CSC2/455 Software Analysis and Improvement
Dominator and SSA Form - II

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URCS

February 11, 2019
Outline

Review

Dominator Trees and Frontiers

Minimal SSA Form

Emitting code for SSA form

Postscript
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Static Single Assignment Form

- Simple algorithm to generate SSA form
  - Introduce $\phi$ functions
  - Rename variables using Reaching Definitions
- Algorithm can generate excessive $\phi$ functions
  - TODAY: Use dominance frontiers to place the minimal number of $\phi$ functions
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Recall: Dominators

- A node $n$ in the CFG dominates a node $m$ iff:
  - $n$ is on all paths from entry to $m$
  - by definition, a node $n$ always dominates itself
  - if $n \neq m$, then $n$ strictly dominates $m$
- Computed using a dataflow-style analysis
  - Each node annotated with a set of its dominators
The dominance frontier of a node $n$ ($\text{DF}(n)$) is a set of nodes $m \in \text{DF}(n)$ iff:

- $n$ does not strictly dominate $m$
- $n$ dominates $q$ where $q \in \text{pred}(m)$

Note that dominance frontiers only contain *join* nodes

I.e. nodes with multiple predecessors

Computing the dominance frontier of each node:

- Iterative Data-flow analysis?
Direct calculation of dominance frontiers using *dominator trees*.
Immediate Dominators

- The *immediate* dominator of a node \( m \) (IDOM(\( m \))) is the node \( n \):
  - such that \( n \) strictly dominates \( m \), and
  - \( n \) does not strictly dominate \( o \) where \( o \in (\text{DOM}(m) - \{m\}) \)
  - in some sense, \( n \) is the “closest” dominator in the CFG to \( m \).

- By definition, ENTRY has no immediate dominator.
Dominator Trees

- Note that each node in the CFG can have only one immediate dominator
  - Can you see why?
- Create a graph $G = (V, E)$, where:
  - $V$ is the set of basic blocks
  - There is an edge $(n, m)$ in $E$ if $n$ is the immediate dominator of $m$ (i.e. $\text{IDOM}(m) = n$)
Example: CFG and its dominator tree
Which node is the immediate dominator of B1?
Computing the dominance frontier

- Find all join nodes in CFG, e.g. $j$
- For all nodes $n$ that dominate predecessors of $j$
  - If $n$ does not strictly dominate $j$, add $j$ to $DF(n)$
- This last step can be operationalized as:
  - Start from a predecessor $p$ in the dominator tree
  - Add $m$ to $DF(p)$
  - Move up the tree and repeat until you reach $IDOM(j)$
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Example: Non-redundant $\phi$ functions

ENTRY

$y_0 = x_0 + 1$
$x_1 = 2$

EXIT

$y_1 = \phi(y_0, y_4)$
$y_1 > 3$

$y_2 = 3$
$a = 3$

$y_3 = \phi(y_1, y_2)$
$y_4 = x_1 + y_3 + 2$
Placing $\phi$ functions

For each definition $d$ in basic block $n$:

Place a $\phi$ function for $d$ in all nodes $m$ where $m \in DF(n)$

Note that each $\phi$ function is also a definition!

Repeat, until no more $\phi$ functions need to be inserted

This is the minimal number of $\phi$ functions for a definition $d$

Can we reduce the overall number of $\phi$ functions?

(Figure 9.9 in Cooper and Turczon)
Other optimizations

- **Dead definitions**
  - Definitions that are not read (i.e. overwritten) do not need $\phi$ functions

- **Two forms:**
  - *Semi-pruned* SSA form, using “globals” names (those variables that are live in to a block)
  - *Pruned* SSA form, using \texttt{LIVEOUT} information
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Renaming variables

- SSA form introduced “subscripts” for each variable
- Should we drop them when generating code?

\[
\begin{align*}
a_0 &= x_0 + y_0 \\
b_0 &= a_0 \\
a_1 &= 17 \\
c_0 &= a_0
\end{align*}
\]
Problem with dropping subscripts

\[
a = x + y \\
b = a \\
a = 17 \\
c = a \quad \# \text{ WRONG!}
\]
Handling subscripts

- Each definition becomes a new variable
  - I.e. Do NOT drop subscripts
- Preserves data dependences
  - Esp. important when we aggressively move code from basic blocks (e.g. very busy expressions, loop invariant code motion, etc.)
Code for $\phi$ functions

- Introduce copies along each incoming edge to a join node

```plaintext
i_2 = 1
i_3 = a + b
i_4 = \phi(i_2, i_3) 
...
```

```plaintext

```

```
Inserting appropriate copies along incoming edges

\[
\begin{align*}
i_2 &= 1 \\
i_4 &= i_2 \\
i_3 &= a + b \\
i_4 &= i_3 \\
\ldots
\end{align*}
\]
Critical edges

- Executing $\phi$ functions by inserting copies into predecessor blocks is not always correct
- If the predecessor of a block contain $\phi$ functions has multiple successors, then the $\phi$ function may execute when it shouldn’t
  - This may be harmless, but not always
- Edges connecting such predecessors to the block contain the $\phi$ function are called critical edges
- Such edges need to be split by inserting a block on that edge
- See the discussion in Cooper and Turczon for more details and an example
More complications

- Excessive copies
  - Copy propagation into $\phi$ functions
  - Note args in resulting $x_1 = \phi(x_0, y_1)$ $\phi$ functions are for different variables
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- Chapter 9 of Cooper and Turczon
  - Section 9.2.1
  - Section 9.3