CSC2/455 Software Analysis and Improvement
Program Analysis/Abstract Interpretation

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URCS

Outline

And now for something (not so completely) different

Program Analysis in Industry

Basic Notions

Illustrating Abstract Interpretation

Postscript

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So far

- Data flow analysis
  - Iterative data flow analysis
  - Region-based analysis
- Loop analysis
- Type Checking
- What next?

Compilers are not the only program analyzers

- Compilers are probably the most used program analyzers
- But are severely time constrained
- Finding *program* errors is not primary goal
  - Syntax errors, type errors
  - Code generation primary goal

Program/Software Analysis

- Software is increasingly mission-critical
- Can kill people!
  - Boeing 737 MAX(?)
  - Therac-25 (X-ray)
  - Industrial Robotics
- (less extreme?) Can lose money
  - Software crashes
  - Data loss
- Can we analyze programs for *functional* correctness?
  - Topic of the next few lectures

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SLAM (Microsoft, early 2000s)

- MS isolated most crashes to buggy drivers
- Static Driver Verifier project
  - Would verify driver code (in C) for correctness
- Used *model checking*
  - Models programs as finite-state machines
  - I used a similar tool (CBMC) to check your assignments

Infer (Facebook)

- Checks C, C++, Objective C, Java and Android code
- Used for checking Facebook's mobile apps
- Open source, `https://fbinfer.com/`
  - Used by Amazon, Mozilla, Uber and Facebook and its affiliates, JD.com, etc.
- Comes with its own language AL (OCaml-derivative?) to describe analyses
  - Analyzes programs in SIL ("Smallfoot Intermediate Language")
- Uses abstract interpretation + *separation logic*
  - Abstract interpretation very similar to data flow analysis frameworks
- CACM Article: Scaling Static Analyses at Facebook
- Good video: Getting the most out of static analyzers

SPARTA (Facebook)

- Language-independent analyzer
  - a C++ framework
- Open source, `https://code.fb.com/open-source/sparta/`
- Used in FB's RedEx tools
  - for analyzing Android binary code (`.dex`)
- Also uses *abstract interpretation*

Other efforts

- Stanford Checker
  - commercialized by Coverity, late 2000s
  - CACM article, "A few billion lines of code later: using static analysis to find bugs in the real world"
- Google's static analysis tools
  - Checker Framework for Java programs
  - Shipshape (abandoned?) (Google Tricorder)
  - CACM article, "Lessons from Building Static Analysis Tools At Google"
- Oracle's Soufflé
  - Soufflé: Logic Defined Static Analysis

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Limitations

- None of these frameworks and tools can escape the fact that analysis is an undecidable problem
- All compute approximations
  - Or risk ending up intractable
- Must be designed to be *sound*
  - Approximations are conservative/safe
- Leads to imprecision (i.e. *incomplete*)
  - May model behaviour not in original programs
  - (recall IDEAL vs MOP vs MFP)
  - leads to false positives

States and Transitions

- A program's state is a mapping of variables to values
- Programs move from one state to another
  - begin execution in subset of (initial) states
- Notions of state *before* a program point (i.e. a statement) and *after* a program point
  - Also sometimes known as *pre-condition* and *post-condition* respectively.
- Relation that maps before-states to after-states is called a *transition* relation (t)
  - $\langle x, y \rangle$ (x is before-state, y is after-state)

Traces

- An execution trace of a program is a sequence of states
  - $s_0 s_1 s_2 \ldots s_n$
- An execution trace may be finite or infinite
  - $s_0 s_1 s_2 \ldots$
- The collection of partial traces can actually happen (i.e. state transitions obey the transition relation) is called the *collecting semantics*
  - I.e. for all $s_i s_j$ in trace, $\langle s_i, s_j \rangle \in t$

Example

```
x = 0
while(x < 100)
   x = x + 1;
```

- states are $\mathbb{Z}$
- initial state is 0
- transition relation is $\{\langle x, x' \rangle | x < 100 \wedge x' = x + 1\}$
- is 0 1 2 3 part of the collecting semantics?
- is 0 2 4 6 part of the collecting semantics?

Patrick Cousot and Radhia Cousot, *Basic Concepts of Abstract Interpretation*

What can we do with the concrete semantics?

- By examining the (concrete) collecting semantics, we can check various "properties"
  - We'll formalize "property" later.
- Problems with using the concrete semantics:
  - The previous example had a single state in initial set $\{0\}$, this is not always true. Consider `x = randint()`
  - How do we deal with infinite loops?
  - How do we deal with alternate paths (i.e. conditionals)?
  - How do we get the concrete semantics for all programs statically?
- Implementation issues:
  - Even if we could get a concrete semantics, how large would it be?

