

CSC2/458 Parallel and Distributed Systems

Automated Parallelization in Software

(Contd.)

Sreepathi Pai

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URCS

Outline

Characterizing loop dependences

Identifying Loop Dependences

Current Loop Optimizations

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Characterizing loop dependences

Identifying Loop Dependences

Current Loop Optimizations

Why characterize dependences?

- The definition of dependence that we have used so far:
 - Two statements have a dependence if:
 - Both access same location (memory or register)
 - And one of the accesses is a write
- This is not sufficient to reason about dependences in loops
- We will extend this definition of dependences to *loop dependence*
 - Study additional characteristics of dependences

Already encountered characteristics of dependences

- True dependence
 - $S_1 \delta S_2$
 - S_1 writes, S_2 reads
- Anti-dependence
 - $S_1 \delta^{-1} S_2$
 - S_1 reads, S_2 writes
- Output dependence
 - $S_1 \delta^o S_2$
 - Both S_1 and S_2 write

Loop-independent dependence

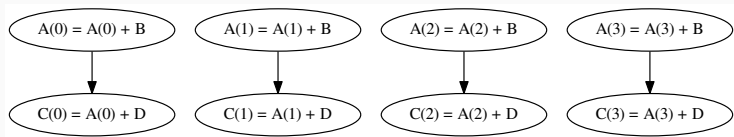
- What are the dependences in the loop body below?
- Can you change the order of the statements in the loop body?

```
DO I = 1, 10
  A(I) = A(I) + B
  C(I) = A(I) + D
ENDDO
```

- Can you change the (execution) order of loop iterations?

Note: FORTRAN uses parentheses in array references: e.g., A(I)

Loop-independent dependences visualized



NOTE: Only dependences from first four iterations visualized.

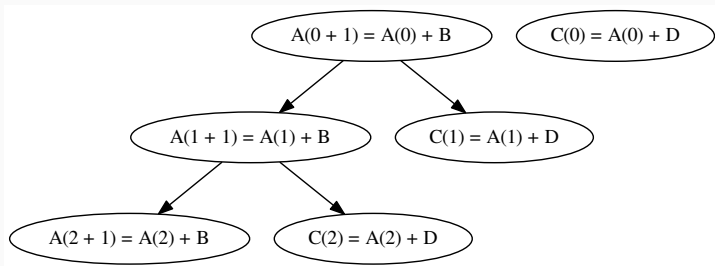
Loop-carried dependences

- What are the dependences in the loop body below?
- Can you change the order of the statements in the loop body?

```
DO I = 1, 10
  A(I + 1) = A(I) + B
  C(I) = A(I) + D
ENDDO
```

- Can you change the (execution) order of loop iterations?

Loop-carried dependences visualized



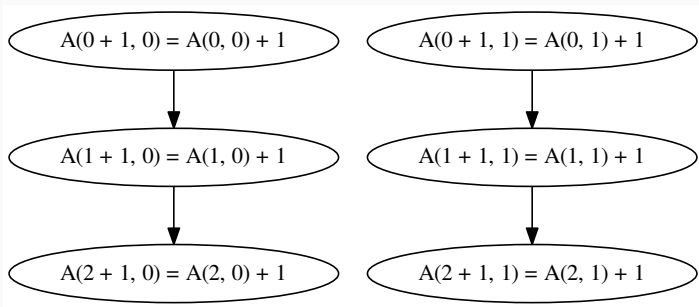
NOTE: Only dependences from first three iterations visualized.

Dependence Level for Loop-Carried Dependences

```
DO I = 1, 10
  DO J = 1, 2
    A(I + 1, J) = A(I, J) + 1
  ENDDO
ENDDO
```

- Can you change the order of inner loop?
- Can you change the order of the outer loop?

Dependencies Visualized



NOTE: Only dependencies from first three iterations visualized.

Loop Dependences

- Loop-independent dependence
 - In same iteration, independent of loops
- Loop-carried dependence
 - Across different iterations of atleast one loop
- Dependence Level of a Loop-carried Dependence
 - The nesting level k of loop that carries the dependence
 - $S_1 \delta_k S_2$

Iteration Spaces

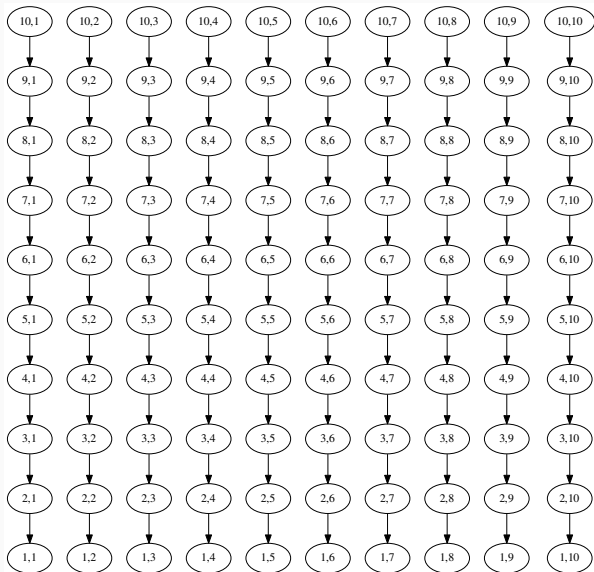
```
DO I = 1, 2
  DO J = 1, 2
    S
  ENDDO
ENDDO
```

- S has four instances (I, J) : $(1, 1)$, $(1, 2)$, $(2, 1)$, $(2, 2)$
- Each of these values represents an *iteration vector*
 - Particular values of loop indices from outermost loop to innermost loop

Iteration Space Example

```
DO J = 1, 10
  DO I = 1, 10
    A(I+1, J) = A(I, J) + X
  ENDDO
ENDDO
```

Iteration Space Figure



Iteration Vector Ordering

For two vectors i and j , each containing n elements, $i < j$ is defined as:

```
def lessthan(i, j, n):
    if n == 1:
        return i[0] < j[0]

    # test prefix for elementwise-equality
    if i[0:n-1] == j[0:n-1]:
        return i[n-1] < j[n-1]
    else:
        return lessthan(i, j, n-1)
```

Can similarly define other order relations.

