

CSC2/455 Software Analysis and Improvement

Program Analysis/Abstract Interpretation

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URCS

Outline

And now for something (not so completely) different

Program Analysis in Industry

Basic Notions

Illustrating Abstract Interpretation

Postscript

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So far

- Data flow analysis
 - Iterative data flow analysis
 - Region-based analysis
- Type Checking
- What next?

Compilers are not the only program analyzers

- Compilers are probably the most used program analyzers
- But are severely time constrained
- Finding *program* errors is not primary goal
 - Syntax errors, type errors
 - Code generation primary goal

Program/Software Analysis

- Software is increasingly mission-critical
- Can kill people!
 - Boeing 737 MAX(?)
 - Therac-25 (X-ray)
 - Industrial Robotics
- (less extreme?) Can lose money
 - Software crashes
 - Data loss
- Can we analyze programs for *functional* correctness?
 - Topic of the next few lectures

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SLAM (Microsoft, early 2000s)

- MS isolated most crashes to buggy drivers
- Static Driver Verifier project
 - Would verify driver code (in C) for correctness
- Used *model checking*
 - Models programs as finite-state machines
 - I used a similar tool (CBMC) to check your assignments



Infer (Facebook)

- Checks C, C++, Objective C, Java and Android code
- Used for checking Facebook's mobile apps
- Open source, <https://fbinfer.com/>
 - Used by Amazon, Mozilla, Uber and Facebook and its affiliates, JD.com, etc.
- Comes with its own language AL (OCaml-derivative?) to describe analyses
 - Analyzes programs in SIL ("Smallfoot Intermediate Language")
- Uses abstract interpretation + *separation logic*
 - Abstract interpretation very similar to data flow analysis frameworks
- CACM Article: Scaling Static Analyses at Facebook
- Good video: Getting the most out of static analyzers

SPARTA (Facebook)

- Language-independent analyzer
 - a C++ framework
- Open source,
<https://code.fb.com/open-source/sparta/>
- Used in FB's RedEx tools
 - for analyzing Android binary code (.dex)
- Also uses *abstract interpretation*



Other efforts

- Stanford Checker
 - commercialized by Coverity, late 2000s
 - CACM article, “A few billion lines of code later: using static analysis to find bugs in the real world”
- Google’s static analysis tools
 - Checker Framework for Java programs
 - Shipshape (abandoned?) (Google Tricorder)
 - CACM article, “Lessons from Building Static Analysis Tools At Google”
- Oracle’s Soufflé
 - Soufflé: Logic Defined Static Analysis

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Limitations

- None of these frameworks and tools can escape the fact that analysis is an undecidable problem
- All compute approximations
 - Or risk ending up intractable
- Must be designed to be *sound*
 - Approximations are conservative/safe
- Leads to imprecision (i.e. *incomplete*)
 - May model behaviour not in original programs
 - (recall IDEAL vs MOP vs MFP)
 - leads to false positives

States and Transitions

- A program's state is a mapping of variables to values
- Programs move from one state to another
 - begin execution in subset of (initial) states
- Notions of state *before* a program point (i.e. a statement) and *after* a program point
 - Also sometimes known as *pre-condition* and *post-condition* respectively.
- Relation that maps before-states to after-states is called a *transition* relation (t)
 - $\langle x, y \rangle$ (x is before-state, y is after-state)

Traces

- An execution trace of a program is a sequence of states
 - $s_0 s_1 s_2 \dots s_n$
- An execution trace may be finite or infinite
 - $s_0 s_1 s_2 \dots$
- The collection of partial traces that can actually happen (i.e. state transitions obey the transition relation) is called the *collecting semantics*
 - I.e. for all $s_i s_j$ in trace, $\langle s_i, s_j \rangle \in t$

Example

```
x = 0
while(x < 100)
  x = x + 1;
```

- states are \mathbb{Z}
- initial state is 0
- transition relation is $\{\langle x, x' \rangle \mid x < 100 \wedge x' = x + 1\}$
- is 0 1 2 3 part of the collecting semantics?
- is 0 2 4 6 part of the collecting semantics?

What can we do with the concrete semantics?

