

# Errata: The Complexity Theory Companion

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**Introduction to the Errors** This document contains the list of errors in

*The Complexity Theory Companion*, L. Hemaspaandra and M. Ogihara, Texts in Theoretical Computer Science, an EATCS Series, Springer-Verlag. Hardcover edition ISBN 3-540-67419-5, 2002. Softcover edition ISBN 3-642-08684-7, 2010.

If you find other errors in the book, please let us know so that they can be added to this list.

**Key** “ $(n, m)$ ” means page  $n$ , line  $m$ . If  $m$  is negative, that counts up from the bottom, e.g., line  $-2$  is the next to last line of the page. If  $m$  is positive, the running header and figures/tables are *not* counted, e.g., line  $m$  is the first line after the running-header and any figures/tables (including their captions). Also, “\hrule”s, i.e., black horizontal lines, are not counted as lines. Ranges are shown using the natural notations, e.g.,  $(1, 5-10)$  and  $(3, -4)-(4, 17)$ .

## List of Errors

- (6,17) “an an” should be replaced by “an”; add a “]” after  $p_k(n)$ .
- (9,8) Eliminate the first occurrence of “then P=NP” in the statement of the theorem.

- (9,-4)  $y \in w_{\max}(x)$  at the end of the displayed formula should be replaced by  $y \leq w_{\max}(x)$ .
- (10,-4)  $u \in \text{Left}[A, p]$  should be  $u \in C$ .
- (16,-2)  $\Delta_1$  should be replaced by  $S_{t+1}$ .
- (19,3) “sparse, for” should be replaced by “sparse.”
- (20,4–5) in both places,  $p_\ell(p_k(|y|))$  should be replaced by  $p_k(p_\ell(|y|))$ .
- (21,3) “formula” should be replaced by “formula without.”
- (21,6) “ formula” should be replaced by “formula with.”
- (24,1) “binary of” should be replaced by “binary.”
- (24,11–12)  $b \leq 2^{|a|}$  should be replaced by  $b < 2^{|a|+1}$ , and  $(2^n)^c = 2^{cn}$  should be replaced by  $(2^{n+1})^c = 2^{cn+c}$ .
- (28,6) “encoding encoding” should be replaced by “encoding.”
- (35,13) “functions functions” should be replaced by “functions.”
- (38,15)  $f(g(y, x_2), x_1) = y$  should be replaced by  $f(g(y, x_2), x_2) = y$ .
- (50,16)  $R_{i-1,G} \cup$  should be replaced by  $R_{i-1,G}(v) \cup$ .
- (52,11)  $L^{=n}$  should be replaced by  $\Sigma^n$ .
- (52,13) Each of the three  $f$ 's should be a  $g$ .
- (54,1) “brute-force search” should be replaced by “brute force.”
- (58,-15 – -8) Replace the statement of Theorem 3.22 with:

**Theorem 3.22 (Cook’s Theorem)** *Let  $N_i$  be a standard enumeration of NPTMs that without loss of generality is such that  $N_i$  runs in time  $n^i + i$ . There is a function  $f_{\text{COOK}} \in \text{FP}$ , mapping from strings to boolean formulas, such that*

1.  $(\forall i) (\forall x) [N_i(x) \text{ accepts} \iff f_{\text{COOK}}(N_i, x, 0^{|x|^i+i}) \text{ is satisfiable}]$ ,

