

Counting Euler tours?

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Euler tour

- 1) every edge exactly once
- 2) end where started

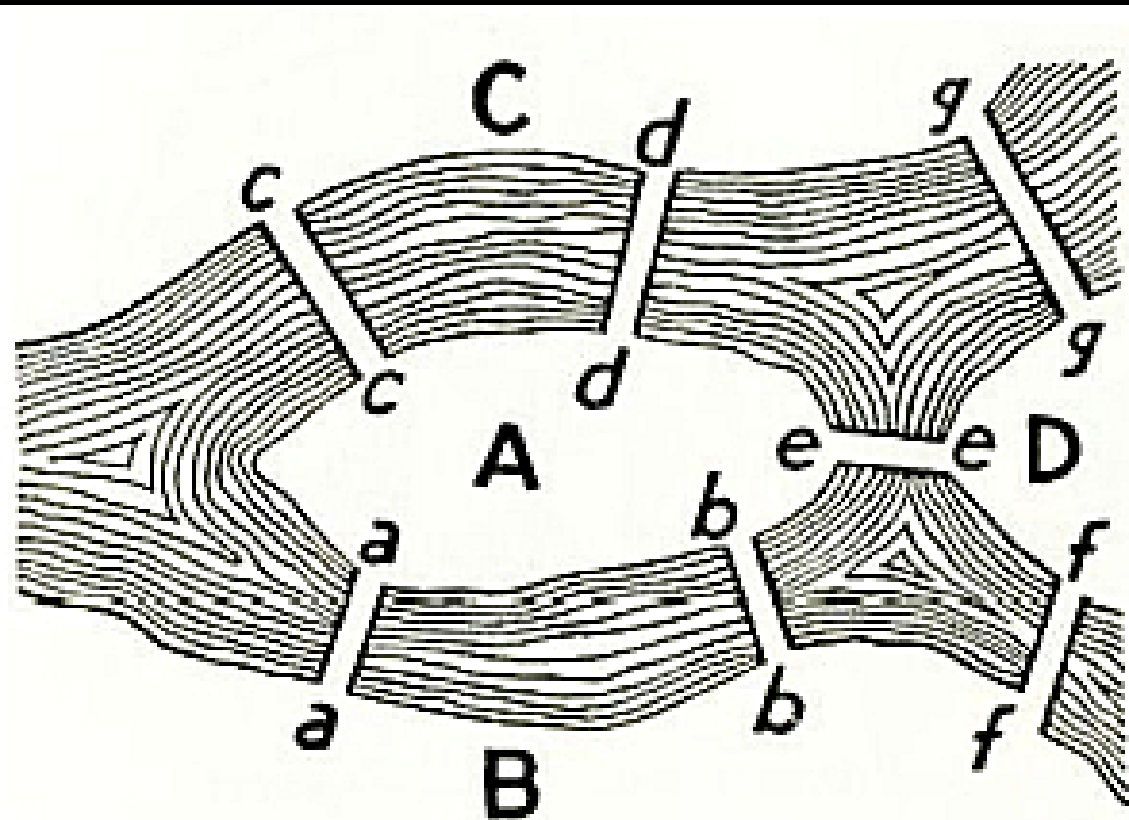
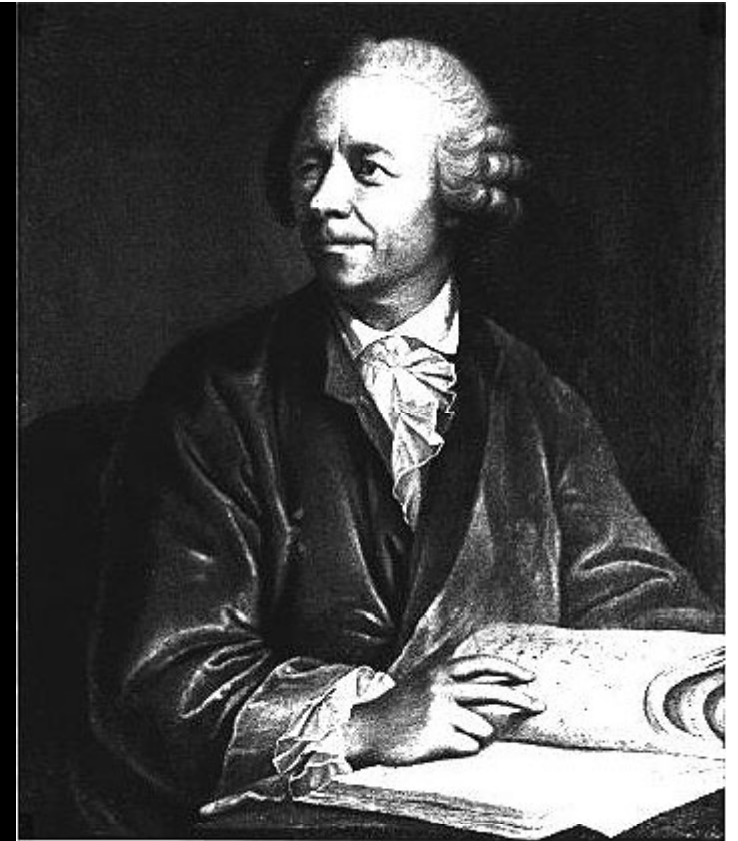