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A Graphical Programming Language[1]

Consider a simple language with the following constructs:

- `init(R)` where `R` is a region in 2D space
  - e.g. `init(`$\{(x, y)|(x, y) \in R\}$`)`
  - This chooses a single point in R non-deterministically
  - A program must always start with `init`
- `translate(dx, dy)` moves the point by dx in the X-direction, and dy in the Y-direction
  - e.g. `translate(1.0, 0.5)` moves the point to the right and up in the Cartesian plane
- `rotate(angle)` rotates the point by `angle` about the origin
  - e.g. `rotate(90)` will move a point on the X-axis to a point on the Y-axis

[1] This exposition is based on Chapter 2 of Rival and Yi (see Postscript)

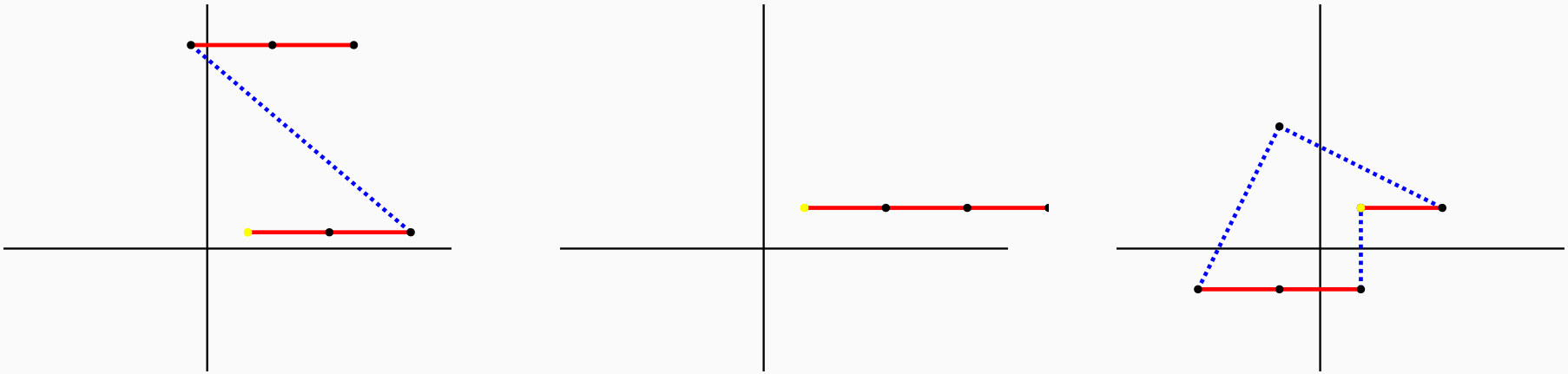
Grammar

```
P =   init(R) ';' Rest
Rest = translate(dx, dy)
       | rotate (a)
       | '{' Rest '}' or '{' Rest '}'
       | iter '{' Rest '}'
       | Rest ';' Rest
       | Empty
Empty =
```

- A program begins with `init` and is followed by statements in *Rest*
- The `or` construct is a non-deterministic choice
  - It executes the block (delimited by braces) on the right or the block on the left
  - Simulates a conditional
- The `iter` either executes the code inside the block or moves to the next statement
  - Simulates a loop
  - Note, `iter` can execute the block forever!

An Example Program

```
init([0, 1]x[0, 1]);
 translate(1, 0);
 iter {
     {
         translate(1, 0);
     } or {
         rotate(90);
     }
 }
```



Property we're interested in

Can x ever become negative?

- Can be represented by the set $x_{neg} = \{(x, y) | x < 0\}$
- Problems with concrete execution approach:
  - Infinite initial states ($R = \{(x, y) | 0 \leq x \leq 1 \wedge 0 \leq y \leq 1\}$
  - Conditionals that perform translation or rotation
  - Loop may be infinite
- But if we could obtain the set of states in all possible concrete executions, say x_{conc}, we need to show
  - $x_{conc} \cap x_{neg} = \emptyset$

Abstract Execution: Approximating Concrete Executions

- Abstract interpretation is a framework for performing program analysis
- Key ideas:
  - Abstract domain: Set of properties we're interested in
  - Abstraction function: Converts a concrete state to an element of the abstract domain
  - Transfer functions: Transforms an abstract state before a statement to an abstract state after the statement
  - Union/Join: Combines abstract states from alternate paths
  - Widen (∇): Combines abstract state across loop iterations

