} & \quad \text{Mammal} \\
& \quad \text{Bird} & \quad \text{Egg-layer} \\
\text{Birds normally fly} & \quad \text{Egg-layers normally are not mammals} (\text{Most birds fly}) & \quad (\text{Most egg-layers are non-mammals})
\end{align*} \]

For the time being, we don't distinguish such "soft" (defeasible) relationships from strict and (like Chicken \( \rightarrow \) Bird).

Two kinds of conflicts:

1. Fly \( \rightarrow \) Has-wings
2. Mammal \( \rightarrow \) Milk-producer

3. Chicken \( \rightarrow \) Bird
4. Furry-animal \( \rightarrow \) Egg-layer

In (a), one feels that the Chicken \( \rightarrow \) Fly edge should "win" because it's more specific, i.e., chickens don't fly.

In (b), we can't really decide whether a platypus is or is not a mammal (unless we already know).

This intuition can be captured with (what we'll call) the **Triangle Rule**:

A direct edge from P to Q preempts the final edge of any longer path from P to Q; (relative to P)

Casually:

- P
- Q

Preempted edge (relative to P)

**Reasoning**

What can we infer from a given root node P, by tracing paths upward?

Well, it depends on whether we're "credulous" or "skeptical".

**Definition (informal):**

A proper extension of a node P in a NMI network (a directed, usually acyclic graph with positive & negative edges) is obtained by adding edges "bottom up" starting at P, subject to:

(a) never adding preemted edges (relative to a node already added); and

(b) never adding both a positive edge and a negative edge entering the same node; and

(c) never adding an edge from a node that has negative entering edges only.
E.g., the proper extensions of the previous example networks, relative to "chicken" and "platypus" are:

Note: "Bottom-up" edge addition assumes that as many ancestors of a node \( Q \) as possible have been added before any edges to \( Q \) are added.*

Credulous reasoning (given a node \( P \) in a network)

Pick any proper extension of \( P \), & "believe" all conclusions reached from it.

Ideally skeptical reasoning

Believe the conclusions reached from \( P \) in all proper extensions.

E.g. All nodes in (a) above are believed, and only "furry-animal" & "egg-layer" are believed in (b), as conclusions from "platypus".

* Also, we add all successor edges of a node that are not ruled out by (a), (b), or (c) (in the definition of proper extension) before adding any edges from any of the successors of the node.
**Things can get a bit tricky...**

Caveat: The B & L text doesn't use "proper" extensions, & defines "credulous" extensions differently.

In (e), in forming a proper extension from \(X\), we first add \(Y, Z, U, V\). Then we can add either \(Y \rightarrow W\) (but not \(U \rightarrow W\)), supporting \(\neg W\), or we can add \(Z \rightarrow W\) (but not \(V \rightarrow W\)), supporting \(W\). So we have 2 credulous extensions, but only \(Y, Z, U, V\) are ideally skeptical conclusions.

In (d), it looks as if \(Z \rightarrow V\) preempts \(U \rightarrow V\); however, \(Z \rightarrow V\) is itself preempted by \(Y \rightarrow V\). But it doesn't matter, because \(Y \rightarrow V\) also preempts \(U \rightarrow V\). So the unique proper extension is

\[ Y \rightarrow Z \rightarrow U \rightarrow V \rightarrow X \]

So in this case the credulous & ideally skeptical conclusions are the same: \(Y, Z, U, V\) are all believed as consequences of \(X\).

**A desirable distinction one can make (not in B&L):**

Soft (defeasible) vs. strict (strong, hard) edges.

- \(P \rightarrow Q\) and \(Q \rightarrow P\) (subset)
- \(P \rightarrow Q\) and \(Q \rightarrow P\) (not subset)
- \(P \rightarrow Q\) and \(Q \rightarrow P\) (cyclic)

Therefore, could write as \(P \leftrightarrow Q\).

In fact, allowing cycles seems potentially useful for cases like

- \(P \leftrightarrow Q\)
- \(P \leftrightarrow Q\)
- \(P \leftrightarrow Q\)

We can then generalize the Triangle rule:

A strict path (of type \(P \rightarrow P_2 \rightarrow P_3 \rightarrow \cdots \rightarrow P_n \rightarrow Q\), or of type \(P \rightarrow P_2 \rightarrow P_3 \rightarrow \cdots \rightarrow P_n \leftrightarrow Q\)) preempts the last edge of any other path of length \(\geq 2\) from \(P\) to \(Q\).

**Final comments:** It would be better if we could take account of probabilities (if known); and if we allowed for multiple-promise rules, n-ary pred's, etc!