FIGURE 98. *Geographic Map:
The Königsberg Bridges.*



Euler tour

- 1) every edge exactly once
- 2) end where started

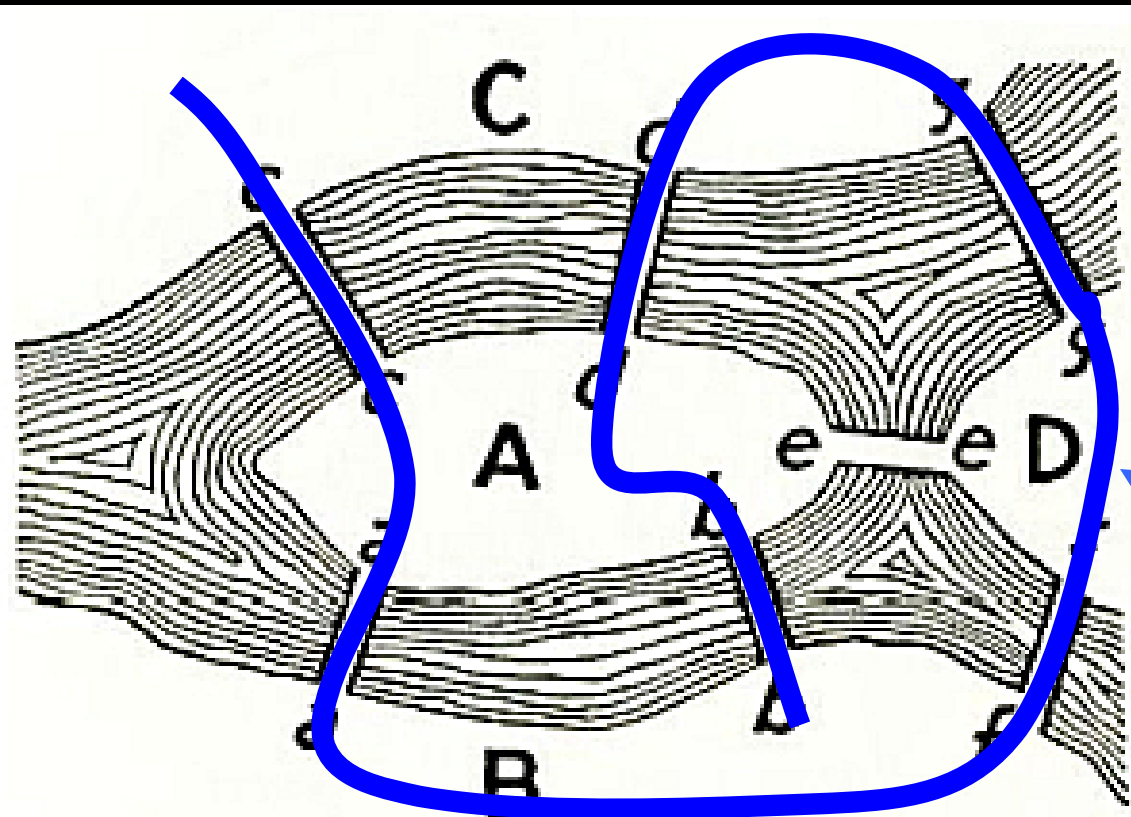
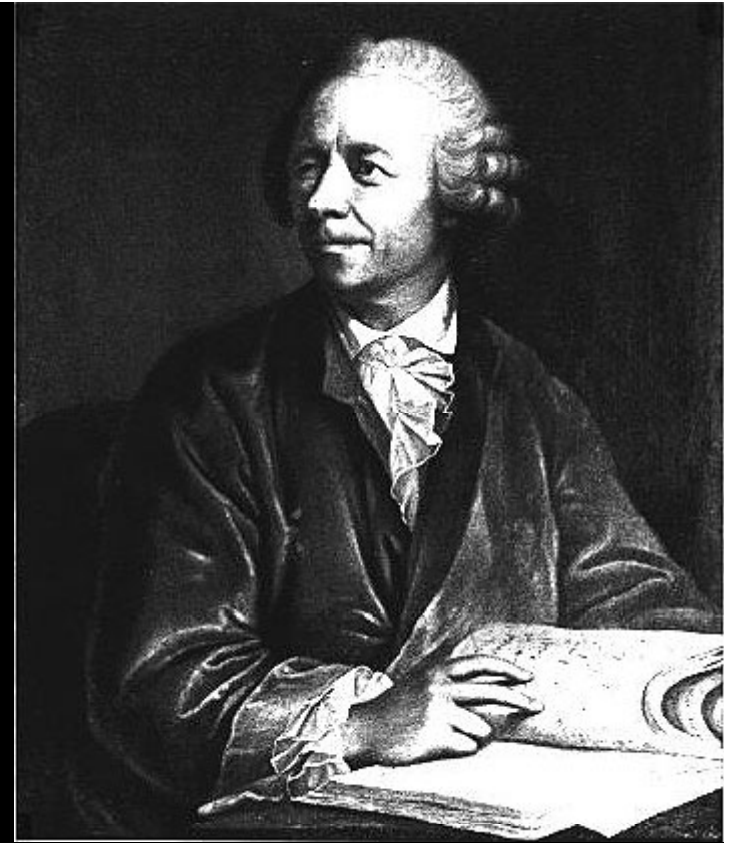


FIGURE 98. *Geographic Map:
The Königsberg Bridges.*



not an Euler tour

Basic facts

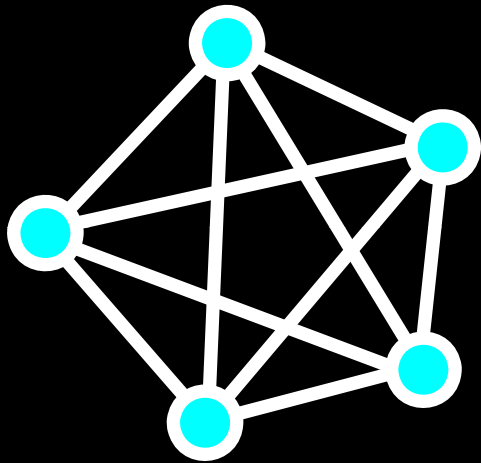
- there exists one if and only if all vertices have even degree (Eulerian graph)
- an Euler tour in an Eulerian graph can be found in linear time.

Can we find a random one?

Can we count their number?

(efficiently)

Can we count their number?



polynomial time
algorithm



264

Can we count their number?

undirected graphs

#P-complete

(Brightwell-Winkler'05)



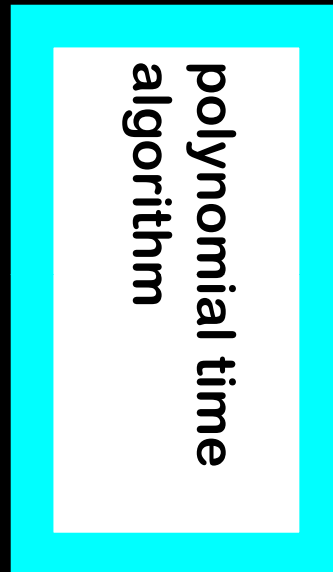
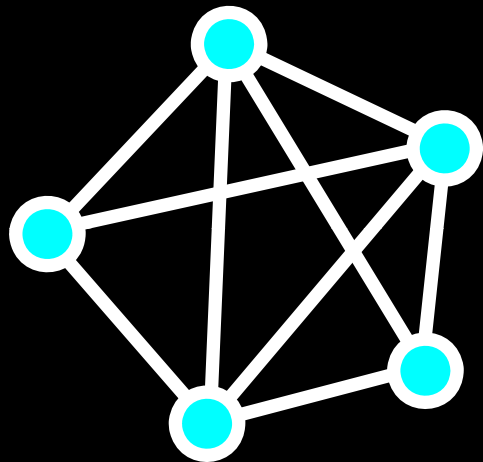
exact counting
in poly-time
unlikely

directed graphs

polynomial-time algorithm known
(using spanning trees)

Can we count their number?

^
approximately



$264 \pm 10\%$

(open question, listed, e.g., in
Approximation algorithms (V.Vazirani))

self-reducible \Rightarrow

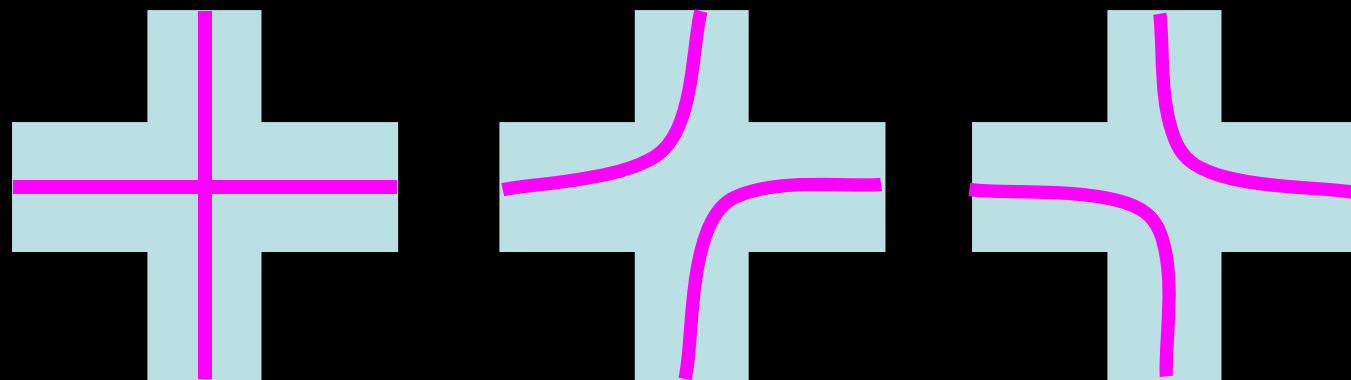
approximate counting \Leftrightarrow approximate sampling
(Jerrum, Valiant, V.Vazirani'86)

Can we find a random one?