- By examining the (concrete) collecting semantics, we can check various “properties”
 - We’ll formalize “property” later.
- Problems with using the concrete semantics:
 - The previous example had a single state in initial set $\{0\}$, this is not always true. Consider `x = randint()`
 - How do we deal with infinite loops?
 - How do we deal with alternate paths (i.e. conditionals)?
 - How do we get the concrete semantics for all programs statically?
- Implementation issues:
 - Even if we could get a concrete semantics, how large would it be?

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A Graphical Programming Language¹

Consider a simple language with the following constructs:

- `init(R)` where R is a region in 2D space
 - e.g. `init({(x,y)|(x,y) ∈ R})`
 - This chooses a single point in R non-deterministically
 - A program must always start with `init`
- `translate(dx, dy)` moves the point by dx in the X-direction, and dy in the Y-direction
 - e.g. `translate(1.0, 0.5)` moves the point to the right and up in the Cartesian plane
- `rotate(angle)` rotates the point by $angle$ about the origin
 - e.g. `rotate(90)` will move a point on the X-axis to a point on the Y-axis

¹This exposition is based on Chapter 2 of Rival and Yi (see Postscript)

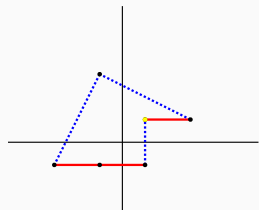
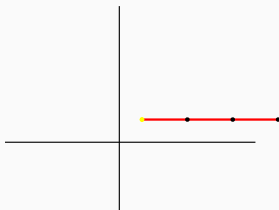
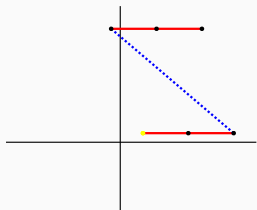
Grammar

```
P =  init(R) ';' Rest
Rest = translate(dx, dy)
      | rotate (a)
      | '{' Rest '}' or '{' Rest '}'
      | iter '{' Rest '}'
      | Rest ';' Rest
      | Empty
Empty =
```

- A program begins with `init` and is followed by statements in *Rest*
- The `or` construct is a non-deterministic choice
 - It executes the block (delimited by braces) on the right or the block on the left
 - Simulates a conditional
- The `iter` either executes the code inside the block or moves to the next statement
 - Simulates a loop
 - Note, `iter` can execute the block forever!

An Example Program

```
init([0, 1]x[0, 1]);  
translate(1, 0);  
iter {  
  {  
    translate(1, 0);  
  } or {  
    rotate(90);  
  }  
}
```



Property we're interested in

Can x ever become negative?

- Can be represented by the set $x_{neg} = \{(x, y) | x < 0\}$
- Problems with concrete execution approach:
 - Infinite initial states ($R = \{(x, y) | 0 \leq x \leq 1 \wedge 0 \leq y \leq 1\}$)
 - Conditionals that perform translation or rotation
 - Loop may be infinite
- But if we could obtain the set of states in all possible concrete executions, say x_{conc} , we need to show
 - $x_{conc} \cap x_{neg} = \emptyset$

Abstract Execution: Approximating Concrete Executions

- Abstract interpretation is a **framework** for performing program analysis
- Key components:
 - Abstract domain: Set of properties we're interested in
 - Abstraction function: Converts a concrete state to an element of the abstract domain
 - Transfer functions: Transforms an abstract state before a statement to an abstract state after the statement
 - Union/Join: Combines abstract states from alternate paths
 - Widen (∇): Combines abstract state across loop iterations

Abstract Domains

- For the property we're interested in, we only need the sign of x
- Potential abstract domain, *signs-x*
 - Only tracks $x < 0$, $x \in R$
 - Can only answer questions about this property, and about x
- Another abstract domain, track signs of all state variables
 - Tracks $\{x < 0, x \geq 0, x \in R\} \times \{y < 0, y \geq 0, y \in R\}$
- Abstract domain for the rest of the lecture: Intervals
 - For each state variable, track l_v and h_v such that $l_v \leq v \leq h_v$
 - For the graphical language, $l_x \leq x \leq h_x$, $l_y \leq y \leq h_y$
 - Thus, our abstraction approximates states using rectangles
 - The sides of the rectangle are parallel to the axes
 - l_v and h_v can be $-\infty$ and ∞ respectively to represent unbounded "rectangles"

