CSC2/455 Software Analysis and Improvement
Program Analysis/Abstract Interpretation

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URCS
And now for something (not so completely) different

Program Analysis in Industry

Basic Notions

Illustrating Abstract Interpretation

Postscript
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Postscript
So far

- Data flow analysis
  - Iterative data flow analysis
  - Region-based analysis
- Loop analysis
- Type Checking
- What next?
Compilers are not the only program analyzers

- Compilers are probably the most used program analyzers
- But are severely time constrained
- Finding *program* errors is not primary goal
  - Syntax errors, type errors
  - Code generation primary goal
Software is increasingly mission-critical

- Can kill people!
  - Boeing 737 MAX(?)
  - Therac-25 (X-ray)
  - Industrial Robotics

- (less extreme?) Can lose money
  - Software crashes
  - Data loss

- Can we analyze programs for functional correctness?
  - Topic of the next few lectures
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MS isolated most crashes to buggy drivers

Static Driver Verifier project
  - Would verify driver code (in C) for correctness

Used model checking
  - Models programs as finite-state machines
  - I used a similar tool (CBMC) to check your assignments
**Infer (Facebook)**

- Checks C, C++, Objective C, Java and Android code
- Used for checking Facebook’s mobile apps
- Open source, [https://fbinfer.com/](https://fbinfer.com/)
  - Used by Amazon, Mozilla, Uber and Facebook and its affiliates, JD.com, etc.
- Comes with its own language AL (OCaml-derivative?) to describe analyses
  - Analyzes programs in SIL (“Smallfoot Intermediate Language”)
- Uses abstract interpretation + *separation logic*
  - Abstract interpretation very similar to data flow analysis frameworks
- CACM Article: Scaling Static Analyses at Facebook
- Good video: Getting the most out of static analyzers
• Language-independent analyzer
  • a C++ framework
• Open source,
  https://code.fb.com/open-source/sparta/
• Used in FB’s RedEx tools
  • for analyzing Android binary code (.dex)
• Also uses abstract interpretation
Other efforts

- Stanford Checker
  - commercialized by Coverity, late 2000s
  - CACM article, “A few billion lines of code later: using static analysis to find bugs in the real world”

- Google’s static analysis tools
  - Checker Framework for Java programs
  - Shipshape (abandoned?) (Google Tricorder)
  - CACM article, “Lessons from Building Static Analysis Tools At Google”

- Oracle’s Soufflé
  - Soufflé: Logic Defined Static Analysis
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Limitations

- None of these frameworks and tools can escape the fact that analysis is an undecidable problem.
- All compute approximations
  - Or risk ending up intractable
- Must be designed to be sound
  - Approximations are conservative/safe
- Leads to imprecision (i.e. incomplete)
  - May model behaviour not in original programs
  - (recall IDEAL vs MOP vs MFP)
  - leads to false positives
States and Transitions

- A program’s state is a mapping of variables to values
- Programs move from one state to another
  - begin execution in subset of (initial) states
- Notions of state **before** a program point (i.e. a statement) and **after** a program point
  - Also sometimes known as *pre-condition* and *post-condition* respectively.
- Relation that maps before-states to after-states is called a *transition* relation \((t)\)
  - \(\langle x, y \rangle \) (\(x\) is before-state, \(y\) is after-state)
An execution trace of a program is a sequence of states

\[ s_0 s_1 s_2 \ldots s_n \]

An execution trace may be finite or infinite

\[ s_0 s_1 s_2 \ldots \]

The collection of partial traces can actually happen (i.e. state transitions obey the transition relation) is called the collecting semantics

\[ \text{l.e. for all } s_i s_j \text{ in trace, } \langle s_i, s_j \rangle \in t \]
Example

```
x = 0
while(x < 100)
x = x + 1;
```

- states are $\mathbb{Z}$
- initial state is 0
- transition relation is $\{\langle x, x'\rangle | x < 100 \land x' = x + 1\}$
- is 0 1 2 3 part of the collecting semantics?
- is 0 2 4 6 part of the collecting semantics?

Patrick Cousot and Radhia Cousot, *Basic Concepts of Abstract Interpretation*
What can we do with the concrete semantics?

- By examining the (concrete) collecting semantics, we can check various “properties”
  - We’ll formalize “property” later.
- Problems with using the concrete semantics:
  - The previous example had a single state in initial set \{0\}, this is not always true. Consider \(x = \text{randint()}\)
  - How do we deal with infinite loops?
  - How do we deal with alternate paths (i.e. conditionals)?
  - How do we get the concrete semantics for all programs statically?
- Implementation issues:
  - Even if we could get a concrete semantics, how large would it be?
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Consider a simple language with the following constructs:

- **init(R)** where \( R \) is a region in 2D space
  - e.g. \( \text{init}(\{(x, y) | (x, y) \in R\}) \)
  - This chooses a single point in \( R \) non-deterministically
  - A program must always start with init

- **translate(dx, dy)** moves the point by \( dx \) in the X-direction, and \( dy \) in the Y-direction
  - e.g. \( \text{translate}(1.0, 0.5) \) moves the point to the right and up in the Cartesian plane

- **rotate(angle)** rotates the point by \( angle \) about the origin
  - e.g. \( \text{rotate}(90) \) will move a point on the X-axis to a point on the Y-axis

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\(^1\)This exposition is based on Chapter 2 of Rival and Yi (see Postscript)
A program begins with `init` and is followed by statements in `Rest`.

