Problem Set 8

CSC 280, 2006

Due before class, Wednesday Apr. 5, 2006

Answers to five of problems 1–8 constitute a complete answer set. Half of the marks received for any additional problems done will count as extra credit.

Problem 1 Prove *by diagonalization* that USES-TAPE2$_{TM}$ $=_{def}$ $\{\langle M, w\rangle \mid M$ is a 2-tape TM that eventually writes a nonblank symbol on its second tape, when given w on its input tape$\}$ is undecidable.

Problem 2

(a) Show that the PCP is undecidable even if we allow only binary strings (say, in $\{0,1\}^*$) in the dominos.

(b) Show that the PCP is decidable if we allow only unary strings (say, in $\{1\}^*$) in the dominos.

Problem 3

(a) Show that if A is Turing-recognizable and $A \leq_m \overline{A}$ then A is decidable. Use a direct proof, exhibiting a decider for A (rather than relying on lemmas/theorems previously seen).

(b) Let A be any Turing-recognizable problem (set). Show that $A \leq_m A_{TM}$.

Problem 4 Let EQ-SUBLANG$_{CFG}$ $= \{\langle G, A, B\rangle \mid G$ is a CFG, A and B are variables of G, and the sets of terminal strings derivable from A and B are the same$\}$.

(a) Show that EQ-SUBLANG$_{CFG}$ $\leq_m$EQ$_{CFG}$; (EQ$_{CFG}$ is the equivalence problem for CFGs); and furthermore, the mapping reduction can be done by a function computable in polynomial time.

(b) Show that EQ$_{CFG}$ $\leq_m$EQ-SUBLANG$_{CFG}$; and furthermore, the mapping reduction can be done by a function computable in polynomial time.

Problem 5 *TM construction, mapping reducibility.*

Two TMs M_1, M_2 are *nearly identical* if $M_1 = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, $M_2 = (Q, \Sigma, \Gamma, \delta', q_0, q_{accept}, q_{reject})$, and δ, δ' are identical functions except perhaps that for one pair (q, a), where $q \in Q$ and $a \in \Gamma$, $\delta(q, a) \neq \delta'(q, a)$. Let EQ$'_{TM}$ be the equivalence problem for nearly identical TMs, i.e.,

EQ$'_{TM}$ $= \{\langle M_1, M_2\rangle \mid M_1, M_2$ are nearly identical TMs and $L(M_1) = L(M_2)\}$.

Show that the general equivalence problem for TMs is mapping-reducible to that for nearly identical TMs, i.e., EQ$_{TM}$ $\leq_m$ EQ$'_{TM}$. (*Hint*: Think of combining two given TMs so as to obtain the behavior of one or the other.)

Problem 6 *Big-O, small-o*

(a) Give the simplest, strongest possible "big-O" bounds on the following functions of n, very briefly indicating your reasoning:

$3n^3 + 14n^2 log_2 n + 1000$; $n^{2.7} + n^3/log_2 n$; $2^{\sqrt{n}} + n^{log_2 n}$; $n^{100} + 2^{(log_2 n)^2}$

(b) Call the 4 functions in (a) $f_1(n), ..., f_4(n)$, and compare them pairwise, stating for which pairs $f_i(n), f_j(n)$ the relation $f_i(n) = o(f_j(n))$ holds (again briefly indicating your reasoning).

(c) State which of the following big-O bounds on functions of n are equivalent, briefly indicating your reasoning:
$O(n^3)$; $O(n)$; $2^{O(log_2 n)}$; $2^{O(3 log_2 n)}$; $n^{O(1)}$; $2^{log_2 O(n)}$; $O(2^n)$; $2^{O(n)}$

Problem 7 Show that palindrome recognition (where palindromes are even- or odd-length strings in some alphabet that 'read' the same forward and backward) is **(a)** $O(n^2)$ on a single-tape TM, **(b)** $O(n)$ on a 2-tape TM, and **(c)** $O(n)$ on a (single-tape) two-headed TM.[1] (We are talking about time complexity, in terms of the length n of input strings.)

(d) Show that any $t(n)$-time two-headed TM has an equivalent $O(t^2(n))$ single-head TM.

Problem 8

(a) Show that the languages recognizable in $O(1)$ time are regular.

(b) Define the *fusion* of two languages A, B as
$fusion(A, B) = \{a_1 b_1 a_2 b_2 ... a_n b_n \mid a_1 a_2 ... a_n \in A,\ b_1 b_2 ... b_n \in B,\ n \geq 1\}$,

i.e, we take two equal-length strings from A and B and form a new string of length $2n$ by alternately using symbols from the A-string and the B-string.

Show that if A, B are recognizable in $O(f(n))$ time, where $f(n)$ grows at least linearly with n (thus allowing the entire input to be scanned), so is *fusion*(A, B).

[1] The two heads can scan the same square, and the machine starts off with both heads scanning the initial square. The transitions are dependent on the current state and the current symbols scanned by each head, and specify 2 symbols written and two moves L or R. If the two heads both try to write on the same square, head #1 'wins'.