Outline

Review

Dominance Frontiers and Dominator Trees

Emitting code for SSA form

The SSA Form and Functional Programming

Postscript
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Postscript
A node \( n \) in the CFG dominates a node \( m \) iff:

- \( n \) is on all paths from entry to \( m \)
- by definition, a node \( n \) always dominates itself
- if \( n \neq m \), then \( n \) strictly dominates \( m \)

Computed using a dataflow-style analysis

- Each node annotated with a set of its dominators
Simple algorithm to generate SSA form
  - Introduce $\phi$ functions
  - Rename variables using Reaching Definitions
Algorithm can generate excessive $\phi$ functions
  - TODAY: Use dominance frontiers to place the minimal number of $\phi$ functions
Also today: Removing $\phi$ functions
  - Machines don’t support $\phi$ functions, so we must emulate them
Maximal SSA Form

- Insert $\phi$ nodes for each definition at every join node
- Rename LHS
- Rename RHS using reaching definitions
Reducing the number of phi nodes

- Why insert $\phi$ nodes at only join nodes?
- Can we skip inserting $\phi$ nodes for a definition at some join node?
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The dominance frontier of a node $n$ ($\text{DF}(n)$) is a set of nodes if a node $m \in \text{DF}(n)$ iff:

- $n$ does not strictly dominate $m$
- $n$ dominates $q$ where $q \in \text{pred}(m)$

Note that dominance frontiers only contain join nodes, i.e. nodes with multiple predecessors.

Computing the dominance frontier of each node:

- Iterative Data-flow analysis?
Direct calculation of dominance frontiers using *dominator trees*.
**Immediate Dominators**

- The *immediate* dominator of a node $m$ ($\text{IDOM}(m)$) is the node $n$:
  - such that $n$ strictly dominates $m$, and
  - $n$ does not strictly dominate $o$ where $o \in (\text{DOM}(m) - \{m\})$
  - in some sense, $n$ is the "closest" dominator in the CFG to $m$.
- By definition, ENTRY has no immediate dominator
Not Strictly Dominates

- $n$ strictly dominates $m$
  - $SDOM(n, m) = n \in DOM(m) \land n \neq m$
- $n$ does not strictly dominate $m$
  - $\neg SDOM(n, m) = n \notin DOM(m) \lor n = m$
- Note that each node in the CFG can have only one immediate dominator
  - Can you see why?
- Create a graph $G = (V, E)$, where:
  - $V$ is the set of basic blocks
  - There is an edge $(n, m)$ in $E$ if $n$ is the immediate dominator of $m$ (i.e. $\text{IDOM}(m) = n$)
Example: CFG and its dominator tree
Computing the dominance frontier

- Find all join nodes in CFG, e.g. $j$
- For all nodes $n$ that dominate predecessors of $j$ (in the CFG)
  - If $n$ does not strictly dominate $j$, add $j$ to $DF(n)$
- This last step can be operationalized over all predecessors $p$ of $j$ in the CFG:
  - Start traversing the dominator tree at $p$
  - If $p$ is IDOM($j$), stop. Otherwise add $j$ to $DF(p)$
  - Repeat by moving up the dominator tree until you reach IDOM($j$)
Example: Non-redundant $\phi$ functions

\[ y_0 = x_0 + 1 \]
\[ x_1 = 2 \]
\[ y_1 = \phi(y_0, y_4) \]
\[ y_1 > 3 \]
\[ y_2 = 3 \]
\[ a = 3 \]
\[ y_3 = \phi(y_1, y_2) \]
\[ y_4 = x_1 + y_3 + 2 \]
Placing $\phi$ functions

- For each definition $d$ in basic block $n$:
  - Place a $\phi$ function for $d$ in all nodes $m$ where $m \in DF(n)$
  - Note that each $\phi$ function is also a definition!
  - Repeat, until no more $\phi$ functions need to be inserted
- This is the minimal number of $\phi$ functions for a definition $d$ structurally
  - Can we further reduce the overall number of $\phi$ functions?
- (Figure 9.9 in Cooper and Turczon)
Other optimizations