4-regular graphs

Markov chain

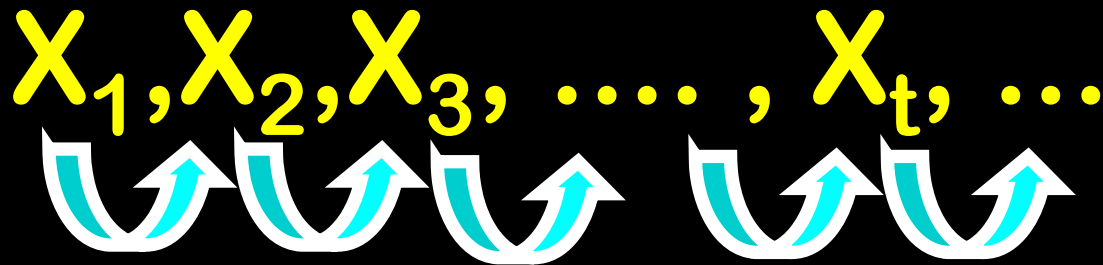
pick a random vertex v
locally change the tour at v



(exactly 2 of these are valid)

Can we find a random one?

Markov chain



OPEN:
is the mixing
time polynomial?

Can we find a random one?

4-regular graphs

Markov chain

pick a random vertex v
locally change the tour at v



(exactly 2 of these are valid)

L_1 distance to uniform
distribution

$$\sum_a \left| P(X_t = a) - \frac{1}{M} \right|$$

mixing time =
 t to get L_1 distance $\leq \epsilon/2$

Can we find a random one?

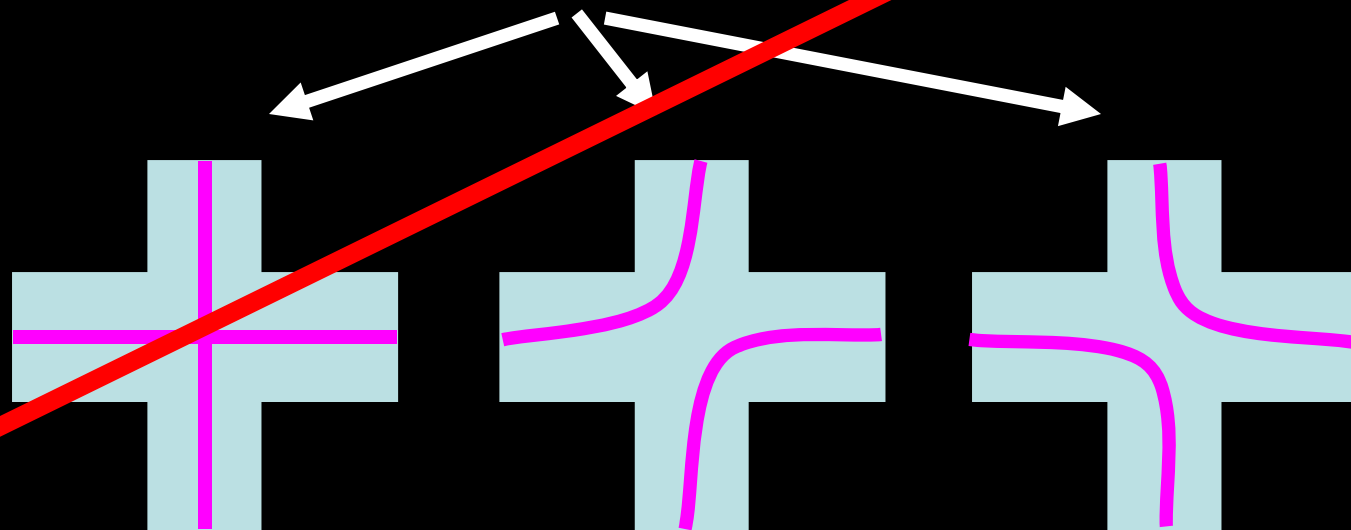
~~4-regular graphs~~
general

?????

Markov chain

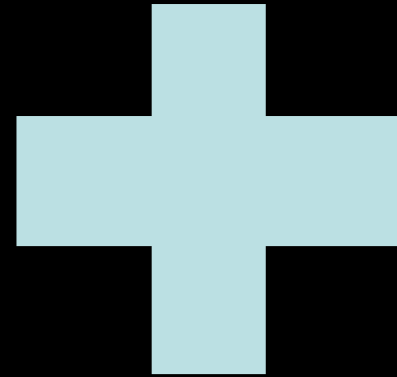
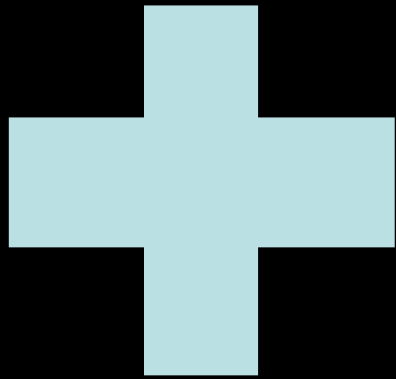
pick a vertex v

locally change the tour at v

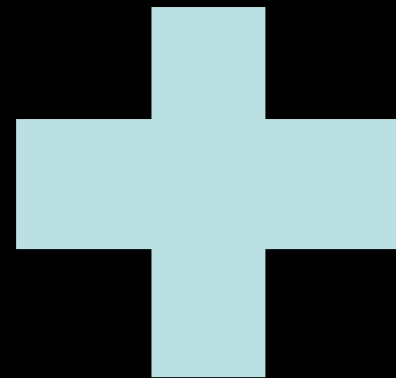
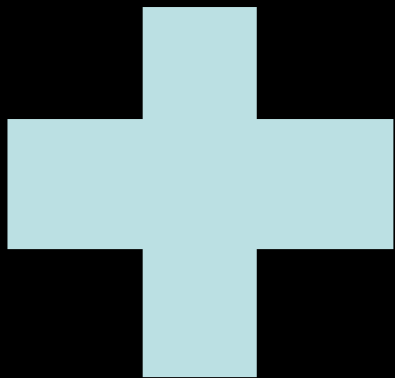


(exactly 2 of these are valid)