Abstract Domains

- For the property we're interested in, we only need the sign of x
- Potential abstract domain, *signs-x*
  - Only tracks $x < 0$, $x \in R$
  - Can only answer questions about this property, and about x
- Another abstract domain, track signs of all state variables
  - Tracks $\{x < 0, x \geq 0, x \in R\} \times \{y < 0, y \geq 0, y \in R\}$
- Abstract domain for the rest of the lecture: Intervals
  - For each state variable, track l_v and h_v such that $l_v \leq v \leq h_v$
  - For the graphical language, $l_x \leq x \leq h_x$, $l_y \leq y \leq h_y$
  - Thus, our abstraction approximates states using rectangles
  - The sides of the rectangle are parallel to the axes
  - l_v and h_v can be $-\infty$ and ∞ respectively to represent unbounded "rectangles"

Abstract Execution Using Intervals: `init`

```
init([0, 1]x[0, 1])
```

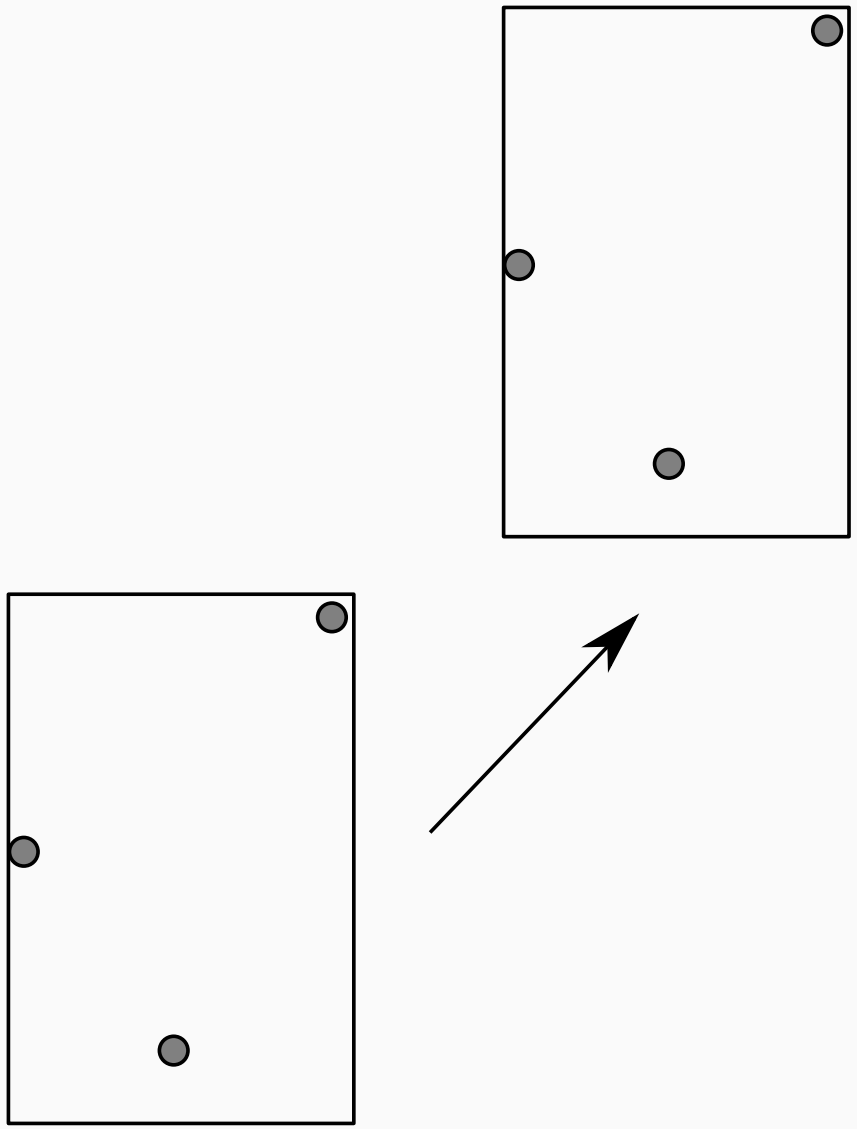
Concrete execution will give us a point in that rectangular region.