The `or` construct is a non-deterministic choice:
- It executes the block (delimited by braces) on the right or the block on the left.
- Simulates a conditional.

The `iter` either executes the code inside the block or moves to the next statement:
- Simulates a loop.
- Note, `iter` can execute the block forever!
An Example Program

```plaintext
init([0, 1]x[0, 1]);
  translate(1, 0);
  iter {
    translate(1, 0);
  } or {
    rotate(90);
  }
```

Can $x$ ever become negative?

- Can be represented by the set $x_{neg} = \{(x, y) | x < 0\}$
- Problems with concrete execution approach:
  - Infinite initial states ($R = \{(x, y) | 0 \leq x \leq 1 \land 0 \leq y \leq 1\}$
  - Conditionals that perform translation or rotation
  - Loop may be infinite
- But if we could obtain the set of states in all possible concrete executions, say $x_{conc}$, we need to show
  - $x_{conc} \cap x_{neg} = \emptyset$
Abstract interpretation is a framework for performing program analysis.

Key ideas:
- Abstract domain: Set of properties we’re interested in.
- Abstraction function: Converts a concrete state to an element of the abstract domain.
- Transfer functions: Transforms an abstract state before a statement to an abstract state after the statement.
- Union/Join: Combines abstract states from alternate paths.
- Widen ($\nabla$): Combines abstract state across loop iterations.
Abstract Domains

- For the property we’re interested in, we only need the sign of $x$
- Potential abstract domain, $\text{signs-}x$
  - Only tracks $x < 0$, $x \in \mathbb{R}$
  - Can only answer questions about this property, and about $x$
- Another abstract domain, track signs of all state variables
  - Tracks $\{x < 0, x \geq 0, x \in \mathbb{R}\} \times \{y < 0, y \geq 0, y \in \mathbb{R}\}$
- Abstract domain for the rest of the lecture: Intervals
  - For each state variable, track $l_v$ and $h_v$ such that $l_v \leq v \leq h_v$
  - For the graphical language, $l_x \leq x \leq h_x$, $l_y \leq y \leq h_y$
  - Thus, our abstraction approximates states using rectangles
  - The sides of the rectangle are parallel to the axes
  - $l_v$ and $h_v$ can be $-\infty$ and $\infty$ respectively to represent unbounded “rectangles”
Abstract Execution Using Intervals: \textit{init}

\texttt{init([0, 1]x[0, 1])}

Concrete execution will give us a point in that rectangular region.

- Our abstract state after \textit{init} will be:
  - \( l_x = 0 \)
  - \( h_x = 1 \)
  - \( l_y = 0 \)
  - \( h_y = 1 \)

- The transfer function for \textit{init} computes the rectangle that covers the region

- In this case, the abstraction is precise
translate(1.0, 0.5)

- Our abstract state after translate will be:
  - $l_x = l_x + 1.0 = 0.5$
  - $h_x = h_x + 1.0 = 2$
  - $l_y = l_y + 0.5 = 0.5$
  - $h_y = h_y + 0.5 = 1.5$

- The transfer function for translate shifts the current abstract state
  - The resulting abstract state is still precise
rotate(45)

- Our rectangle after rotate will be:
  - \( l_{x_1}, l_{y_1} = \text{rotate}((l_x, l_y), 45) \)
  - \( h_{x_1}, l_{y_2} = \text{rotate}((h_x, l_y), 45) \)
  - \( l_{x_2}, h_{y_1} = \text{rotate}((l_x, h_y), 45) \)
  - \( h_{x_2}, h_{y_2} = \text{rotate}((h_x, h_y), 45) \)

- The transfer function for rotate rotates the corners of the rectangle
  - The result is still a rectangle, but cannot always be represented using intervals
  - So it is not an abstract state
Finding a new interval in the abstract domain

- Let $c'$ be a co-ordinate after rotate computed as in the previous slide, then
  - $l_x = \min(l_{x1}, l_{x2}, h_{x1}, h_{x2})$
  - $h_x = \max(l_{x1}, l_{x2}, h_{x1}, h_{x2})$
  - $l_y = \min(l_{y1}, l_{y2}, h_{y1}, h_{y2})$
  - $h_y = \max(l_{y1}, l_{y2}, h_{y1}, h_{y2})$

- Obviously, in general, this new interval contains more states than the rotated rectangle
  - We have lost precision

- But, the new interval/rectangle we have calculated is the “best fit”
A note on domains

