
CSC 284/484 Advanced Algorithms - homework 1
applied homework due: February 6th, 11:59pm EST
theoretical homework due: February 6th, 12:30pm EST

Do not search for a solution online, do not use any written material when writing any part of the code (for example, no copy-paste, no open textbook when writing code, no reediting of an old source file from an old project, etc). You can discuss your solution(s) or problems/issues you are facing with your classmates. Do not share code.

PROBLEM 1 (applied)—implement any max-flow algorithm

- Sufficient: Implement any s - t -max-flow algorithm.
 - Test your algorithm on the attached test data.
 - Write a little note indicating:
 - * Whether your code passed all the tests (if not where is the problem).
 - * Timing information (how long your code runs on each test).
- Better: Implement two s - t -max-flow algorithms.
 - Test your algorithms on the attached test data.
 - Write a little note indicating:
 - * Whether your code passed all the tests (if not where is the problem).
 - * Timing information (how long your code runs on each test).
 - * Compare the two algorithms.
- Even Better: Try all kinds of heuristics for the algorithms... see which one is the best on the test data.
- Your note should be typeset (LaTeX, Word, etc); send your code and note to the instructor and the TA (breber@cs.rochester.edu) (subject line: CSC 284/484 HOMEWORK 1).
- Allowed languages: C, C++, Java, Python (I discourage you from using Python—it is better for you to learn how to code algorithms in more efficient languages).

INPUT FORMAT: The first line contains k , the number of problems. Then descriptions of the problems follow. The first line contains n (the number of vertices) and m (the number of edges). The second line contains $s, t \in \{1, \dots, n\}$ (the source and the sink). Then m lines follow. Each line contains three integers u, v, c , where $u, v \in \{1, \dots, n\}$ and $1 \leq c \leq 1,000,000$; this means that there is edge (u, v) with capacity c in the graph. We have $2 \leq n \leq 1,000$ and $1 \leq m \leq 10,000$.

OUTPUT FORMAT: The output contains one line for each problem—the value of the maximum s - t flow.

EXAMPLE INPUT:

```
2
2 1
1 2
1 2 10
2 1
2 1
1 2 10
```

EXAMPLE OUTPUT:

```
10
0
```

PROBLEM 2 (theoretical)—explore alternative definition of cuts

Definition 0.1 Let $G = (V, E)$ be a directed graph. Let $s, t \in V$ be two distinct vertices. An s - t alternative-cut is a pair of disjoint sets A, B such that $s \in A$, $t \in B$, $A \cup B = V$, every vertex in A is reachable from s using paths contained in A , and every vertex in B can reach t using paths contained in B .

Prove or disprove

Conjecture 1 Let $G = (V, E)$ be a directed graph with edge capacities $c : E \rightarrow \mathbb{R}_{\geq 0}$. Let $s, t \in V$ be two distinct vertices. Assume that every $v \in V$ is reachable from s . Assume that t can be reached from every $v \in V$. We have

$$\max_{s-t \text{ flows } f} |f| = \min_{s-t \text{ alternative-cuts } A, B} c(A, B).$$

Typeset your solution (LaTeX, Word, etc) and send pdf to the instructor by email (subject line: CSC 284/484 HOMEWORK 1). In addition to sending the pdf print your solution and bring it to class of Feb. 6.

PROBLEM 3 (theoretical)—solve a problem using flows

We are given a directed graph $G = (V, E)$, two vertices s and t and a positive integer K . We want to find s - t -walks P_1, \dots, P_K such that no two walks have the same i -th edge (for any i) and the length of the longest walk is minimized.

(We describe the problem more formally. Let walk P_j be $v_{j,0}, \dots, v_{j,\ell_j}$ where $v_{j,0} = s$ and $v_{j,\ell_j} = t$. We want to minimize $\max_{j \in [K]} \ell_j$. The condition on the walks is: for any distinct $j, h \in [K]$ and any $i \in [\min\{\ell_j, \ell_h\}]$ we have $(v_{j,i-1}, v_{j,i}) \neq (v_{h,i-1}, v_{h,i})$.)

Show how to solve the problem using flows.

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