Counting A-trails in a map



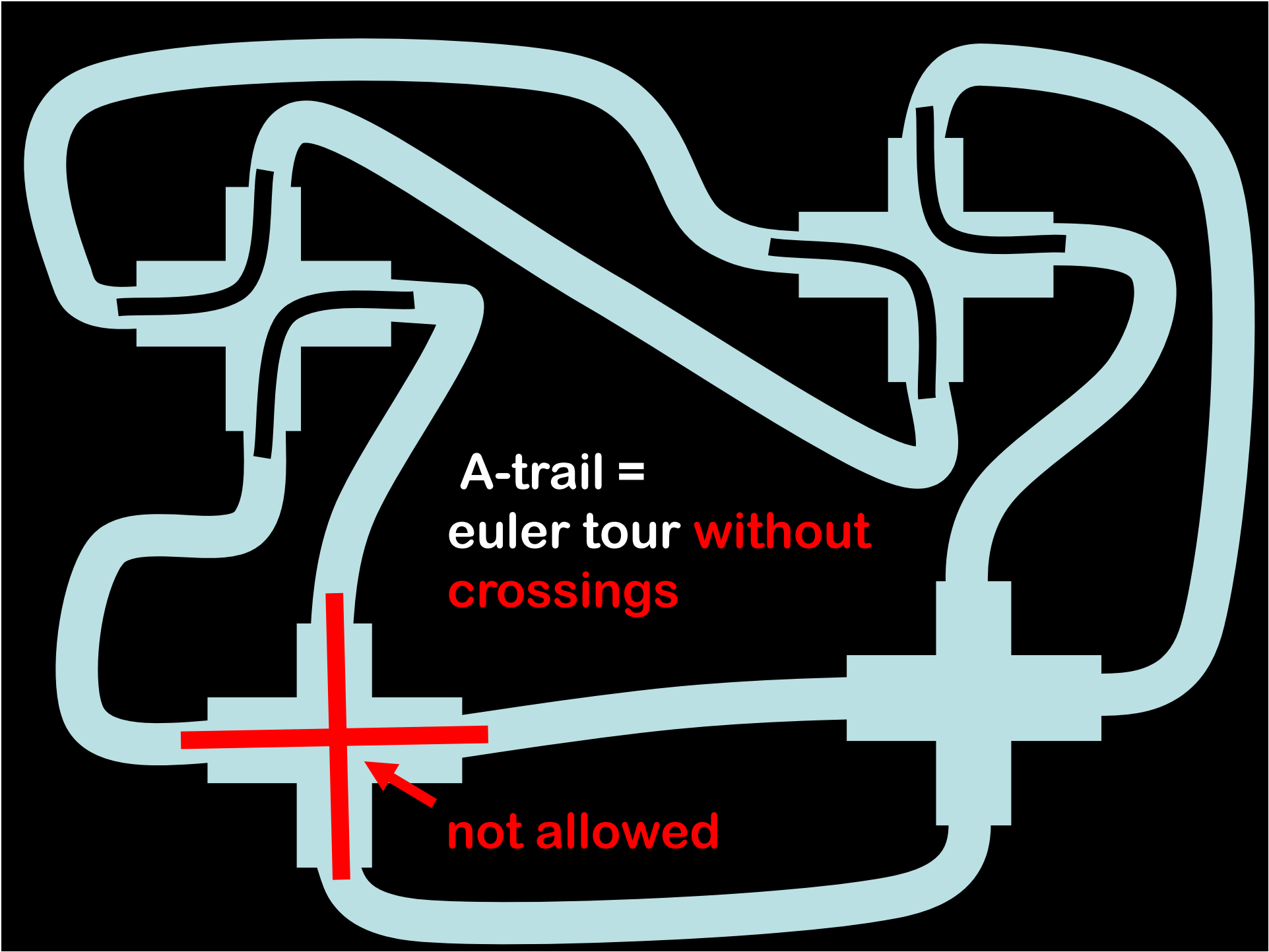
vertices with “rotations”