- Intervals (and signs) are non-relational domains
  - They can’t capture relations between $x$ and $y$
  - e.g., a property that $x > y$
- Intervals also can’t capture complex regions
- A more complicated abstract domain: convex polyhedra
  - A list of linear inequalities
  - Region is the feasible region (i.e. points that satisfy all the inequalities)
  - Convex polyhedra support relational properties, e.g. $x - y < 2$
- During program analysis you must choose the domain that most efficiently captures the property of interest
Handling Compound Statements

- Let the transfer function be called \texttt{analysis(stmt, pre-condition)}
- So far, we’ve defined:
  - \texttt{analysis(init, I)} (where \( I \) is the state on entry)
  - \texttt{analysis(translate, a)} where \( a \) is the state before translate
  - \texttt{analysis(rotate, a)} where \( a \) is the state before rotate
- Let’s now define \texttt{analysis(p1 ; p2, a)}
  - The analysis of two statements \( p_1 \) and \( p_2 \), with pre-condition \( a \)
  - Note that pre-condition for \( p_2 \) is the post-condition of \( p_1 \), so
  - \texttt{analysis(p1 ; p2, a)} is then \texttt{analysis(p2, analysis(p1, a))}
Handling conditionals: Union (Join)

{\text{translate}(1.0, 0)} \text{ or } \{\text{translate}(0, 1.0)}

- Program could execute either left part or right part
- Two possible abstract states, disjoint
- Resulting abstract state must incorporate both
  - \text{Union} the two abstract states
- analysis(p1 \text{ or } p2, a) is:
  - analysis(p1, a) \text{ } \cup \text{ } \text{analysis}(p2, a)
- The resulting abstract state is also, in general, imprecise
Union for Intervals

- For $a_1 \cup a_2$
  - $l_x = \min(l_{x1}, l_{x2})$
  - $h_x = \max(h_{x1}, h_{x2})$
  - $l_y = \min(l_{y1}, l_{y2})$
  - $h_y = \max(h_{y1}, h_{y2})$
Handling Loops

iter {b}

can be written as:

{}, # loop executes 0 times
{} or {b}, # loop executes 1 time
{} or {b} or {b;b}, # loop executes 2 times
{} or {b} or {b;b} or {b;b;b}, # loop executes 3 times
...

This can be written (recursively) as: $p_{k+1} = p_k \text{or}\{p_k; b\}$, where:

- $p_0$ is {}
- $p_1$ is {} or {b}
- $p_2$ is {} or {b} or {b; b} and so on...
Iterating

analysis(iter \{b\}, a)

can be defined as an interative algorithm:

\[
R = a \\
\text{do} \\
\quad T = R \\
\quad R = \text{union}(R, \text{analysis}(b, R)) \quad \# \text{analysis for or} \\
\text{while } R \neq T
\]

Will this always terminate?
Example

\begin{verbatim}
init([0, 1] x [0, 1])
translate(1, 0)
iter { translate(0, 1) }
\end{verbatim}

- The analysis of this code will not terminate!
- Abstract state before iter
  - \(1 \leq x \leq 2, 0 \leq y \leq 1\)
- After first union:
  - \(1 \leq x \leq 2, 0 \leq y \leq 1 \cup 1 \leq x \leq 2, 1 \leq y \leq 2\)
  - Result: \(1 \leq x \leq 2, 0 \leq y \leq 2\)
- After second union: \(1 \leq x \leq 2, 0 \leq y \leq 3\)
- And so on ...
  - the \(h_y\) bound keeps increasing without bound
The Widen operator

- We note that the interval $0 \leq y \leq \infty$ would overapproximate $0 \leq y \leq n$ where $n$ is not $\infty$.
- If we obtained this interval in our abstract state, we could terminate because $h_y$ would “stop increasing”.
  - $0 \leq y \leq \infty$ already includes all possible abstract states of the form $0 \leq y \leq n$.
  - Union would no longer return a different result ensuring termination.
- So widen ($\triangledown$) is an operator that overapproximates unions.
  - Its primary purpose is to ensure convergence.
Analyzing loops using widen

\begin{verbatim}
R = a
do
   T = R
   R = widen(R, analysis(b, R))
until inclusion(R, T)  # i.e. R is included in T
return T
\end{verbatim}
def widen(a, b):
    out = union(a, b)

    if a.lx != b.lx:
        out.lx = -inf

    if a.hx != b.hx:
        out.hx = inf

    if a.ly != b.ly:
        out.ly = -inf

    if a.hy != b.hy:
        out.hy = inf
Putting it all together

- Each time analysis is applied, we obtain an abstract state $a$
- If $a$ overlaps with $x_{neg}$, then we have detected a violation of our property
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This lecture follows the exposition in Chapter 2 of the book “An Introduction to Static Analysis: An Abstract Interpretation Perspective” by Xavier Rival and Kwangkeun Yi, MIT Press, 2020

- The library has print copies
- Suggest buying this book – it is self-contained and reasonably priced

- Prof. Cousot has a number of tutorials, I’ll post links to them in the next lecture
  
  - Prof. Patrick Cousot and Prof. Radhia Cousot invented abstract interpretation