2.  $(\exists g_{\text{COOK}} \in \text{FP}) (\forall i) (\forall x) [g_{\text{COOK}}(f_{\text{COOK}}(N_i, x, 0^{|x|^{i+i}})) = \langle N_i, x \rangle]$ ,  
and
  3.  $(\exists h_{\text{COOK}} \in \text{FP}) (\forall i) (\forall x) (\forall a) [if a is a satisfying assignment of  $f_{\text{COOK}}(N_i, x, 0^{|x|^{i+i}}$ ), then  $h_{\text{COOK}}(N_i, x, a, 0^{|x|^{i+i}}$  outputs an accepting computation path of  $N_i(x)$ ].$
- (59, 6) “be such that” should be replaced by “be such that  $N_i$  runs in time  $n^i + i$ ,”.
- (59, 14–23) Each occurrence of  $f_{\text{COOK}}$  and  $h_{\text{COOK}}$  should have as an additional, final argument  $0^{|x|^{i+i}}$ .
- (68,28) “unambiguous” should be replaced by “ambiguous.”
- (69,22–23)  $s$  should be replaced by  $x$ .
- (70,6)  $\in$  should be replaced by  $\notin$ .
- (70,-6)  $\langle x, y \rangle$  should be replaced by  $\langle x, u \rangle$ .
- (70,-1)  $p(n)$  should be replaced by  $\mu(n)$ .
- (71,7)  $p(n)$  should be replaced by  $\mu(n)$  and  $\mu(|x|)$  should be replaced by  $\mu(n)$ .
- (72,14) Add the following before the last sentence. The oracle access of the machine  $N$  is many-one in the sense that  $N$  accepts if and only if the oracle’s answer to its query is positive.
- (72,24) Add the following before the sentence starting with “This implies”: The oracle access of the machine  $\hat{N}$  is many-one in the sense that  $\hat{N}$  accepts if and only if the oracle’s answer to its query is positive.
- (72,25)  $A$  should be replaced by  $A'$ .
- (72,28) Replace the two sentences, start from “Since” and ending with “ $< \frac{1}{2}$ ” by the following: The function  $(1 - 1/x)^x$  is strictly increasing for  $x > 0$ . The function converges to  $\frac{1}{e} < \frac{1}{2}$ . So, the probability that  $T^{B'}$  on input  $x$  rejects is at most  $\frac{1}{2}$ .

- (73,19) Replace the first sentence as follows: It is not hard to see that the proof of part 2 of Proposition 4.6 below relativizes, i.e., we have, for each  $A$ ,  $\text{BPP}^{\text{BPP}^A} = \text{BPP}^A$ .
- (76,-9)  $N_1$  should be replaced by  $N_0$ .
- (77,15–16) Replace each occurrence of  $p$  by  $q$ .
- (77,-12) Replace  $C$  by  $B$ .
- (77,-8) Replace each occurrence of  $s(|x|)$  by  $|x\#y|$ .
- (82,7) Replace  $\text{PP}^A$  by  $\text{PP}$ .
- (80,7) “leftmost” should be replaced by “rightmost.”
- (101, 19) “multiplication multiplication” should be replaced by “multiplication.”
- (112, 4) Lemma number should be 6.28 rather than 6.31.
- (116, the figure) Replace the “Step 3” in line -2 by “Step 4.”
- (117,-20) Insert  $x$  after “input.”
- (119,12–13) Replace “part 4” by “parts 1 and 2.”
- (119,-3) Replace  $|A|^2$  by  $2^{|A|^2}$ .
- (119,-2) Replace  $s = 2\lceil \log |A| \rceil + 1$  by  $2|A|^2 + 1$ .
- (120,8) Remove “Set  $A_n$  to the  $n \times n$  matrix such that for.”
- (120,14) In the displayed formula, replace  $(i - j)^{-1}$  with  $(k - j)^{-1}$ .
- (123,25) Delete “exists.”
- (123,26) Add “steps” at the end of line, after “ $p(|x|)$ .”
- (123,27)  $V$  should be  $V'$ .
- (123,-7) The second  $V$  should be  $V'$ .

- (123,-7) Replace “prover” with “provers.”
- (124,-10) Add “has” after  $V$ .
- (127,-6)  $p(|x|) + 2$  should be  $p(|x|) + 1$ .
- (134,16) Replace “witnessing” with “witnessing that.”
- (126,-15) Replace  $w''$  by  $w'$ .
- (138, 9) “grater” should be “greater.”
- (139, 5) “ $x = 0$ ” should be “ $X = 0$ .”
- (149, -1) “ $x \in Z_Q$ ” should be “ $x \in (Z_Q)^s$ ,”
- (156, 8) The proof of Proposition 6.38 has some flaws. The hypothesis of the proposition requires the condition  $2d + 1 \leq R$ . The corrected statement and the proof can be found on p.270 of [RS96] (Lemma A.3).
- (168,-16)  $\mathcal{M}_5$  should be replaced by  $\mathcal{M}_k$ .
- (169,14) Remove “define.”
- (169,15-16) Replace each occurrence of  $\mu_s^{x_{i_j}}$  by  $\mu_s^{x_{i_s}}$ .
- (169,16) “ $r > s$ ” should be replaced by “ $r < s$ .”
- (171,9)  $e(e^{-1}(g_{s_2} \circ g_{s_1}))$  should be replaced by  $e(e^{-1}(g_{s_2}) \circ e^{-1}(g_{s_1}))$ .
- (171,14)  $G_{s_2, s_2}^{(r)}$  should be replaced by  $G_{s_1, s_2}^{(r)}$ .
- (171,-8)  $\ell$  should be replaced by  $\log \ell$ .
- (178,6)  $y$  should be replaced by  $u$ .
- (178,9)  $y$  should be replaced by  $u$ .
- (179,-1)  $\theta_3^{-1}\gamma$  should be replaced by  $\theta_3^{-1} \circ \gamma$ .
- (181,6)  $R_\zeta$  should be  $P(\hat{u})$ .

