CSC2/455 Software Analysis and Improvement
Type Inference - II

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URCS
Outline

Type Systems for Realistic Languages

Type Checking ChocoPy

Postscript
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The type system we studied in the last class was for a functional language

- Programs consist of expressions
- All well-formed expressions have a type

Some languages, notably imperative languages (Python, C, etc.), differentiate between statements and expressions

- Expressions have types
- Statements do not have types
- “Procedures” do not have return values
Additional judgements for imperative languages

- $\Gamma \vdash C$
  - $C$ is a well-formed command (or statement) in $\Gamma$

In general, block-structured languages may have $C$ be a block as well.
Subtyping

- Most languages have a subtype relation
  - C has *unsigned short* is a subtype of *unsigned int*
  - Subtypes can be used wherever the "supertype" is used
- Usually denoted by $A \leq B$
  - $A$ is a subtype of $B$
  - $A \leq A$, obviously
- Type hierarchy, a tree of types
  - Usually object as the root
  - A edge from $B$ to $A$ if $A < B$
Type Systems for Realistic Languages

Type Checking ChocoPy

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ChocoPy

- ChocoPy is simpler version of Python3
  - Developed as a “toy” language for compiler courses
- Fully-specified
  - Syntax (BNF)
  - Type System
  - Semantics (Operational Semantics)
- We will only look at its Type System
ChocoPy Types

- **Basic types:** object, int, bool, str
- **List type:** `[T]`, where T is another type
  - e.g. `[int]`
  - T be a list as well, this is an example of a *recursive type*
  - `[[int]]`
- **Special types**
  - `<None>`
  - `<Empty>`, indicating an empty list `[]`
A ≤ A, for all types A
C ≤ A if C is a subclass of A
  • int, bool, str are all subclasses of object
[T] ≤ object
  • but no relationship between two different list types
<None> ≤ object
<Empty> ≤ object
A type $T_1$ is assignment compatible ($\leq_a$) to $T_2$ iff values of type $T_1$ can be used wherever values of $T_2$ are expected.

- $T_1 \leq T_2$ (note: this is subtyping $\leq$)
- $T_1$ cannot be $<$None$>$ if $T_2$ is int, bool or str
- $T_1$ can be $<$Empty$>$ only if $T_2$ is $[T]$
- $T_1$ can be $[<None>]$ if $T_2$ is $[T]$, where $<None> \leq_a T$
  - Note that means $T$ can only be object (*)
The join $\sqcup$ of two types $A$ and $B$ denoted as $A \sqcup B$ is defined as:

- $B$ if $A \leq_a B$
  - This is commutative, so $B \sqcup A$ is also $B$ in this case
- $C$ where $C$ is the least common ancestor of $A$ and $B$ in the type hierarchy defined by $\leq$
  - So $\text{int} \sqcup \text{str}$ is $\text{object}$
  - Note this is the least upper bound or LUB (analogous to GLB in meet lattices)
Type Environment

- Judgements of the form:
  - $O, M, C, R \vdash ...$
- Where the type environment consists of:
  - $O$, "local" environment
  - $M$, method/attribute environment, i.e. for A.x-style code
  - Both are maps
- And:
  - $C$, the name of the current class, can be $\bot$ if outside class
  - $R$, the return type of the current function, can be $\bot$ if outside function
def contains(items:[int], x:int) -> bool:
    i:int = 0
    while i < len(items):
        if items[i] == x:
            return True
        i = i + 1
    return False

The general type for a function \( f \) is \( O(f) = \{T_1 \times T_2 \times \ldots \times T_n \rightarrow T_0; x_1, x_2, \ldots, x_n; v_1 : T'_1, v_2 : T'_2, \ldots, v'_m : T'_m\} \)

- \( T_1 \times T_2 \times \ldots \times T_n \rightarrow T_0 \) is the type of arguments and the return type
  - Here, \([\text{int}] \times \text{int} \rightarrow \text{bool}\)

- \( x_1, x_2, \ldots, x_n \) are names of the arguments: \( \text{items}, x \)

- \( v_1 : T'_1, \ldots, v'_m : T'_m \) are types of variables/functions declared inside the function
  - Here \( i : \text{int} \)
class Point1D:
    x: int = 0

    # the quotes are used since Point1D is being defined ...
    def distance(self: "Point1D", o: "Point1D") -> int:
        ...

- Attributes are simply \( M(C, a) = T \), where \( C \) is the class, \( a \) is the attribute, and \( T \) is its type
  - So, \( M(\text{Point1D}, x) = \text{int} \)
- And methods \( M(C, m) \) are just like functions in the previous slide
  - So,
    \[
    M(\text{Point1D}, \text{distance}) = \{ \text{Point1D} \times \text{Point1D} \to \text{int}; \ldots \}\]
Type Judgements in ChocoPy

- $O, M, C, R \vdash e : T$
  - Expression $e$ has type $T$ in the environment $O, M, C, R$
- $O, M, C, R \vdash b$
  - $b$ is a well-formed (block) statement in $O, M, C, R$
  - can also be a single statement, obviously

