

DEFAULT LOGIC

Default Logic is based on inference rules

of form
$$\frac{\alpha(x) : \beta(x)}{\gamma(x)}$$

premise (or prerequisite) $\alpha(x) : \beta(x)$ consistency check (justification) $\beta(x)$
conclusion $\gamma(x)$

E.g., (1)
$$\frac{\text{Bird}(x) : \text{Flies}(x)}{\text{Flies}(x)}$$
 Assumes "If it didn't fly I would know it"!

"Birds generally fly"

"If x is a bird, and it is consistent to assume that x flies, conclude x flies"

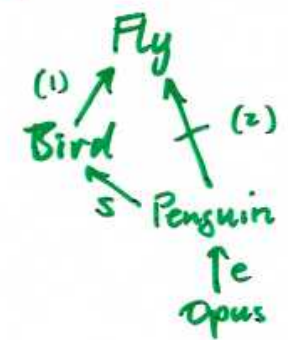
Normal defaults:
$$\frac{\alpha(x) : \beta(x)}{\beta(x)}$$

E.g., add another rule to (1):

(2)
$$\frac{\text{Penguin}(x) : \neg \text{Flies}(x)}{\neg \text{Flies}(x)}$$

If we also have $\forall x. \text{Penguin}(x) \Rightarrow \text{Bird}(x)$ and $\text{Penguin}(\text{Opus})$, what follows?

cf., default inheritance hierarchies:



see B&L ch. 10

Two inference paths:

$\text{Penguin}(\text{Opus}) \vdash_{\text{FOL}} \text{Bird}(\text{Opus}) \xrightarrow{\text{def. (1)}} \text{Fly}(\text{Opus})$
 $\text{Penguin}(\text{Opus}) \xrightarrow{\text{def. (2)}} \neg \text{Fly}(\text{Opus})$

Note: Once we've applied one of (1), (2), we can no longer apply the other - the consistency test fails!

Compare with CWA, which

- also licenses certain conclusions, based on consistency... but disregards new conclusions that may already have been added
- no premises (less "cautious"), only -ve conclusions

We can approximate the effect of CWA with default rules of form

$$\frac{\neg P(x)}{P(x)} \quad , \quad \frac{\neg Q(x)}{Q(x)} \quad \text{Note: no premise (or premise = T)}$$

E.g., facts $\{P(A) \vee Q(A)\}$, defaults as above

With CWA, we got an inconsistent augmentation (both $\neg P(A)$ & $\neg P(Q)$ are added).

With default logic, once we've added $\neg P(A)$, we can't apply the 2nd default rule, and similarly if we use the 2nd rule first.

"Extensions" of a default theory

Given a db of facts Δ & default rules D , an extension $E[\Delta; D]$ of the default theory is a smallest set s.t.

- $\Delta \subseteq E[\Delta; D]$
- All logical consequences of $E[\Delta; D]$ are in $E[\Delta; D]$
- If D contains a rule of form $\frac{\alpha(x) : \beta(x)}{\gamma(x)}$ & $\alpha(\tau) \in E[\Delta; D]$ & $\neg\beta(\tau) \notin E[\Delta; D]$ then $\gamma(\tau) \in E[\Delta; D]$

Inconsistent extensions - can occur for non-normal default theories.

E.g., $\{ \}$, $\left\{ \frac{P(A)}{\neg P(A)}, \frac{\neg P(A)}{P(A)} \right\}$
facts

Multiple extensions

Normal default theories always have extensions, & these are always consistent (if Δ is!) (see theorem p. 154). G&N (not mentioned in B&L)

however, they can still have multiple extensions.

E.g., earlier example of Opus the penguin: one extension contains $\text{Fly}(\text{Opus})$, the other $\neg \text{Fly}(\text{Opus})$.

We can modify the rules to get a unique extension:

(1') $\frac{\text{Bird}(x) : \neg \text{Penguin}(x) \wedge \text{Flies}(x)}{\text{Flies}(x)}$ "semi-normal"

or use (1'') $\frac{\text{Bird}(x) \wedge \neg \text{Penguin}(x) : \text{Flies}(x)}{\text{Flies}(x)}$ normal, but requires more knowledge

(2) as before $\frac{\text{Penguin}(x) : \neg \text{Flies}(x)}{\neg \text{Flies}(x)}$

Given $\text{Penguin}(\text{Opus})$, we can't apply (1') or (1'').

Applying (2) then leads to $\neg \text{Flies}(\text{Opus})$.