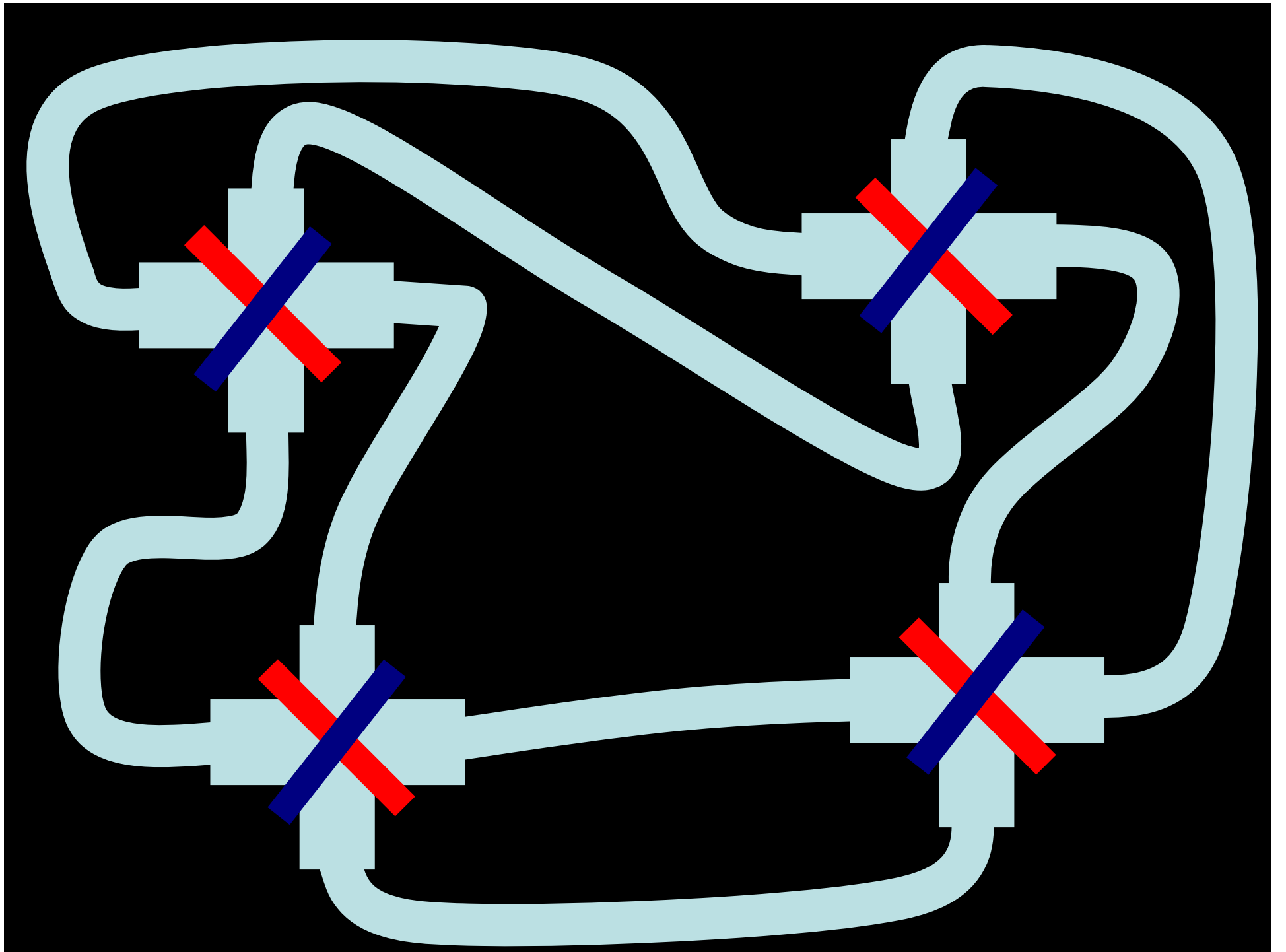
map

**vertices with "rotations"
and a graph**

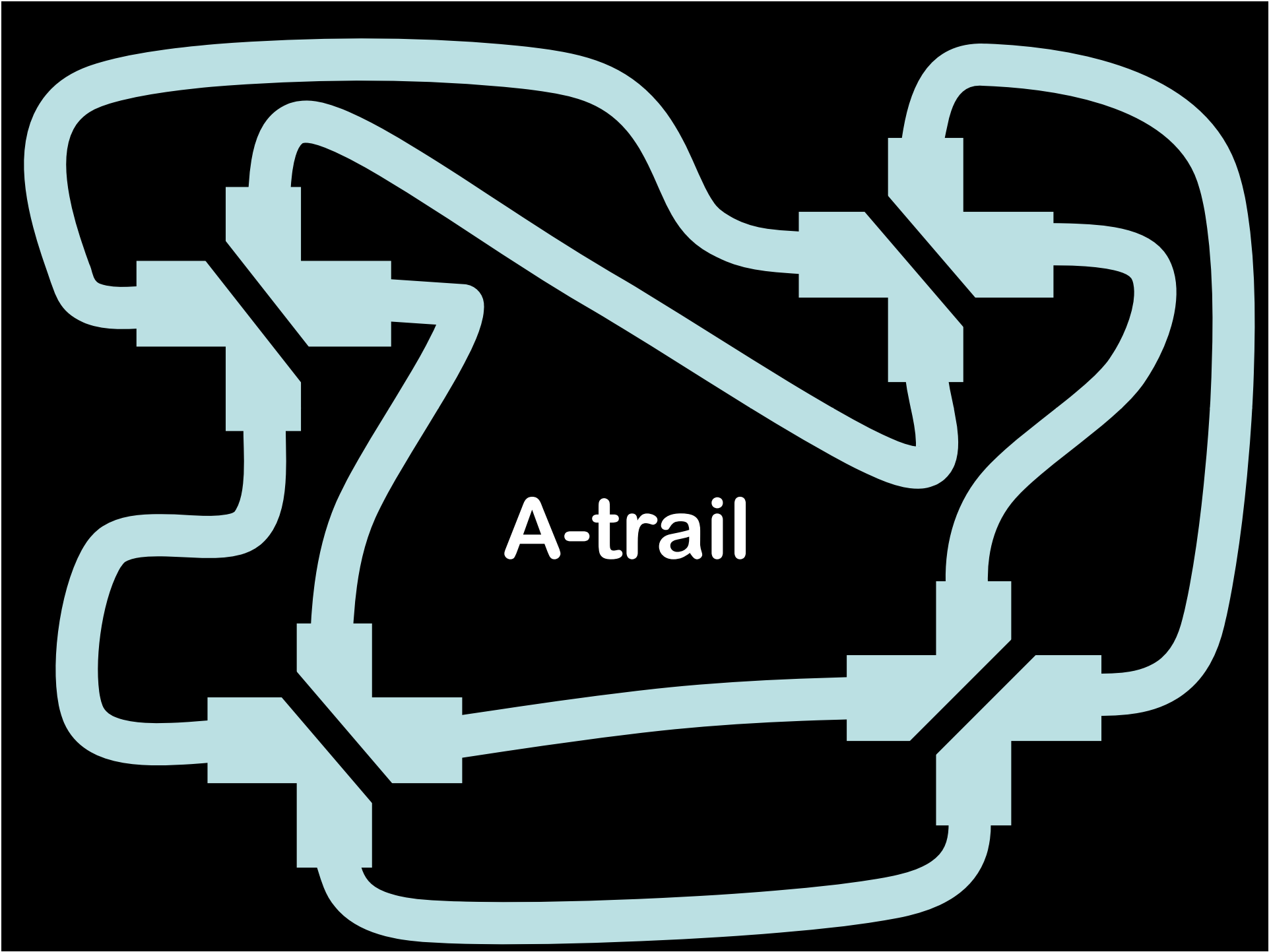


A-trail =
euler tour **without**
crossings

not allowed

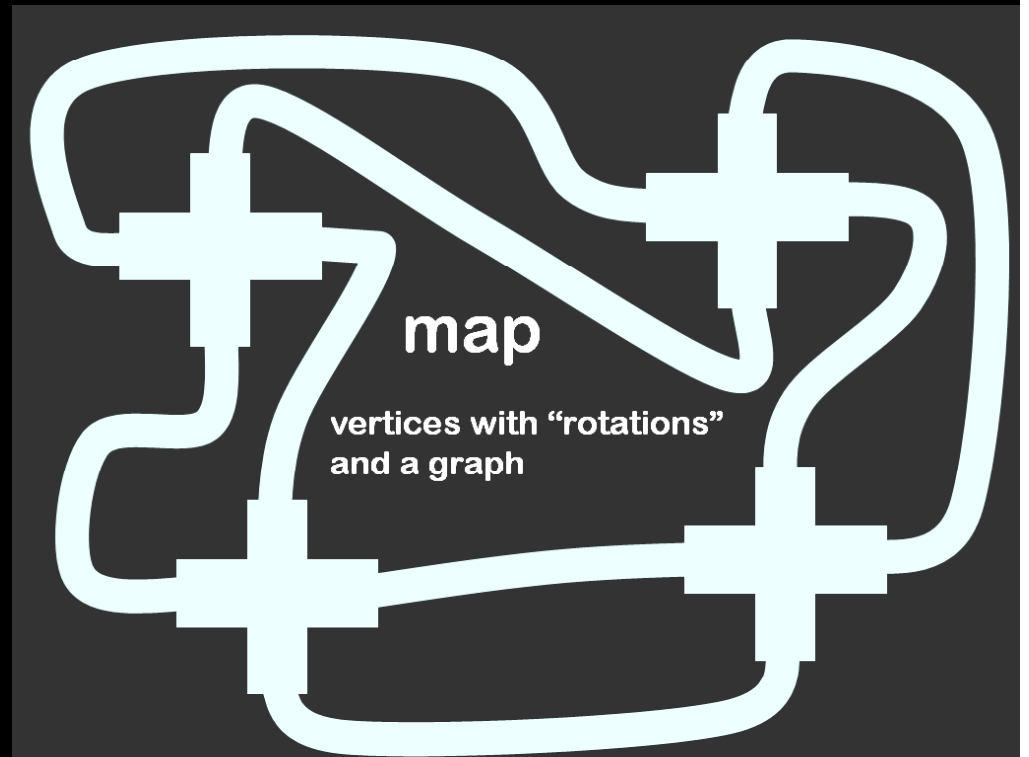


A-trail



Can we (approximately) count their number?

for planar maps
yes (Kotzig'68)



for general maps?

Our results: A-trails in 4-reg enough

approximate sampling/counting
of A-trails in 4-regular maps



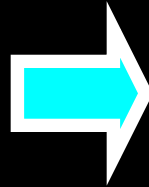
approximate sampling/counting
of Euler tours in Eulerian graphs

(**AP-reduction**

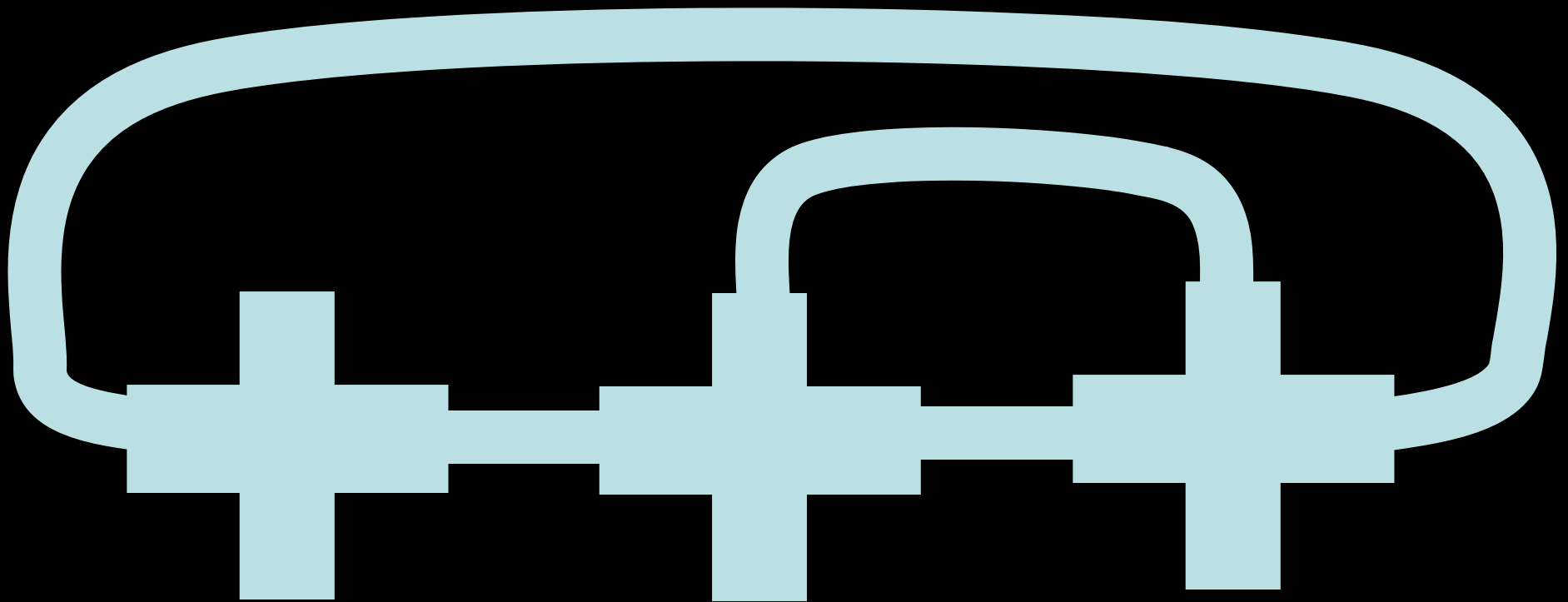
(Goldberg, Dyer, Greenhill, Jerrum'04))

