

Problem Set 9

CSC 280, 2006

Due before class, Wednesday Apr. 12, 2006

Answers to five of problems 1–8 constitute a complete answer set. Half of the marks received for any additional problems done will count as extra credit.

Problem 1 Show that P is closed under (the Kleene star) *. *Hint:* Use a dynamic programming approach (working from the shortest substrings of the input to longer ones), building a table that indicates for each substring whether it is composed of strings in the language under consideration.

Problem 2

(a) Show that P is closed under complement.

(b) Carefully state what is wrong with the following argument “showing” that NP is closed under complement:

Given that $A \in \text{NP}$, we know that there is a nondeterministic TM M that decides A in time $\leq p(n)$ for some polynomial p . Switch the accept and reject state of M to obtain M' . Since M halts in time $\leq p(n)$ for all inputs of length n , so does M' ; so M' is a polynomial-time nondeterministic decider for \overline{A} , and hence $\overline{A} \in \text{NP}$.

Problem 3 Prove that $\text{E}_{CFG} \in \text{P}$.

Problem 4 Prove that $\text{CYCLIC} \in \text{P}$, where

$$\text{CYCLIC} = \{ \langle G \rangle \mid G \text{ is a directed graph containing a cycle.} \}$$

Problem 5 *Classes in NP.* Be careful to use appropriate measures of problem size.

(a) Show that $\text{SET-COVER} \in \text{NP}$. SET-COVER is the problem of determining whether a string x is of form $\langle k, S_1, \dots, S_m \rangle$ where $1 \leq k \leq m$ and S_1, \dots, S_m are finite sets of positive integers and there is a size- k subcollection S_{i_1}, \dots, S_{i_k} of these sets whose union is the same as the union of S_1, \dots, S_m .

(b) A *constraint satisfaction problem* $\text{CSP}_{\mathcal{V}, \mathcal{P}, \mathcal{Q}}$ is defined for any given finite set of values \mathcal{V} , a set of unary predicates \mathcal{P} over values \mathcal{V} (where each unary predicate can be thought of as subset of \mathcal{V}), and a set \mathcal{Q} of binary predicates over \mathcal{V} (where each binary predicate can be thought of as a set of pairs of elements of \mathcal{V}). With such an interpretation of parameters $\mathcal{V}, \mathcal{P}, \mathcal{Q}$, $\text{CSP}_{\mathcal{V}, \mathcal{P}, \mathcal{Q}}$ consists of strings of form $\langle P_1(x_1), \dots, P_k(x_k); Q(y_1, z_1), \dots, Q(y_m, z_m) \rangle$, where $x_1, \dots, x_k, y_1, \dots, y_m, z_1, \dots, z_m$ are (not necessarily distinct) variables (perhaps coded as sequential integers), $P_1, \dots, P_k \in \mathcal{P}$, and $Q_1, \dots, Q_m \in \mathcal{Q}$; and there is a way of assigning some value in \mathcal{V} to each of the distinct variables among the $x_1, \dots, x_k, y_1, \dots, y_m, z_1, \dots, z_m$ such that the given unary and binary constraints are satisfied. (Often a problem instance $\langle P_1(x_1), \dots, P_k(x_k); Q(y_1, z_1), \dots, Q(y_m, z_m) \rangle$ is thought of as a labelled directed graph, where each variable is a node and each node has 0 or more labels imposing unary constraints upon its value, and there are labeled directed edges imposing binary constraints pairwise on values of nodes. An example of a CSP is the graph coloring problem for 3 colors, 3COLOR . Here \mathcal{V} is the set of 3 colors, there are no unary predicates, i.e., we do not impose constraints on which nodes may have which colors – any node may have

any of the 3 possible colors, and the only binary predicate is “ \neq ”, i.e., two nodes related by this predicate must have different colors; if the colors are a, b, c , then \neq can be thought of as the relation $\{(a, b), (a, c), (b, a), (b, c), (c, a), (c, b)\}$. Note that because “ \neq ” is symmetric, edge direction doesn’t matter in this case.)

Show that for any $\mathcal{V}, \mathcal{P}, \mathcal{Q}$, $\text{CSP}_{\mathcal{V}, \mathcal{P}, \mathcal{Q}} \in \text{NP}$.

Problem 6 *Another problem in NP.*

Show that the *linear integer programming problem* is in NP. In linear integer programming, a set of variables x_1, \dots, x_k is constrained by a set of linear inequalities, each of form $a_{i,1}x_1 + \dots + a_{i,k}x_k \text{ rel } b_i$, $i = 1, \dots, m$, where $\text{rel} \in \{\leq, \geq\}$, and the $a_{i,j}$ and b_i are integer constants. The problem is to determine whether it is possible to assign integer values to the variables so as to satisfy the m inequalities.

Problem 7 *Polynomial-time verifiers.*

Suppose that we redefine a polynomial-time verifier V for A as a TM such that for some polynomial p , $A = \{w \mid \text{for some } c, V \text{ accepts } \langle w, c \rangle \text{ in time } p(n), \text{ where } n = |w| + |c|\}$.

Would it still be true that $A \in \text{NP}$ if and only if A has a polynomial-time verifier? Fully justify your answer to both the “if” and the “only if” part.

Problem 8 Show that 3SAT is polynomial-time reducible to a constraint satisfaction problem, as defined in 5(b), for a certain choice of $\mathcal{V}, \mathcal{P}, \mathcal{Q}$. *Hint:* Let variables of the constraint satisfaction problem correspond to clauses, with possible values 000, 001, ..., 111.