- (189,6-7) Replace “for all  $u \in \Sigma^*$  and  $v \in \Sigma^{p(|u|)}, q(|\langle u, v \rangle|) \leq r(|v|)$ ” by “for all  $u \in \Sigma^*$  and  $v \in \Sigma^{t(|u|)}, p(|\langle u, v \rangle|) \leq r(|u|)$ .”
- (200,-4) Insert a space after “restriction.”
- (204,18) Replace  $o$  by  $\mathcal{O}$ .
- (205,-9) The  $r$  should be  $rd$  and each occurrence of  $b$  should be  $bd$ .
- (209,3)  $G[\rho \neq 1]$  should be replaced by  $G_1[\rho \neq 1]$ .
- (210,7) The second occurrence of  $G_1[\rho \neq 0]$  should be replaced by  $G_1[\rho \neq 1]$ .
- (217,2–3) Each occurrence of the variable  $t$  should be replaced by an  $r$ .
- (217,11) “defined in the above” should be replaced by “defined above.”
- (217,-8) “hen” should be replaced by “then.”
- (235,9) “is closed? under” should be replaced by “is closed under”
- (236,20) Replace “ $\{x \mid z \in \Sigma^{p(|x|)} \wedge y \leq_{lex} z\}$ ” by “ $\|\{z \mid z \in \Sigma^{p(|x|)} \wedge y \leq_{lex} z\}\|$ .”
- (241,-11) “if” is missing before “and only if.”
- (243,4) Replace “ $\mathcal{S}_{m,r} \leq 1$ ” by “ $\mathcal{S}_{m,r}(z) \leq 1$ .”
- (243,-5) “ $Q_{m,r}$ ” should be “ $Q_m$ .”
- (249,3) Replace “ $2^{-p(|x|)+2}$ ” by “ $2^{-p(|x|)-2}$ .”
- (250,19) Replace “with the located on” by “with the head located on.”
- (250,-12) Insert “just ended” after “of queries.”
- (252,-16) In the displayed formula, replace “ $2s^*(x, i) - t(x)$ ” by “ $2s^*(x, i) - t(x)2^{q(p(|x|))} + t(x)$ .”
- (267,-1) NPNP should be replaced by NP.

- (268,9–18) Replace the statement of Cook’s Theorem by the one given in this errata listing under the page (58,-15–8) entry.
- (272,-5) Replace “for some  $k$ ” with “for some  $k \geq 1$ .”
- (288, the figure) Replace “ $|y| \leq$ ” by “ $|y| =$ ” in each of the four places it appears in the figure.
- (293,13) “ $\text{PP}^{\text{NP}}$ ” should be replaced by “ $\text{NP}^{\text{PP}}$ .”
- (293,-14) “ $\text{SPP}^{\text{SPP}} = \text{SPP}^{\text{FewP}} = \text{PP}$ ” should be replaced by “ $\text{SPP}^{\text{SPP}} = \text{SPP}^{\text{FewP}} = \text{SPP}$ .”
- (295, the table) “faction” should be replaced by “fraction” on line –2 of Figure A.22.
- (300,9) The quantity  $\frac{3}{4}$  at the very end of the definition of IP should be replaced by  $\frac{1}{4}$ .
- (300,16) The quantity  $\frac{3}{4}$  at the very end of the definition of MIP should be replaced by  $\frac{1}{4}$ .
- (305,-10) The left “]” should be moved right to be beside the right “]”.
- (305,-7) The left “]” should be moved right to be beside the right “]”.
- (305,-3) The left “]” should be moved right to be beside the right “]”.
- (306,-19) The left “]” should be moved right to be beside the right “]”.
- (307,-18)  $\leq_d$  should be  $\leq_d^p$ .

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