Loop dependence

Statement $S1$ (source) depends on statement $S2$ (sink) if:

- There exist iteration vectors i and j such that $i < j$ or $i = j$
- There is a path from $S1$ to $S2$ in the loop
- $S1$ accesses memory M in iteration i
- $S2$ accesses memory M in iteration j
- and one of the accesses is a write

Distance Vectors

$$d(i,j)_k = j_k - i_k$$

- Where $i, j, d(i, j)$ are n -element vectors
- i_k indicates k -th element of i

Example distance vector: $(0, 1)$

Direction Vectors

$$D(i, j)_k =$$

- " $<$ ", if $d(i, j)_k > 0$
- " $=$ ", if $d(i, j)_k = 0$
- " $>$ ", if $d(i, j)_k < 0$

Example direction vector for $(0, 1)$: $(=, <)$

Information we need to track

For every pair of memory references:

- Iteration Vectors i and j which have a dependence, or
- Unique Distance Vectors $d(i, j)$, or
- Unique Direction Vectors $D(i, j)$

- Which of these indicates a loop-independent dependence?
 - $(=, =)$
 - $(=, <)$
- Of the loop-carried dependence in example above, what level is the loop-carried dependence?

Theorems

WARNING: Informal language

- Direction Vector Transform (Theorem 2.3 in AK)
 - If a transformation reorders loop iterations, and preserves the leftmost non-"=" component as "<", all dependences are preserved.
- Theorem 2.4 in AK
 - If a level- k dependence exists, and a transformation reorders loop iterations while not reordering the level- k loop
 - And does not move loops inside k outside the loop and vice versa
 - It preserves all level- k dependences.
- Iteration Reordering (Theorem 2.6 in AK)
 - Iterations of a level k loop can be reordered if there is no level k dependence.

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Generalizing Loop Indices

```
DO I_1 = ...
  DO I_2 = ...
    ...
    DO I_N = ...
      A(f1, f2, f3, ..., fM) = ...
      ... = A(g1, g2, g3, ..., gM)
    ENDDO
  ENDDO
ENDDO
```

where A is M -dimensional array, and fX and gX are *index functions* of the form

- $fX(I_1, I_2, \dots, I_N)$
- $gX(I_1, I_2, \dots, I_N)$
- $1 \leq X \leq M$

Dependence using Iteration Vectors

Let α and β be iteration vectors:

- $\alpha = (i_1, i_2, i_3, \dots, i_N)$
- $\beta = (i'_1, i'_2, i'_3, \dots, i'_N)$

Then a dependence exists if:

- (vectors) $\alpha < \beta$
- $fX(\alpha) = gX(\beta)$, for $1 \leq X \leq M$

Example

```
DO J = 1, 10
  DO I = 1, 10
    A(I+1, J) = A(I, J) + X
  ENDDO
ENDDO
```

- $f1(J, I) = I + 1, f2(J, I) = J$
- $g1(J, I) = I, g2(J, I) = J$
- For $\alpha = (0, 0)$ (i.e. $J = 0, I = 0$) and $\beta = (0, 1)$ (i.e. $J = 0, I = 1$):
 - $f1(\alpha) = g1(\beta)$, i.e. $1 = 1$
 - $f2(\alpha) = g2(\beta)$, i.e. $0 = 0$
 - Many other values of α and β also satisfy these equations.

Dependence Testing

Do iteration vectors α and β exist such that:

- (vectors) $\alpha < \beta$
- $fX(\alpha) = gX(\beta)$, for $1 \leq X \leq M$

How can we find α and β if they exist?

Restrictions on Index functions

- fX and gX must be decidable
- fX and gX must be "analyzable"
 - to avoid brute force search
- fX and gX must be a linear functions of loop indices:
 - i.e. for $fX(i_1, i_2, i_3, \dots, i_N)$
 - $fX = a_1i_1 + a_2i_2 + \dots + a_ni_n + e$
 - e is optional loop invariant calculation

Dependence Testing on Restricted Index Functions

- Given that fX and gX are linear functions of loop indices
- Do iteration vectors α and β exist such that:
 - (vectors) $\alpha < \beta$
 - $fX(\alpha) = gX(\beta)$, for $1 \leq X \leq M$

How can we find α and β if they exist?

What is this problem better known as?

Dependence Testing

- Integer Linear Programming is NP-complete
- Lots of heuristics invented
 - Profitable to know if no solution exists since it implies no dependence!
 - See Chapter 3 of AK (we will not cover this in this course, take CSC 2/455)

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Current Focus in Compilers

- GCC begin supporting vectorization for C around 4.9
 - `-ftree-vectorize` or `-O3`
 - Can get it to tell you why vectorization failed.
- LLVM also supports vectorization
 - See "Polly" at <http://polly.llvm.org>

More focus on optimization by loop transformation

- More emphasis on *Scheduling*
 - Which iteration of loop executes where
- Classical loop transformations
 - Loop tiling
 - Loop fusion
 - etc.
- Unifying theory and infrastructure
 - polyhedral.info