Abstract Execution Using Intervals: `init`

```
init([0, 1]x[0, 1])
```

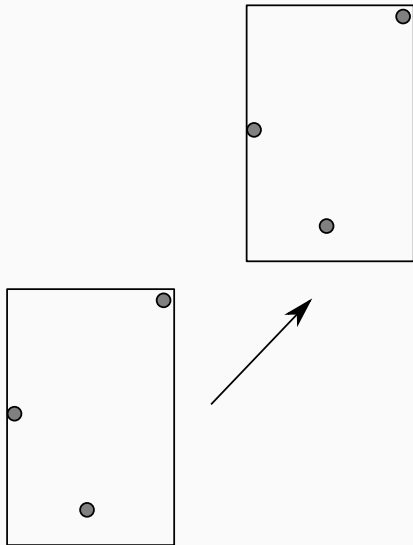
Concrete execution will give us a point in that rectangular region.

- Our abstract state after `init` will be:
 - $l_x = 0$
 - $h_x = 1$
 - $l_y = 0$
 - $h_y = 1$
- The transfer function for `init` computes the rectangle that covers the region
- In this case, the abstraction is precise

Abstract Execution Using Intervals: `translate`

`translate(1.0, 0.5)`

- Our abstract state after `translate` will be:
 - $l_x = l_x + 1.0 = 0.5$
 - $h_x = h_x + 1.0 = 2$
 - $l_y = l_y + 0.5 = 0.5$
 - $h_y = h_y + 0.5 = 1.5$
- The transfer function for `translate` shifts the current abstract state
 - The resulting abstract state is still precise



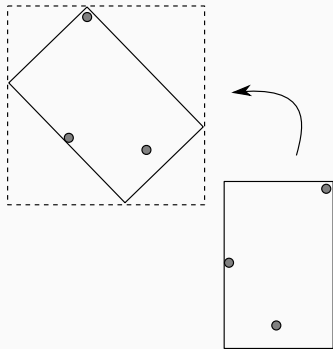
Abstract Execution Using Intervals: rotate

rotate(45)

- Our rectangle after rotate will be:
 - $l_{x_1}, l_{y_1} = rotate((l_x, l_y), 45)$
 - $h_{x_1}, l_{y_2} = rotate((h_x, l_y), 45)$
 - $l_{x_2}, h_{y_1} = rotate((l_x, h_y), 45)$
 - $h_{x_2}, h_{y_2} = rotate((h_x, h_y), 45)$
- The transfer function for rotate rotates the corners of the rectangle
 - The result is still a rectangle, but cannot always be represented using intervals
 - So it is not an abstract state

Finding a new interval in the abstract domain

- Let c' be a co-ordinate after rotate computed as in the previous slide, then
 - $l_x = \min(l_{x_1}, l_{x_2}, h_{x_1}, h_{x_2})$
 - $h_x = \max(l_{x_1}, l_{x_2}, h_{x_1}, h_{x_2})$
 - $l_y = \min(l_{y_1}, l_{y_2}, h_{y_1}, h_{y_2})$
 - $h_y = \max(l_{y_1}, l_{y_2}, h_{y_1}, h_{y_2})$
- Obviously, in general, this new interval contains more states than the rotated rectangle
 - We have lost precision
- But, the new interval/rectangle we have calculated is the “best fit”



A note on domains

- Intervals (and signs) are non-relational domains
 - They can't capture relations between x and y
 - e.g., a property that $x > y$
- Intervals also can't capture complex regions
- A more complicated abstract domain: *convex polyhedra*
 - A list of linear inequalities
 - Region is the feasible region (i.e. points that satisfy all the inequalities)
 - Convex polyhedra support relational properties, e.g. $x - y < 2$
- During program analysis you must choose the domain that most efficiently captures the property of interest

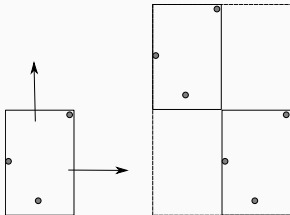
Handling Compound Statements

- Let the transfer function be called `analysis(stmt, pre-condition)`
- So far, we've defined:
 - `analysis(init, I)` (where I is the state on entry)
 - `analysis(translate, a)` where a is the state before `translate`
 - `analysis(rotate, a)` where a is the state before `rotate`
- Let's now define `analysis(p1 ; p2, a)`
 - The analysis of two statements $p1$ and $p2$, with pre-condition a
 - Note that pre-condition for $p2$ is the post-condition of $p1$, so
 - `analysis(p1 ; p2, a)` is then `analysis(p2, analysis(p1, a))`