A-trails in 4-reg enough

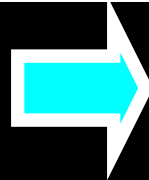
approximate sampling/counting
of A-trails in 4-regular maps



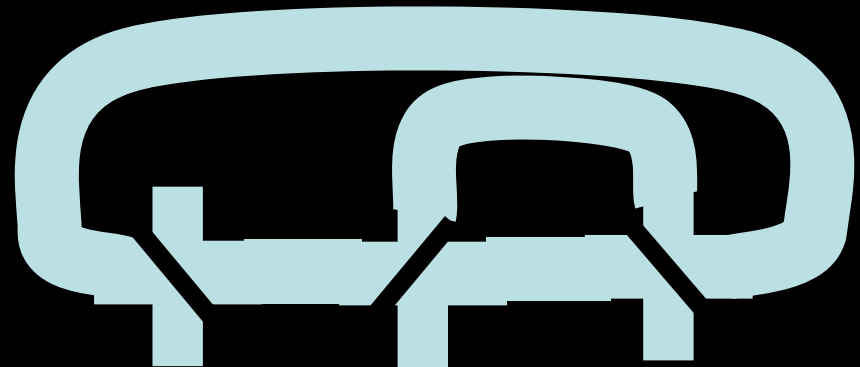
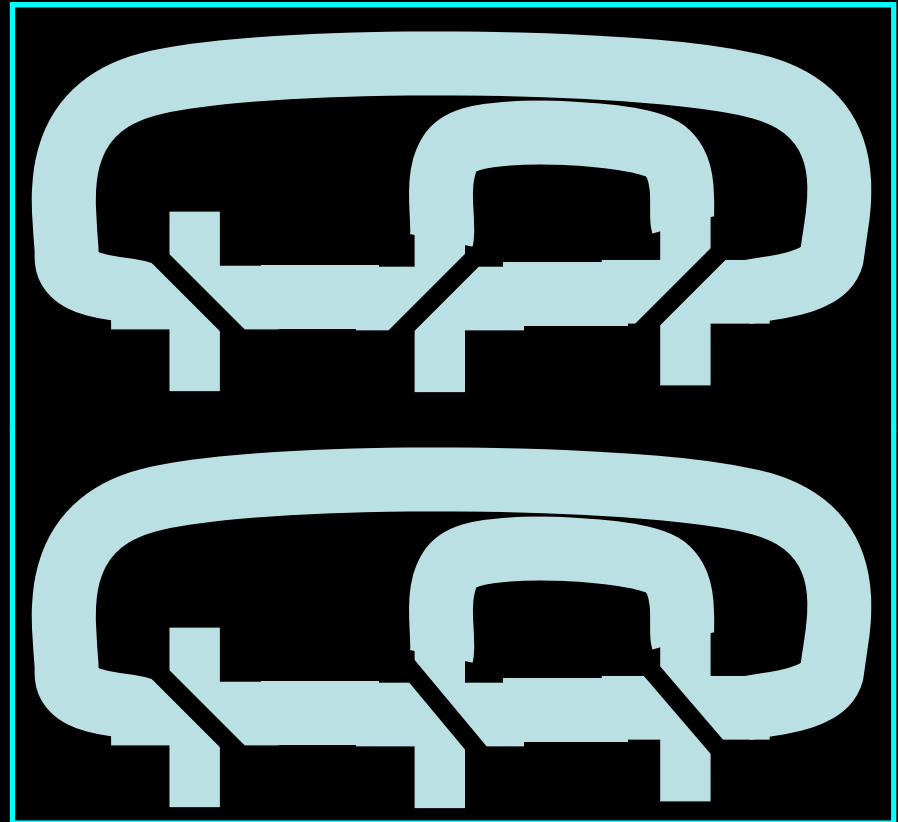
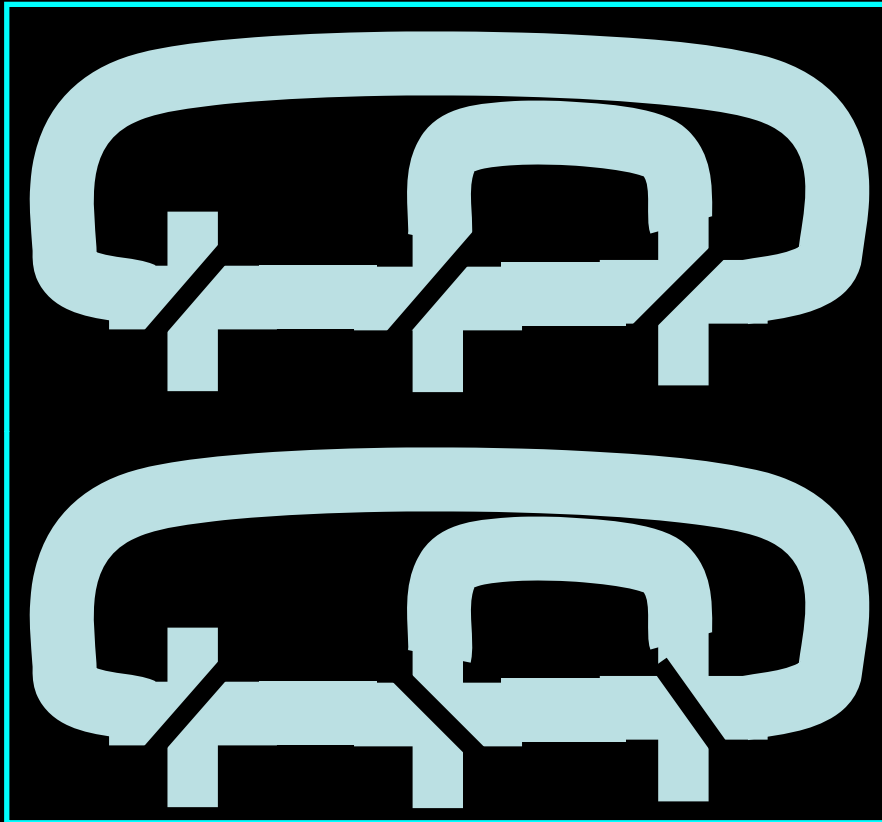
approximate sampling/counting
of Euler tours in 4-regular graphs



