Problem Set 10

CSC 280, 2006

Due before class, Wednesday Apr. 19, 2006

Answers to four of problems 1–7 constitute a complete answer set. Half of the marks received for any additional problems done will count as extra credit.

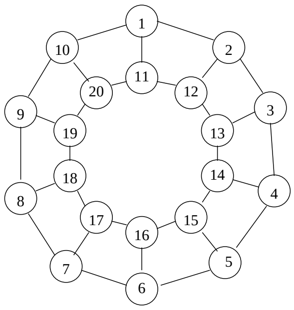
Problem 1 *More reductions*

(a) Show that SUBSET-SUM $\leq_p$ KNAPSACK, where the knapsack problem is that of determining whether a given list of positive integers $x_1, ..., x_k$ (with repetition allowed) can be partitioned into two subsets (strictly, *multi*sets) whose sums are equal.

(b) Show that INDEPENDENT-SUBSET $\leq_p$ CLIQUE, where

INDEPENDENT-SUBSET $= \{\langle G, k\rangle | G$ is an undirected graph containing k vertices such that there is no edge in G joining any pair of these vertices$\}$.

Problem 2 *Graph properties* Given the following graph, answer the questions below, providing a few words of explanation/justification where appropriate.



(a) Does this graph have a Hamilton circuit?

(b) What is the largest independent subset?

(c) Is the graph 2-colorable?

(d) What is the smallest vertex cover?

(e) What is the smallest edge cover (defined as a set of edges such that every vertex is an endpoint of one of these edges)?

Problem 3 *Cook-Levin theorem.*

(a) Give the boolean formula, as would be needed in the Cook-Levin construction, expressing the fact that for a NTM with $\delta(p, a) = \{(q, b, L), (r, c, R)\}$, the following (local) changes in configuration are possible at a particular time step $i-1$ to i and at a particular tape cell j (and its two neighbors):

$...dpa... \longrightarrow \{...qdb..., ...dcr...\}.$

(b) Convert the above formula to CNF. (*Suggestion*: for conciseness, you might replace the variable symbols used above by simpler ones.)

(c) How many clauses would this have if converted to 3CNF? Justify your answer (use the conversion given in the notes and in class, not the one in Sipser).

Problem 4 *Reducing CNF-SAT to SUBSET-SUM.*

Describe how one could generalize the 3SAT to SUBSET-SUM reduction covered in class to apply to CNF clauses of arbitrary size.

Problem 5 *Another NP-complete graph problem*
Note: a weighted graph (for our purposes) is one whose edges are labelled with nonnegative integer weights. A simple path (or cycle) is one that doesn't intersect itself. The *cost* of a path (or cycle) is the sum of weights of the edges traversed.

WEIGHTED-CYCLE $=_{def} \{\langle G, k\rangle | G$ is an undirected weighted graph that contains a simple cycle of cost $k\}$.

Note: This problem is of interest for example in designing a road race that needs to be of a particular length, has no overlapping or self-intersecting segments, and starts and finishes at the same cross-roads.

Show that WEIGHTED-CYCLE is NP-complete, using a reduction from SUBSET-SUM to WEIGHTED-CYCLE. *Hint*: One approach is to think in terms of loops that branch outward from a central region, like petals of a flower. But there are other equally good approaches.

Problem 6 Show that SAT-SAT $=_{def} \{\langle\phi\rangle | \phi$ is a boolean formula with 2 or more satisfying assignments$\}$ is NP-complete.

Problem 7 Let INTEGRAL-ROOTS be the problem of deciding whether an arbitrary polynomial with integer coefficients has integral roots, i.e., whether we can choose integer values of the variables for which the polynomial is 0. (For example, $x^2y - 2xy - 2x^2 + 4x + y - 2$ is a positive instance of this problem, with roots $x = 1, y = 2$; $xyz^2 - xz^2 - yz^2 - 4xy + z^2 + 4x + 4y - 4$ is another positive instance, with roots $x = 1, y = 1, z = 2$.)

Show that every problem in NP is polynomial-time reducible to INTEGRAL-ROOTS. Does this show that INTEGRAL-ROOTS is NP-complete? Why or why not?