- If reading the ChocoPy reference, make sure to distinguish the colon in type judgements $e : T$ from the colon in Python syntax
  - I’ll use : to indicate the latter, and : to indicate the former
Expressions

\[
O, M, C, R \vdash \text{True} : \text{bool}
\]

\[
O, M, C, R \vdash \text{False} : \text{bool}
\]

\[
i \text{ is an integer literal}
\]

\[
O, M, C, R \vdash i : \text{int}
\]

\[
s \text{ is a string literal}
\]

\[
O, M, C, R \vdash s : \text{str}
\]

\[
O(id) = T \quad T \text{ is not a function type}
\]

\[
O, M, C, R \vdash id : T
\]
Variable Definitions

\[
O(id) = T \quad O, M, C, R \vdash e_1 : T_1 \quad T_1 \leq_a T \quad (\text{VAR-INIT})
\]

\[
O, M, C, R \vdash id : T = e_1
\]
\[
O, M, C, R \vdash s_1 \\
O, M, C, R \vdash s_2 \\
\vdots \\
O, M, C, R \vdash s_n \\
\frac{O, M, C, R \vdash s_1 \ \text{NEWLINE} \ s_2 \ldots s_n \ \text{NEWLINE}}{(\text{STMT-DEF-LIST})}
\]
Operators

\[
\begin{align*}
O, M, C, R & \vdash e_1 : \text{int} \\
O, M, C, R & \vdash e_2 : \text{int} \\
op & \in \{+, -, *, //, \%\} \\
\frac{O, M, C, R \vdash e_1 \ op \ e_2 : \text{int}}{(\text{ARITH})}
\end{align*}
\]

\[
\begin{align*}
O, M, C, R & \vdash e_1 : \text{int} \\
O, M, C, R & \vdash e_2 : \text{int} \\
\ni & \in \{<, <=, >, >=, ==, !=\} \\
\frac{O, M, C, R \vdash e_1 \ ni \ e_2 : \text{bool}}{(\text{COMPARE-INT})}
\end{align*}
\]

\[
\begin{align*}
O, M, C, R & \vdash e_0 : \text{bool} \\
O, M, C, R & \vdash e_1 : T_1 \\
O, M, C, R & \vdash e_2 : T_2 \\
\frac{O, M, C, R \vdash e_1 \ if \ e_0 \ else \ e_2 : T_1 \sqcup T_2}{(\text{IF-EXPR})}
\end{align*}
\]
Return and Class Definitions

\[
O, M, C, R \vdash e : T \quad T \leq_a R
\]

\[
\frac{}{O, M, C, R \vdash \text{return } e}
\]

\[
O, M, C, R \vdash b
\]

\[
\frac{}{O, M, \bot, R \vdash \text{class } C(S) : b}
\]
Function Definitions

```python
x: str = "hello, world"

def somefn(x: int, y: str):
    # what is the type of x here?

• Function argument names:
  • shadow names outside the function (like x here)
  • or are introduced in the definition (like y)

• The notation for shadowing or introducing a variable is:
  • \( O[T/x] \) which is defined as \( O[T/x](x) = T \) and
    \( O[T/x](y) = O(y) \) where \( y \neq x \)
  • i.e., \( O[T/x] \) is a new map with \( x \) mapped to \( T \) and the other
    mappings unchanged
  • repeated applications are possible: \( O[T_1/x][T_2/y] \)
```
\[ T = \begin{cases} T_0, & \text{if } \rightarrow \text{ is present}, \\ \langle \text{None} \rangle, & \text{otherwise}. \end{cases} \]

\[ O(f) = \{ T_1 \times \cdots \times T_n \rightarrow T; \ x_1, \ldots, x_n; \ v_1 : T'_1, \ldots, v_m : T'_m \} \]

\[ n \geq 0 \quad m \geq 0 \]

\[ O[T_1/x_1] \cdots [T_n/x_n][T'_1/v_1] \cdots [T'_m/v_m], \ M, C, T \vdash b \]

\[ O, M, C, R \vdash \text{def } f(x_1 : T_1, \ldots, x_n : T_n) [\rightarrow T_0]^? : b \]
\[ T = \begin{cases} 
T_0, & \text{if } \rightarrow \text{ is present,} \\
\langle \text{None} \rangle, & \text{otherwise.} 
\end{cases} \]

\[ M(C, f) = \{ T_1 \times \cdots \times T_n \rightarrow T; x_1, \ldots, x_n; v_1 : T'_1, \ldots, v_m : T'_m \} \]

\[ n \geq 1 \quad m \geq 0 \]

\[ C = T_1 \]

\[ O\left[\frac{T_1/x_1}{T_n/x_n}\right] \cdots \left[\frac{T_n/x_n}{T'_1/v_1}\right] \cdots \left[\frac{T_m/v_m}{T'_m}\right], M, C, T \vdash b \]

\[ O, M, C, R \vdash \text{def } f(x_1: T_1, \ldots, x_n: T_n) \overset{\rightarrow}{\Rightarrow} T_0 : b \]
Outline

Type Systems for Realistic Languages

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Postscript
• A self-contained introduction to type systems
  • Luca Cardelli, Type Systems, Handbook of Computer Science and Engineering, 2nd Ed

• An updated version (available only through the library)
  • Stephanie Weirich, Type Systems, Handbook of Computer Science and Engineering, 3rd Ed

• Rohan Padhye, Koushik Sen, Paul Hilfinger, ChocoPy v2.2: Language manual and reference

• Leandro TC Melo, Rodrigo G Riberio, Breno CF Guimaraes, Fernando Magno Quintao, Type Inference for C: Applications to the Static Analysis of Incomplete Programs, ACM TOPLAS
  • The psychec tool