CSC2/455 Software Analysis and Improvement
Live variable analysis

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URCS
Outline

Review

Live Variable Analysis

Live Variables using Iterative Data flow Analysis

Postscript
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What we know how to do so far

- Basic blocks to Control Flow Graphs
  - Section 5.4.3 in Cooper and Turczon
- Local optimizations (Value Numbering)
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Motivation

```c
void example(int x) {
    int a = 0; /* def 1 */
    int b = 0; /* def 2 */
    if(x > 1) {
        a = b; /* def 3 */
    } else {
        a = x; /* def 4 */
    }
    return a;
}
```

Must the compiler emit code to initialize a to 0?
void example(int x) {
    int a = 0; /* def 1 */
    int b = 0; /* def 2 */

    if(x > 1) {
        a = b; /* def 3 */
    } else {
        a = x; /* def 4 */
    }

    return a;
}

a is defined (i.e. written) thrice in the code above. Which definitions are “alive” at return a?
• A variable $x$ is live at a definition (i.e. a write) if there is a subsequent read of the variable along a program path from that definition.
  • Alternatively: is this variable overwritten along all paths before a subsequent read?

• Can we identify live variables just by analyzing basic blocks (i.e. local analysis)?
void test() {
    int i;

    // begin delay
    for(i = 0; i < 10000; i++);
    // end delay
}

test():

    push    rbp
    mov     rbp, rsp
    mov     DWORD PTR [rbp-4], 0

.L3:
    cmp     DWORD PTR [rbp-4], 9999
    jg      .L4
    add     DWORD PTR [rbp-4], 1
    jmp     .L3

.L4:
    nop
    pop     rbp
    ret

Code courtesy Compiler Explorer at godbolt.org (this is more convenient than gcc -Wa if you have the Internet)
test():
    ret

Code courtesy Compiler Explorer at godbolt.org
Example 1

```c
x = 0; /* d1 */
if(y > 0)
    x = 2; /* d2 */
else
    x = 3; /* d3 */
printf("%d\n", x);
```
Example 2

x = 0  /* d1 */
i = 0  /* d2 */
while(i++ < 10) { /* d3 */
    if(i % 3 == 0) {
        x = 0; /* d4 */
    }
    x = x + 1; /* d5 */
}

• Which definition(s) of x are live at entry of loop?
• Which definition(s) of x are live in the loop?
• Which definition(s) of x are live at exit of loop?
Relook at the Loop example

i = 1;
L1:
    if(!(i < 10000)) goto L2;
i++;
goto L1;
L2:
    return

Where in the CFG must i be “dead” to eliminate it?
Some Definitions

For a basic block $s$:

- $\text{DEF}(s)$ (for “defined”) is the set of variables written by $s$
- $\text{USE}(s)$ is the set of variables read by $s$

For the blocks below, what are the DEF and USE sets?

- **BB\_eg\_0**:  
  $i = i + 1$

- **BB\_eg\_1**:  
  $a = x;  
  x = 3;$

- **BB2\_eg\_2**:  
  $t = a;$
What happens to a definition of \( a \) in a basic block that is before \( \text{BB}_\text{eg}_3 \) on some path through the CFG?

What can we say about definitions of \( y \) in other basic blocks that are before \( \text{BB}_\text{eg}_3 \) on some path through the CFG?
More definitions

- Upwardly exposed $\text{UE}(s)$ are those variables that are read \textit{before} they are written to in basic block $s$.
  - e.g. $x$ and $y$ in BB\_eg\_3 in the previous slide
- A variable is alive in a basic block if it is in the set:
  - ?
- A variable is killed in a basic block if it is in the set:
  - ?
Finding out if a definition is live

- Start from the definition, and see if it “reaches” a use
  - i.e., it is not killed on some path (in the CFG) to a use
- Start from a use, and walk backwards to see if you can reach the definition
  - Repeat for all uses
Example #1

ENTRY

x = 0

y > 0

x = 2  x = 3

printf("%d", x)

EXIT
Example #2

ENTRY

x=0
i=0

EXIT

i++ < 10

i % 3 == 0

x = 0

x = x + 1

EXIT
When does enumerating paths work in practice?
• Setup data flow equations
• Solve these data flow equations
Equations for Live Variables

- Each basic block has “incoming” edges (from predecessors) and “outgoing” edges (to successors).
- When is an “incoming” variable live “into” a block?
  - it is in UE for that block
- When is a variable live out of a block \( n \)?
  - it is live into a successor block \( m \)
  - first try: \( \text{LIVEOUT}(n) = \bigcup_m \text{UE}(m) \) (where \( m \in \text{succ}(n) \))
  - what are we missing?
\[
\text{LIVEOUT}(n) = \bigcup_{m \in \text{succ}(n)} \text{UE}(m) \cup (\text{LIVEOUT}(m) \cap \overline{\text{DEF}(m)})
\]

The variables live out of block \(n\) must:

- be upwardly exposed in a successor \(m\)
- OR be live out of \(m\) and NOT be killed by \(m\) (i.e. \(\overline{\text{DEF}(m)}\))
  - note this is equivalent to \((\text{LIVEOUT}(m) - \text{DEF}(m))\)

How would you begin solving this equation?
Iterative Data flow Analysis Algorithm for Live Variables

- Live variables is a *backwards dataflow analysis*
  - “Facts” flow from a node’s successors to it
- Initialization:
  - Compute \( \text{DEF}(s) \) for all blocks \( s \)
  - Compute \( \text{UE}(s) \) for all blocks \( s \)
  - Set \( \text{LIVEOUT}(s) = \emptyset \) for all \( s \)
- Repeat (iterate):
  - Compute \( \text{LIVEOUT}(s) \) for all blocks \( s \)
  - Until no set \( \text{LIVEOUT}(s) \) changes – i.e. until a fixpoint is reached.
Termination and correctness

- **Termination**
  - Note \( \text{LIVEOUT}(s) \) only increases in size or remains the same in every iteration
  - What is the maximum size of \( \text{LIVEOUT}(s) \)?

- **Correctness**
  - When the algorithm terminates, all computed \( \text{LIVEOUT}(s) \) sets satisfy the equation
  - and hence meet the definition of “liveness”. 
How does the iterative solution differ from the path-based solution?
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References

- Chapter 8 of Cooper and Turczon
  - Section 8.6.1
- Also recommended:
  - Aho, Lam, Sethi and Ullman, Chapter 9, Section 9.2.5
- Spare time reading (esp. for PhD students):
  - Frances E. Allen, John Cocke, 1971, A Catalogue of Optimizing Transforms