CSC2/455 Software Analysis and Improvement
Type Inference - II

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URCS
Type Systems for Realistic Languages

Type Checking ChocoPy

Postscript
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Postscript
The type system we studied in the last class was for a functional language

- Programs consist of expressions
- All well-formed expressions have a type

Some languages, notably imperative languages (Python, C, etc.), differentiate between *statements* and *expressions*

- Expressions have types
- Statements do not have types
- “Procedures” do not have return values
Additional judgements for imperative languages

- $\Gamma \vdash C$
  - $C$ is a well-formed command (or statement) in $\Gamma$

In general, block-structured languages may have $C$ be a block as well.
Subtyping

- Most languages have a subtype relation
  - C has \textit{unsigned short} is a subtype of \textit{unsigned int}
  - Subtypes can be used wherever the "supertype" is used
- Usually denoted by $A \leq B$
  - $A$ is a subtype of $B$
  - $A \leq A$, obviously
- Type hierarchy, a tree of types
  - Usually \texttt{object} as the root
  - A edge from $B$ to $A$ if $A < B$
Outline

Type Systems for Realistic Languages

Type Checking ChocoPy

Postscript
ChocoPy is a simpler version of Python3
  - Developed as a “toy” language for compiler courses
• Fully-specified
  - Syntax (BNF)
  - Type System
  - Semantics (Operational Semantics)
• We will only look at its Type System
ChocoPy Types

- Basic types: object, int, bool, str
- List type: [T], where T is another type
  - e.g. [int]
  - T be a list as well, this is an example of a recursive type
  - [[]

- Special types
  - <None>
  - <Empty>, indicating an empty list []
ChocoPy Type Hierarchy

- $A \leq A$, for all types $A$
- $C \leq A$ if $C$ is a subclass of $A$
  - $\text{int, bool, str}$ are all subclasses of $\text{object}$
- $[T] \leq \text{object}$
  - but no relationship between two different list types
- $<\text{None}> \leq \text{object}$
- $<\text{Empty}> \leq \text{object}$
A type $T_1$ is assignment compatible ($\leq_a$) to $T_2$ iff values of type $T_1$ can be used wherever values of $T_2$ are expected.

- $T_1 \leq T_2$ (note: this is subtyping $\leq$)
- $T_1$ cannot be $<$None$>$ if $T_2$ is int, bool or str
- $T_1$ can be $<$Empty$>$ only if $T_2$ is $[$T$]$  
- $T_1$ can be $[$None$]$ if $T_2$ is $[$T$]$, where $<$None$> \leq_a T$
  - Note that means T can only be object (*)
The join $\sqcup$ of two types $A$ and $B$ denoted as $A \sqcup B$ is defined as:

- $B$ if $A \leq_a B$
  - This is commutative, so $B \sqcup A$ is also $B$ in this case
- $C$ where $C$ is the least common ancestor of $A$ and $B$ in the type hierarchy defined by $\leq$
  - So $\text{int} \sqcup \text{str}$ is $\text{object}$
  - Note this is the *least upper bound* or LUB (analogous to GLB in meet lattices)
Judgements of the form:

- $O, M, C, R \vdash \ldots$

Where the type environment consists of:

- $O$, “local” environment
- $M$, method/attribute environment, i.e. for A.x-style code
- Both are maps

And:

- $C$, the name of the current class, can be $\bot$ if outside class
- $R$, the return type of the current function, can be $\bot$ if outside function
def contains(items:[int], x:int) -> bool:
    i:int = 0
    while i < len(items):
        if items[i] == x:
            return True
        i = i + 1
    return False

The general type for a function \( f \) is 
\[
O(f) = \{ T_1 \times T_2 \times \ldots \times T_n \rightarrow T_0; x_1, x_2, \ldots, x_n; v_1 : T'_1, v_2 : T'_2, \ldots, v'_m : T'_m \}
\]

- \( T_1 \times T_2 \times \ldots \times T_n \rightarrow T_0 \) is the type of arguments and the return type
  - Here, \([\text{int}] \times \text{int} \rightarrow \text{bool}\)
- \( x_1, x_2, \ldots, x_n \) are names of the arguments: \( \text{items}, x \)
- \( v_1 : T'_1, \ldots, v'_m : T'_m \) are types of variables/functions declared inside the function
  - Here \( i : \text{int} \)
class Point1D:
    x: int = 0

    # the quotes are used since Point1D is being defined ...
def distance(self: "Point1D", o: "Point1D") -> int:
    ...

- Attributes are simply $M(C, a) = T$, where $C$ is the class, $a$ is the attribute, and $T$ is its type
  - So, $M(\text{Point1D}, x) = \text{int}$
- And methods $M(C, m)$ are just like functions in the previous slide
  - So,
    
```plaintext
    M(\text{Point1D}, \text{distance}) = \{ \text{Point1D} \times \text{Point1D} \rightarrow \text{int}; ... \}
```
Type Judgements in ChocoPy

