

In-Class Semi-Workshop Problems for 2023/2/13 and 2023/2/15
(last updated 2023/2/3/631pm)

CSC 280: Computer Models & Limitations

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Each WS should please come into both the 2023/2/13 and the 2023/2/15 classes with sheet(s) of paper (it must be paper: NOT on a laptop—MY laptop will be using the one HDMI feed to project problems/etc.), to project on the doc cam, that give a solution to the problem that is congruent mod 10 to their ‘280’ recitation number, and should have predetermined what person from their WS will present those slides to the class (make sure at least two people per group—except in the size-1 WS—bring in such slides and are ready to present, in case one of the two misses class). So 280-11 presents problem 1, 280-2 presents problem 2, and so on.

But, and this is important (e.g., as preparation for and building skills for part of material in scope on the first midterm), I urge everyone to try all the problems. And at your WS sessions on 2/8 and 2/9 definitely do (guided by your WS leader) your WS’s assigned problem and then as many of the others as possible (making sure to try to get in some problem(s) that involve using the pumping lemma for CFLs—I’ve pointed the WS leaders to one in particular). (Side note to just the 280-8 WS leader: Quickly do one such pumping lemma problem *first*, so that (the quite hard) Problem 8 does not keep you from reaching doing that. If you don’t then have the time to solve Problem 8 in 280-8, don’t worry... in that case I’ll solve it myself in class.)

If you want a challenge, or like lovely solutions, try problem 8, as its solution is really quite beautiful, or at least, it has at least one beautiful solution, that takes a satisfying twist/insight to get to.

1 Problems (Some Are Challenging)

1. Give a context-free grammar for the set of all strings over the alphabet $\{a, b\}$ that are NOT palindromes.

2. Is this context-free? Justify your answer.
 $\{a^i b^j c^k d^\ell \mid i + j = k + \ell \wedge i, j, k, \ell \geq 0\}$.

3. Is this context-free? Justify your answer.
 $\{a^i b^j c^k d^\ell \mid i + k = j + \ell \wedge i, j, k, \ell \geq 0\}$.

4. Is this context-free? Justify your answer.
 $\{a^i b^j c^k d^\ell \mid i = k \wedge j = \ell \wedge i, j, k, \ell \geq 0\}$.

5. Is this context-free? Justify your answer.
 $\{a^i b^j c^k d^\ell \mid i = \ell \wedge j = k \wedge i, j, k, \ell \geq 0\}$.

6. Is this context-free? Justify your answer.
 $\{a^i b^j c^k d^\ell \mid i = j \wedge k = \ell \wedge i, j, k, \ell \geq 0\}$.

7. Prove or disprove: If L and L' are CFLs, then $(L - L') \cup (L' - L)$ is a CFL. (I.e., prove or disprove that the CFLs are closed under symmetric difference.)

8. Give a context-free grammar for the set $\overline{\{ww \mid w \in \{a, b\}^*\}}$, i.e., $\{a, b\}^* - \{ww \mid w \in \{a, b\}^*\}$. (Cute background/context but it is not relevant to solving this: $F = \{ww \mid w \in \{a, b\}^*$ is NOT a CFL. But on this (quite difficult) problem, you are asked to prove that F 's complement is a CFL. So sometimes the complement of a CFL is not a CFL—although in contrast we know that the complement of any regular set is itself regular.)

9. Use the pumping lemma to prove that this is not context-free: $\{www \mid w \in \{a, b\}^*\}$.