- Dead definitions
  - Definitions that are not read (i.e. overwritten) do not need $\phi$ functions
- Two forms:
  - Semi-pruned SSA form, using "globals" names (those variables that are live in to a block)
  - Pruned SSA form, using LIVEOUT information
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Renaming variables

- SSA form introduced “subscripts” for each variable
- Should we drop them when generating code?

\[
\begin{align*}
a_0 &= x_0 + y_0 \\
b_0 &= a_0 \\
a_1 &= 17 \\
c_0 &= a_0
\end{align*}
\]
Problem with dropping subscripts

\[ a = x + y \]
\[ b = a \]
\[ a = 17 \]
\[ c = a \quad # \text{WRONG!} \]
Handling subscripts

- Each definition becomes a new variable
  - i.e. Do NOT drop subscripts
- Preserves data dependences
  - Esp. important when we aggressively move code from basic blocks (e.g. very busy expressions, loop invariant code motion, etc.)
Introduce copies along each incoming edge to a join node

\[
i_2 = 1
i_4 = \phi(i_2, i_3)
\ldots
i_3 = a + b
\]

\[
i_4 = \phi(i_2, i_3)
\]

Code for $\phi$ functions
Inserting appropriate copies along incoming edges

\[ i_2 = 1 \]
\[ i_4 = i_2 \]

\[ i_3 = a + b \]
\[ i_4 = i_3 \]
Critical edges

- Executing $\phi$ functions by inserting copies into predecessor blocks is not always correct.
- If such a predecessor block has multiple successors, then the $\phi$ function may execute when it shouldn’t.
  - This may be harmless, but not always.
- Edges connecting such predecessors to the block containing the $\phi$ function are called *critical* edges.
Critical Edges: Example

\[ i_2 = 1 \]
\[ i_4 = i_2 \]

\[ i_3 = a + b \]
\[ i_4 = i_3 \]

...A...

...B...
Splitting critical edges

- Such edges need to be *split* by inserting a block on that edge.
- See the discussion in Cooper and Turczon for more details and an example.
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Purely Functional Programs

- Everything is a value
- No "assignment", just binding values to names
- No control flow such as jumps
  - Must be emulated using functions
Example: Factorial

def fact(N):
    res = 1
    for i in range(1, N+1):
        res *= i
    return res

def fac(N):
    return 1 if N <= 1 else N * fac(N - 1)
```python
def fact(N):
    res = 1
    i = 1
    if i > N goto loop_end

    loop_head:
        res = res * i
        i = i + 1
        if i <= N goto loop_head

    loop_end:
        return res
```
def fact(N):
    res_0 = 1
    i_0 = 1
    if i_0 > N goto loop_end

    loop_head:
        res_1 = phi(res_0, res_2)
        i_1 = phi(i_0, i_2)

        res_2 = res_1 * i_1
        i_2 = i_1 + 1
        if i_2 <= N goto loop_head

    loop_end:
        res_3 = phi(res_0, res_2)
        return res_3
def fact(N):
    res_0 = 1
    i_0 = 1

    def loop_head(res_1, i_1):
        res_2 = res_1 * i_1
        i_2 = i_1 + 1
        return loop_head(res_2, i_2) if i_2 <= N else loop_end(res_2)

    def loop_end(res_3):
        return res_3

    return loop_end(res_0) if i_0 > N else loop_head(res_0, i_0)

- Each basic block is converted to a function
- Parameters to this function are the LHS of the $\phi$ functions in that BB
- Arguments picked from arguments of $\phi$ function depending on the path the BB was on.
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References

- Chapter 9 of Cooper and Turczon
  - Section 9.2.1
  - Section 9.3
- Optional:
  - Various authors, The SSA book
  - Andrew W. Appel, SSA is functional programming