- Our abstract state after `init` will be:
  - $l_x = 0$
  - $h_x = 1$
  - $l_y = 0$
  - $h_y = 1$
- The transfer function for `init` computes the rectangle that covers the region
- In this case, the abstraction is precise

Abstract Execution Using Intervals: `translate`

```
translate(1.0, 0.5)
```

- Our abstract state after `translate` will be:
  - $l_x = l_x + 1.0 = 0.5$
  - $h_x = h_x + 1.0 = 2$
  - $l_y = l_y + 0.5 = 0.5$
  - $h_y = h_y + 0.5 = 1.5$
- The transfer function for `translate` shifts the current abstract state
  - The resulting abstract state is still precise



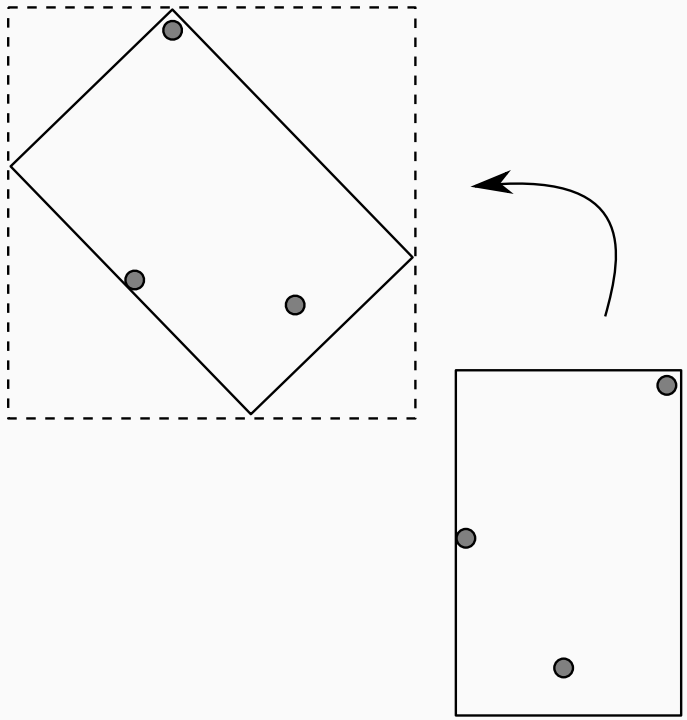
Abstract Execution Using Intervals: `rotate`

```
rotate(45)
```

- Our rectangle after `rotate` will be:
  - $l_{x_1}, l_{y_1} = rotate((l_x, l_y), 45)$
  - $h_{x_1}, l_{y_2} = rotate((h_x, l_y), 45)$
  - $l_{x_2}, h_{y_1} = rotate((l_x, h_y), 45)$
  - $h_{x_2}, h_{y_2} = rotate((h_x, h_y), 45)$
- The transfer function for `rotate` rotates the corners of the rectangle
  - The result is still a rectangle, but cannot always be represented using intervals
  - So it is not an abstract state

Finding a new interval in the abstract domain

- Let c' be a co-ordinate after `rotate` computed as in the previous slide, then
  - $l_x = min(l_{x_1}, l_{x_2}, h_{x_1}, h_{x_2})$
  - $h_x = max(l_{x_1}, l_{x_2}, h_{x_1}, h_{x_2})$
  - $l_y = min(l_{y_1}, l_{y_2}, h_{y_1}, h_{y_2})$
  - $h_y = max(l_{y_1}, l_{y_2}, h_{y_1}, h_{y_2})$
- Obviously, in general, this new interval contains more states than the rotated rectangle
  - We have lost precision
- But, the new interval/rectangle we have calculated is the "best fit"



A note on domains

- Intervals (and signs) are non-relational domains
  - They can't capture relations between x and y
  - e.g., a property that $x > y$
- Intervals also can't capture complex regions
- A more complicated abstract domain: *convex polyhedra*
  - A list of linear inequalities
  - Region is the feasible region (i.e. points that satisfy all the inequalities)
  - Convex polyhedra support relational properties, e.g. $x - y < 2$
- During program analysis you must choose the domain that most efficiently captures the property of interest

