Problem Set 11

CSC 280, 2006

Due before class, Wednesday Apr. 26, 2006

Do one of problems 1 & 2, one of problems 3 & 4, and two of problems 5–7. Half of the marks received for any extra problems done will count as extra credit.

Problem 1 Show that if P = NP, a polynomial time algorithm exists that, given a boolean formula ϕ, actually produces a satisfying assignment for ϕ if it is satisfiable. (*Hint*: This is a simple problem, if you consider that if P = NP, then there is a polynomial time "subroutine" for the satisfiability problem, that you can use to actually find a satisfying assignment. Expect to use the subroutine more than once...)

Problem 2 Show that if P = NP, we can find all the factors of any integer x in polynomial time. *Note*: An NTM that merely verifies in polynomial time that x *does* have some nontrivial factor (as in the COMPOSITES problem) won't help as a subroutine for actually *finding* the factors of x!

Problem 3 Consider the following problem. Given a graph G, with positive integer distances on the edges, and two integers k and d, is there a way to select k vertices of G on which to locate "firehouses", so that no vertex is at distance more than d from a firehouse? Show that this problem is NP-complete. *Suggestion*: Show this is in NP, and then do a reduction from a known NP-complete problem. One possibility for the latter is as follows. Start with a 3CNF formula, and derive from it the same graph as was used in reducing 3SAT to VERTEX-COVER, but giving each edge a length of 1. This graph consists of "triangles" for clauses and "linked pairs" for literals $x_i, \overline{x}_i$, $i = 1, ..., n$ (and connections between the triangles and pairs). Now add a further node y_i for each pair, connecting it to x_i and $\overline{x}_i$ with 2 edges, each of length 2. Thus we have n new nodes and $2n$ new edges. Let the corresponding "firehouse" problem have parameters $k = n$ (i.e., n firehouses) and $d = 2$ (maximum distance to nearest firehouse). Show that this "works", with proofs in both of the required directions. (The trickier direction is showing that a "firehouse solution" implies satifiability of the 3CNF formula. Argue that for each triple of nodes $x_i, \overline{x}_i, y_i$ – i.e., the linked pairs that were expanded to triples – one of the 3 nodes must be a firehouse; and if it is y_i that is a firehouse (as may happen), then either of $x_i, \overline{x}_i$ can be chosen to be 1, i.e., true; whereas if one of $x_i, \overline{x}_i$ is a firehouse, that is the one chosen to be true, to obtain a satisfying assignment.)

Problem 4 Let $U = \{\langle M, x, 1^t \rangle \mid M$ is an NTM that accepts input x within t steps$\}$. Show that U is NP-complete. (*Hint*: Show that U is in NP, and then show that for any arbitrary problem A in NP, A can be reduced to U.)

Problem 5 Show that PSPACE is closed under $\cup, \cap$, and Kleene *.

Problem 6 Show that SPACE($f(n)$) is the same for $f(n) \geq n$, whether we use a 1-tape TM model or a 2-tape model with a read-only input tape (with complexity measured in terms of the number of tape cells used on the second tape).

Problem 7

(a) Show that any PSPACE-hard language (one such that every language in PSPACE is polynomial-time reducible to it) is also NP-hard.

(b) Show that if every NP-hard language is also PSPACE-hard, then PSPACE = NP.