approximate sampling/counting
of A-trails in 4-regular maps

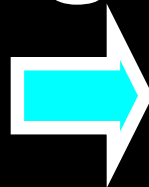


approximate sampling/counting
of Euler tours in 4-regular graphs

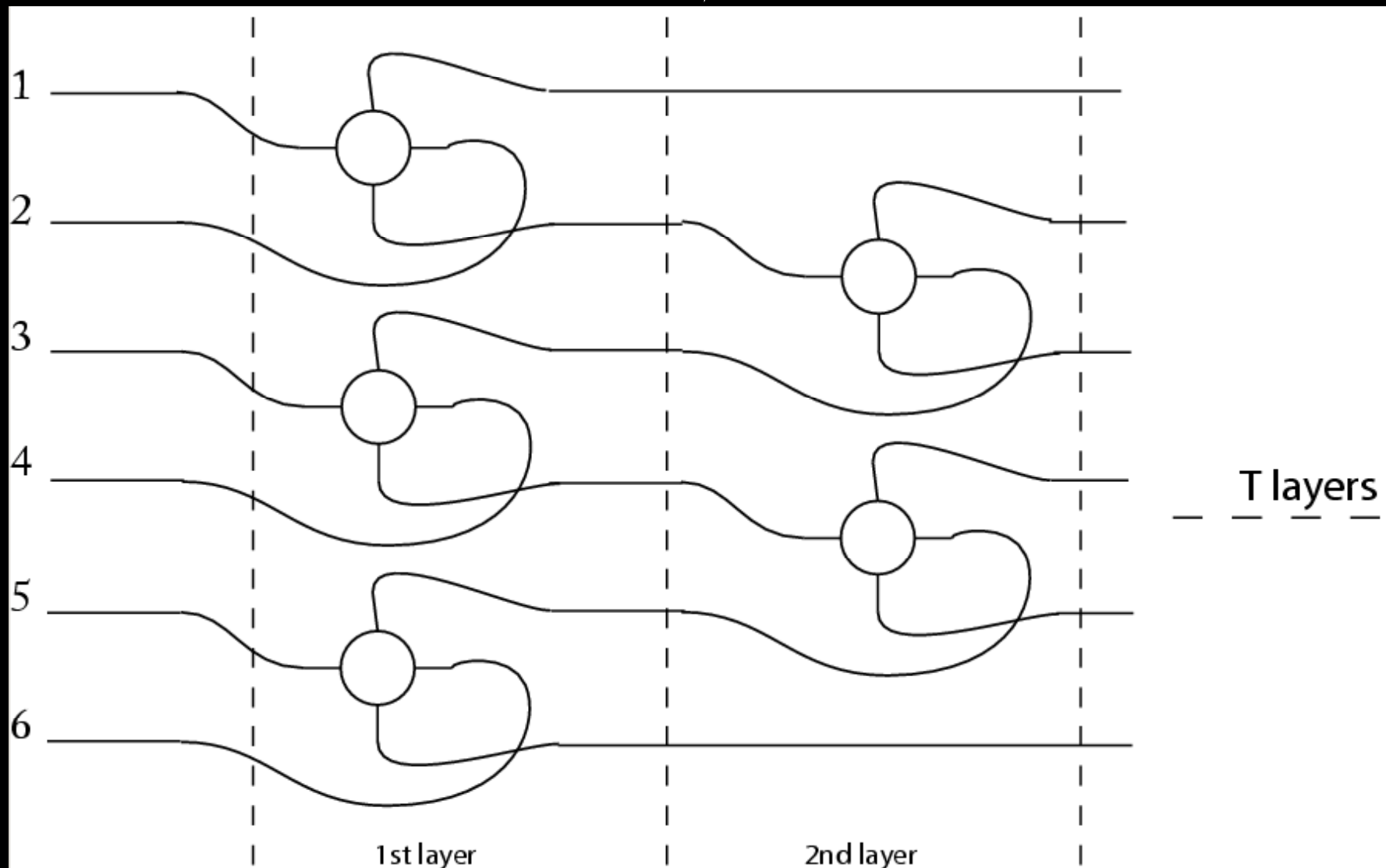


A-trails in 4-reg enough

approximate sampling/counting
of A-trails in 4-regular maps



approximate sampling/counting
of Euler tours in Eulerian graphs



1 2 3 4 5 6

2 1 3 4 6 5

2 3 1 4 6 5

3 2 4 1 6 5

3 4 2 6 1 5

.....

Theorem (Wilson'04):

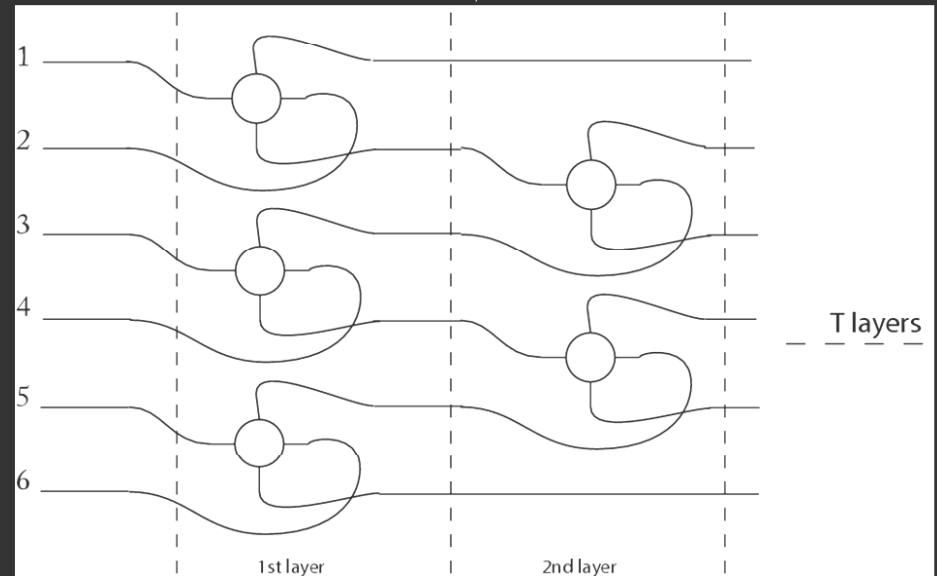
in $O(d^3 \ln^2 d \ln(1/\varepsilon))$ steps get $\varepsilon/2$ L_1 distance from uniform on permutations.