- $O, M, C, R \vdash e : T$
  - Expression $e$ has type $T$ in the environment $O, M, C, R$
- $O, M, C, R \vdash b$
  - $b$ is a well-formed (block) statement in $O, M, C, R$
  - can also be a single statement, obviously
- If reading the ChocoPy reference, make sure to distinguish the colon in type judgements $e : T$ from the colon in Python syntax
  - I’ll use : to indicate the latter, and : to indicate the former
Expressions

\[
\begin{align*}
O, M, C, R & \vdash \text{True} : \text{bool} \\
O, M, C, R & \vdash \text{False} : \text{bool}
\end{align*}
\]

- \text{i is an integer literal} \Rightarrow \frac{}{(\text{INT})} \Rightarrow \frac{}{(\text{INT})} \Rightarrow \frac{}{(\text{INT})}

- \text{s is a string literal} \Rightarrow \frac{}{(\text{STR})} \Rightarrow \frac{}{(\text{STR})} \Rightarrow \frac{}{(\text{STR})}

\[
O(id) = T \quad T \text{ is not a function type} \Rightarrow \frac{}{(\text{VAR-READ})} \Rightarrow \frac{}{(\text{VAR-READ})} \Rightarrow \frac{}{(\text{VAR-READ})}
\]

\[
O, M, C, R \vdash id : T
\]
Variable Definitions

\[ O(id) = T \quad O, M, C, R \vdash e_1 : T_1 \quad T_1 \leq_a T \]

\[ O, M, C, R \vdash id : T = e_1 \quad (\text{VAR-INIT}) \]
\[(O, M, C, R \vdash s_1) \quad (O, M, C, R \vdash s_2) \quad \ldots \quad (O, M, C, R \vdash s_n)\]

\[\Rightarrow (O, M, C, R \vdash s_1 \ \text{NEWLINE} \ s_2 \ldots s_n \ \text{NEWLINE})\]
Operators

\[ O, M, C, R \vdash e_1 : int \]
\[ O, M, C, R \vdash e_2 : int \]
\[ op \in \{+, -, *, //, \%\} \]
\[ \frac{}{O, M, C, R \vdash e_1 \ op \ e_2 : int} \] (ARITH)

\[ O, M, C, R \vdash e_1 : int \]
\[ O, M, C, R \vdash e_2 : int \]
\[ \&\& \in \{<, <=, >, >=, ==, !=\} \]
\[ \frac{}{O, M, C, R \vdash e_1 \ \&\& \ e_2 : bool} \] (COMPARE-INT)

\[ O, M, C, R \vdash e_0 : bool \]
\[ O, M, C, R \vdash e_1 : T_1 \]
\[ O, M, C, R \vdash e_2 : T_2 \]
\[ \frac{}{O, M, C, R \vdash e_1 \ if \ e_0 \ else \ e_2 : T_1 \sqcup T_2} \] (IF-EXPR)
\[
\frac{O, M, C, R \vdash e : T \quad T \leq_a R}{O, M, C, R \vdash \text{return } e} \quad \text{(RETURN-E)}
\]

\[
\frac{O, M, C, R \vdash b}{O, M, \bot, R \vdash \text{class } C(S) : b} \quad \text{(CLASS-DEF)}
\]
x: str = "hello, world"

def somefn(x: int, y: str):
    # what is the type of x here?

- Function argument names:
  - shadow names outside the function (like x here)
  - or are introduced in the definition (like y)
- The notation for shadowing or introducing a variable is:
  - $O[T/x]$ which is defined as $O[T/x](x) = T$ and $O[T/x](y) = O(y)$ where $y \neq x$
  - i.e., $O[T/x]$ is a new map with $x$ mapped to $T$ and the other mappings unchanged
  - repeated applications are possible: $O[T_1/x][T_2/y]$
\[ T = \begin{cases} 
T_0, & \text{if } \rightarrow \text{ is present,} \\
\langle \text{None} \rangle, & \text{otherwise.}
\end{cases} \]

\[ O(f) = \{T_1 \times \cdots \times T_n \rightarrow T; \ x_1, \ldots, x_n; \ v_1 : T'_1, \ldots, v_m : T'_m\} \]

\[ n \geq 0 \quad m \geq 0 \]

\[ O[T_1/x_1] \cdots [T_n/x_n][T'_1/v_1] \cdots [T'_m/v_m], \ M, C, T \vdash b \]

\[ O, M, C, R \vdash \text{def } f(x_1:T_1, \ldots, x_n:T_n) \ [\rightarrow T_0]^?: b \]
Method Definition

\[
T = \begin{cases} 
T_0, & \text{if } \rightarrow \text{ is present,} \\
<\text{None}>, & \text{otherwise.}
\end{cases}
\]

\[
M(C, f) = \{T_1 \times \cdots \times T_n \rightarrow T; x_1, \ldots, x_n; v_1 : T'_1, \ldots, v_m : T'_m\}
\]

\[
n \geq 1 \quad m \geq 0
\]

\[
C = T_1
\]

\[
O[T_1/x_1] \ldots [T_n/x_n][T'_1/v_1] \ldots [T'_m/v_m], M, C, T \vdash b
\]

\[
O, M, C, R \vdash \text{def } f(x_1 : T_1, \ldots, x_n : T_n) \xrightarrow{\rightarrow} T_0 : b
\]
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References

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