Handling conditionals: Union (Join)

`{translate(1.0, 0)}` or `{translate(0, 1.0)}`

- Program could execute either left part or right part
- Two possible abstract states, disjoint
- Resulting abstract state must incorporate both
 - *Union* the two abstract states
- `analysis(p1 or p2, a)` is:
 - `analysis(p1, a) ∪ analysis(p2, a)`
- The resulting abstract state is also, in general, imprecise



Union for Intervals

- For $a_1 \cup a_2$
 - $l_x = \min(l_{x_1}, l_{x_2})$
 - $h_x = \max(h_{x_1}, h_{x_2})$
 - $l_y = \min(l_{y_1}, l_{y_2})$
 - $h_y = \max(h_{y_1}, h_{y_2})$

Handling Loops

```
iter {b}
```

can be written as:

```
{ } # loop executes 0 times  
{ } or {b} # loop executes 1 time  
{ } or {b} or {b;b} # loop executes 2 times  
{ } or {b} or {b;b} or {b;b;b} # loop executes 3 times  
...
```

This can be written (recursively) as: $p_{k+1} = p_k \text{OR} \{p_k; b\}$, where:

- p_0 is $\{\}$
- p_1 is $\{\}$ or $\{b\}$
- p_2 is $\{\}$ or $\{b\}$ or $\{b; b\}$ and so on...

Iterating

```
analysis(iter {b}, a)
```

can be defined as an iterative algorithm:

```
R = a
do
  T = R
  R = union(R, analysis(b, R)) # analysis for or
while R != T
```

Will this always terminate?

Example

```
init([0, 1] x [0, 1])  
translate(1, 0)  
iter { translate(0, 1) }
```

- The analysis of this code will not terminate!
- Abstract state before `iter`
 - $1 \leq x \leq 2, 0 \leq y \leq 1$
- After first union:
 - $1 \leq x \leq 2, 0 \leq y \leq 1 \cup 1 \leq x \leq 2, 1 \leq y \leq 2$
 - Result: $1 \leq x \leq 2, 0 \leq y \leq 2$
- After second union: $1 \leq x \leq 2, 0 \leq y \leq 3$
- And so on ...
 - the h_y bound keeps increasing without bound

The Widen operator

- We note that the interval $0 \leq y \leq \infty$ would overapproximate $0 \leq y \leq n$ where n is not ∞
- If we obtained this interval in our abstract state, we could terminate because h_y would “stop increasing”
 - $0 \leq y \leq \infty$ already includes all possible abstract states of the form $0 \leq y \leq n$
 - Union would no longer return a different result ensuring termination
- So widen (∇) is an operator that overapproximates unions
 - Its primary purpose is to ensure convergence

Analyzing loops using widen

```
R = a
do
  T = R
  R = widen(R, analysis(b, R))
until inclusion(R, T) # i.e. R is included in T
return T
```

Widen for Intervals

```
def widen(a, b):  
    out = union(a, b)  
  
    if a.lx != b.lx:  
        out.lx = -inf  
  
    if a.hx != b.hx:  
        out.hx = inf  
  
    if a.ly != b.ly:  
        out.ly = -inf  
  
    if a.hy != b.hy:  
        out.hy = inf
```

Putting it all together

- Each time analysis is applied, we obtain an abstract state a
- If a overlaps with x_{neg} , then we have detected a violation of our property

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References

- This lecture follows the exposition in Chapter 2 of the book “An Introduction to Static Analysis: An Abstract Interpretation Perspective” by Xavier Rival and Kwangkeun Yi, MIT Press, 2020
 - The library has print copies
 - Suggest buying this book – it is self-contained and reasonably priced
- Prof. Cousot has a number of tutorials, I’ll post links to them in the next lecture
 - Prof. Patrick Cousot and Prof. Radhia Cousot invented abstract interpretation