A-trails in 4-reg enough

approximate sampling/counting
of A-trails in 4-regular maps



approximate sampling/counting
of Euler tours in Eulerian graphs



even-odd sweeping MC

Exact: A-trails in 4-reg enough

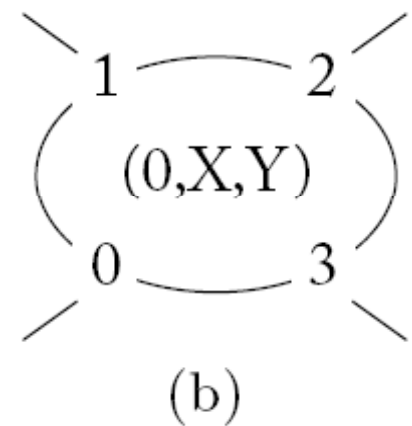
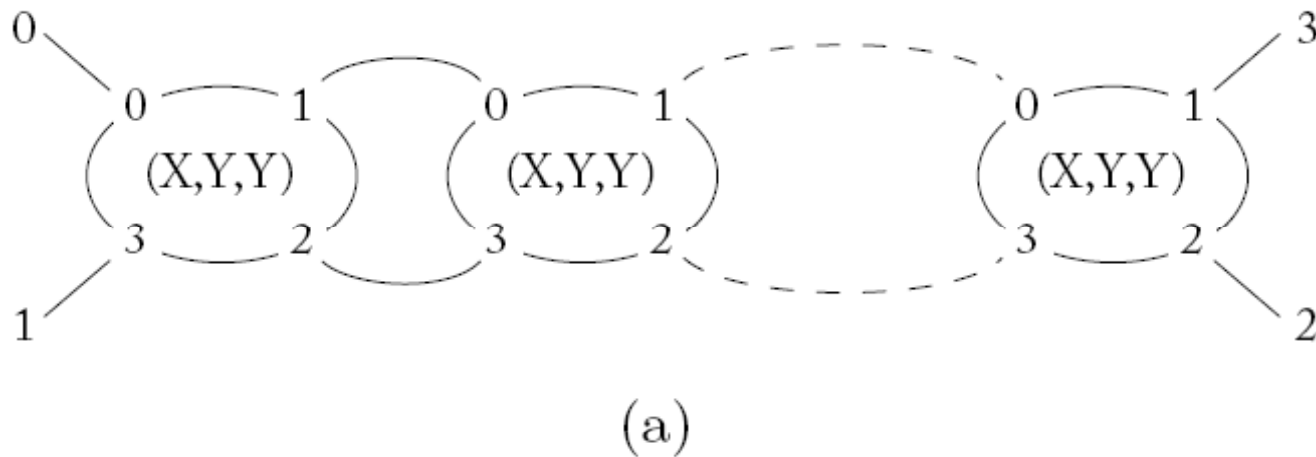
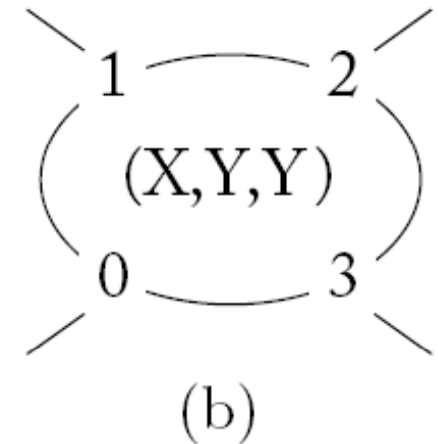
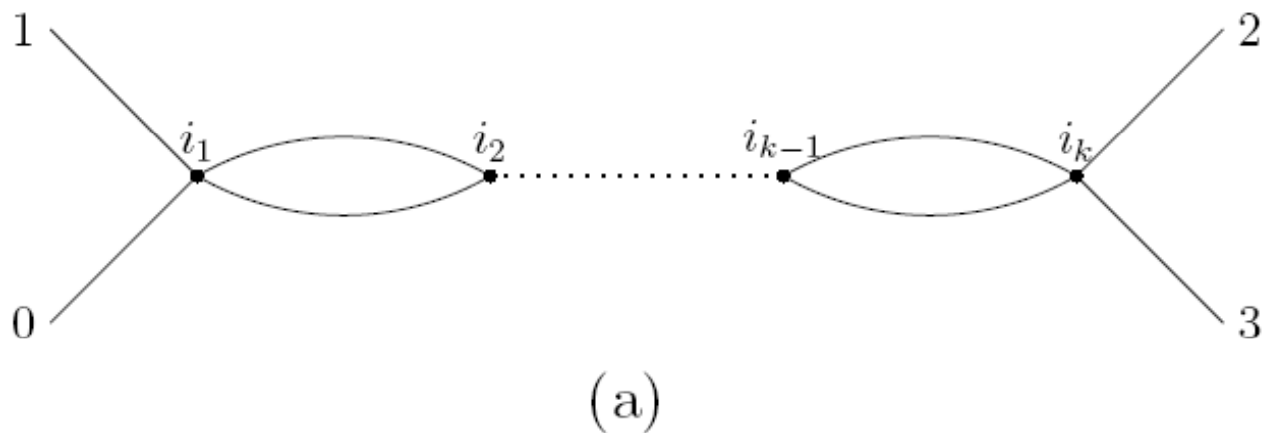
**exact counting
of A-trails in 4-regular maps**



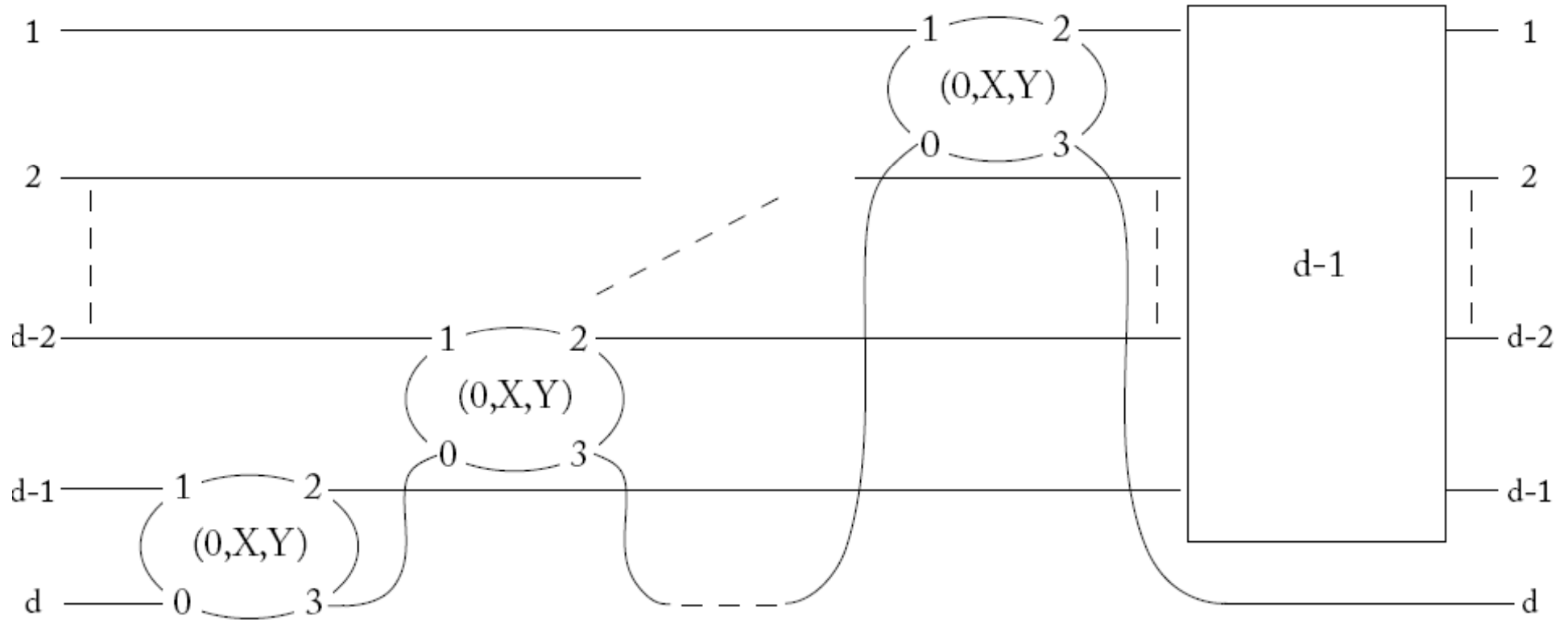
**exact counting
of Euler tours in Eulerian graphs**

**(corollary: counting A-trails
in 4-regular graphs #P-complete)**

Exact: A-trails in 4-reg enough



Exact: A-trails in 4-reg enough



Our results: A-trails in 4-reg enough

approximate sampling/counting
of A-trails in 4-regular maps



approximate sampling/counting
of Euler tours in Eulerian graphs

(**AP-reduction**

(Goldberg, Dyer, Greenhill, Jerrum'04))

Questions:

AP reduction from Euler tours in Eulerian graphs to Euler tours in 4-regular graphs?

Approximate sampling/counting of Euler tours/A-trails?

Which subsets of the hypercube can be sampled from?