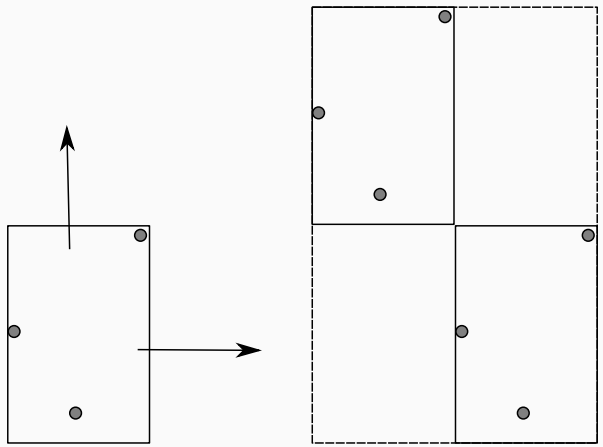
Handling Compound Statements

- Let the transfer function be called `analysis(stmt, pre-condition)`
- So far, we've defined:
  - `analysis(init, I)` (where *I* is the state on entry)
  - `analysis(translate, a)` where *a* is the state before translate
  - `analysis(rotate, a)` where *a* is the state before rotate
- Let's now define `analysis(p1 ; p2, a)`
  - The analysis of two statements `p1` and `p2`, with pre-condition `a`
  - Note that pre-condition for p2 is the post-condition of p1, so
  - `analysis(p1 ; p2, a)` is then `analysis(p2, analysis(p1, a))`

Handling conditionals: Union (Join)

```
{translate(1.0, 0)} or {translate(0, 1.0)}
```

- Program could execute either left part or right part
- Two possible abstract states, disjoint
- Resulting abstract state must incorporate both
  - *Union* the two abstract states
- `analysis(p1 or p2, a)` is:
  - `analysis(p1, a)` $\cup$ `analysis(p2, a)`
- The resulting abstract state is also, in general, imprecise



Union for Intervals

- For $a_1 \cup a_2$
  - $l_x = min(l_{x_1}, l_{x_2})$
  - $h_x = max(h_{x_1}, h_{x_2})$
  - $l_y = min(l_{y_1}, l_{y_2})$
  - $h_y = max(h_{y_1}, h_{y_2})$

Handling Loops

```
iter {b}
```

can be written as:

```
{}                                  # loop executes 0 times
{} or {b}                           # loop executes 1 time
{} or {b} or {b;b}                  # loop executes 2 times
{} or {b} or {b;b} or {b;b;b} # loop executes 3 times
 ...
```

This can be written (recursively) as: $p_{k+1} = p_k \text{or} \{p_k; \text{b}\}$, where:

- p_0 is $\{\}$
- p_1 is $\{\}$ `or` $\{\text{b}\}$
- p_2 is $\{\}$ `or` $\{\text{b}\}$ `or` $\{\text{b; b}\}$ and so on...

Iterating

```
analysis(iter {b}, a)
```

can be defined as an interative algorithm:

```
R = a
do
   T = R
   R = union(R, analysis(b, R)) # analysis for or
while R != T
```

Will this always terminate?

Example

```
init([0, 1] x [0, 1])
translate(1, 0)
iter { translate(0, 1) }
```

- The analysis of this code will not terminate!
- Abstract state before `iter`
  - $1 \leq x \leq 2,\ 0 \leq y \leq 1$
- After first union:
  - $1 \leq x \leq 2,\ 0 \leq y \leq 1 \cup 1 \leq x \leq 2,\ 1 \leq y \leq 2$
  - Result: $1 \leq x \leq 2,\ 0 \leq y \leq 2$
- After second union: $1 \leq x \leq 2,\ 0 \leq y \leq 3$
- And so on ...
  - the h_y bound keeps increasing without bound

The Widen operator

- We note that the interval $0 \leq y \leq \infty$ would overapproximate $0 \leq y \leq n$ where n is not ∞
- If we obtained this interval in our abstract state, we could terminate because h_y would "stop increasing"
  - $0 \leq y \leq \infty$ already includes all possible abstract states of the form $0 \leq y \leq n$
  - Union would no longer return a different result ensuring termination
- So widen (∇) is an operator that overapproximates unions
  - Its primary purpose is to ensure convergence

Analyzing loops using widen

```
R = a
do
   T = R
   R = widen(R, analysis(b, R))
until inclusion(R, T)  # i.e. R is included in T
return T
```

Widen for Intervals

```
def widen(a, b):
   out = union(a, b)

   if a.lx != b.lx:
      out.lx = -inf

   if a.hx != b.hx:
      out.hx = inf

   if a.ly != b.ly:
      out.ly = -inf

   if a.hy != b.hy:
      out.hy = inf
```

Putting it all together

- Each time `analysis` is applied, we obtain an abstract state a
- If a overlaps with x_{neg}, then we have detected a violation of our property

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References

- This lecture follows the exposition in Chapter 2 of the book "An Introduction to Static Analysis: An Abstract Interpretation Perspective" by Xavier Rival and Kwangkeun Yi, MIT Press, 2020
  - The library has print copies
  - Suggest buying this book – it is self-contained and reasonably priced
- Prof. Cousot has a number of tutorials, I'll post links to them in the next lecture
  - Prof. Patrick Cousot and Prof. Radhia Cousot invented abstract interpretation