$E[\Delta; \delta]$ is then the deductive closure of
 $\{ \text{Penguin}(\text{Opus}), \forall x. \text{Penguin}(x) \Rightarrow \text{Bird}(x), \neg \text{Flies}(\text{Opus}) \}$,

i.e., $\{ \text{Penguin}(\text{Opus}), \forall x. \text{Penguin}(x) \Rightarrow \text{Bird}(x), \text{Bird}(\text{Opus}), \neg \text{Flies}(\text{Opus}), \forall x. \text{Bird}(x) \Rightarrow \text{Bird}(x), \text{Opus} = \text{Opus}, \dots \}$

Semimonotonicity:

Adding new normal defaults to a normal default theory requires no withdrawal of implicit beliefs. (p 154 G&N)

However, the multiple extension problem leads to some peculiar results...

E.g.: the Invitation Problem (Kyburg)

"People generally live where their spouses live":

$$\frac{\text{Live-at}(\text{spouse}(x), y) : \text{Live-at}(x, y)}{\text{Live-at}(x, y)}$$

"People generally live where they work":

$$\frac{\text{Work-at}(x, y) : \text{Live-at}(x, y)}{\text{Live-at}(x, y)}$$

Facts: $\text{spouse}(\text{John}) = \text{Mary}, \text{Live-at}(\text{Mary}, \text{Buffalo}),$
 $\text{Work-at}(\text{John}, \text{Syracuse}),$
 $\text{Live-at}(x, y) \wedge \text{Live-at}(x, z) \Rightarrow y = z$

Also assume unique names.

We can derive either $\text{Live-at}(\text{John}, \text{Syracuse})$ or $\text{Live-at}(\text{John}, \text{Buffalo})$, but (with UNA) not both. This is unhelpful if John has invited us and we're trying to decide whether to head E or W from Rochester!

comment on credulous & skeptical reasoning

E.g. The Lottery Paradox (Kyburg)

1,000,000 tickets have been sold. Generally, a given ticket won't win:

$$\frac{\text{Ticket}(x) : \neg \text{Win}(x)}{\neg \text{Win}(x)}$$

But some ticket will win:

$$(\exists x) \text{Ticket}(x) \wedge \text{Win}(x) \wedge [x = T_1 \vee \dots \vee x = T_{1,000,000}];$$

$$\text{Ticket}(T_1), \dots, \text{Ticket}(T_{1,000,000}).$$

We can "argue" that $T_{1,000,000}$ will win:

Apply default rule to $T_1, T_2, \dots, T_{999,999}$, obtaining $\neg \text{Win}(T_1), \neg \text{Win}(T_2), \dots, \neg \text{Win}(T_{999,999})$. Hence (by the existential assertion), $\text{Win}(T_{1,000,000})!$

Similarly we can argue any of the T_i wins, i.e., there are 1,000,000 extensions.

(One approach to such problems: allow some arguments to defeat others (e.g., Loui, Pollock))

E.g., Dropouts & Employment - unwanted rule chaining

- (1) $\frac{\text{Dropout}(x) : \text{Adult}(x)}{\text{Adult}(x)}$ "Most dropouts are adult"
- (2) $\frac{\text{Adult}(x) : \text{Employed}(x)}{\text{Employed}(x)}$ "Most adults are employed"

Given that $\text{Dropout}(\text{Joe})$, we conclude $\text{Adult}(\text{Joe})$ & hence $\text{Employed}(\text{Joe})$.

Again, we could use more complex versions of (2):

$$(2') \frac{\text{Adult}(x) : \neg \text{Dropout}(x) \wedge \text{Employed}(x)}{\text{Employed}(x)} \quad \text{semi-normal}$$

or

$$(2'') \frac{\text{Adult}(x) \wedge \neg \text{Dropout}(x) : \text{Employed}(x)}{\text{Employed}(x)} \quad \text{requires more knowledge}$$

plus

$$(3) \frac{\text{Adult}(x) : \neg \text{Dropout}(x)}{\neg \text{Dropout}(x)} \quad \text{or more rules}$$

$$\text{Adult}(\text{Mary}) \stackrel{(3)}{\vdash} \neg \text{Dropout}(\text{Mary}) \stackrel{(2'')}{\vdash} \text{Employed}(\text{Mary})$$

Other criticisms of default logic :

- as for CWA, drawing default conclusions requires a consistency check, which in general is undecidable.
- default rules enable us to reason with default knowledge, but not about it.

E.g., can't argue as follows:

Quakers are generally pacifists,
 pacifists are ^{always} generally Democrats

\therefore Quakers are ^{generally} often Democrats

Another approach, "conditional logics", have the opposite problem (defaults are interpretable as facts, but not properly usable as rules)

Probabilistic inference methods can be used to provide a semantic basis for some kinds of default reasoning