CSC2/455 Software Analysis and Improvement
Foundations of Data Flow Analysis

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URCS

Outline

Review

The Meaning of It All

Data flow analysis

Postscript

Outline

Analyses so far

- Live variables
- Available expressions
- Very busy expressions
- Dominators
    - Dominance frontiers (direct)
- Solved using Iterative data flow analysis

Outline

The Reaching Definitions Problem

```
if (x > 5)
   x#0 = 5;
else
   x#1 = 0;

if (x)
   x#2 = 3;

u = x;
```

Which definitions reach `u = x`?

The Ideal Solution: Example

Compute reaching definitions for each *possible execution* path to `u = x`, and combine the results using $\cup$:

Execution Path 1

```
x#0 = 5
x#2 = 3
u = x
```

Execution Path 2

```
x#1 = 0
u = x
```

So only $\{$`x#2, x#1`$\}$ reach `u`.

The Ideal Solution

- Let $P = ENTRY, B0, B1, ..., B_k$ be an execution path to the entry of block B_k
- Let f_P be the *composite* transfer function we want to evaluate
- The ideal solution is defined as the *meet* ($\wedge$) over all P

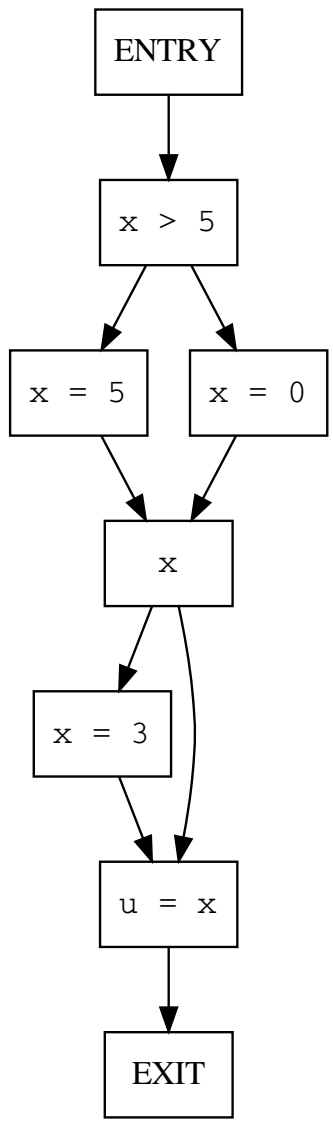
$$\text{IDEAL}[B_k] = \bigwedge_{P \text{ is path from ENTRY to } B_k} f_P(P)$$

Note, for reaching definitions $\wedge = \cup$.

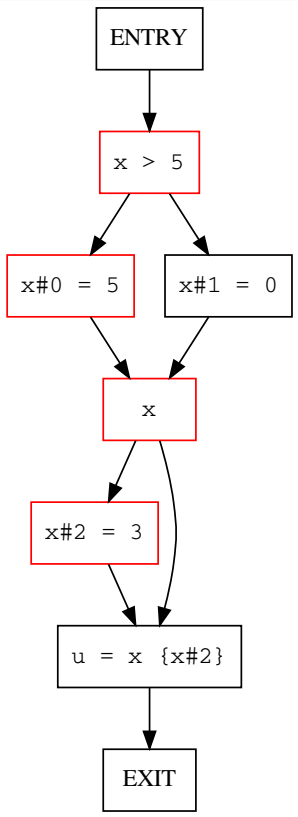
What is the problem with trying to find all execution paths?

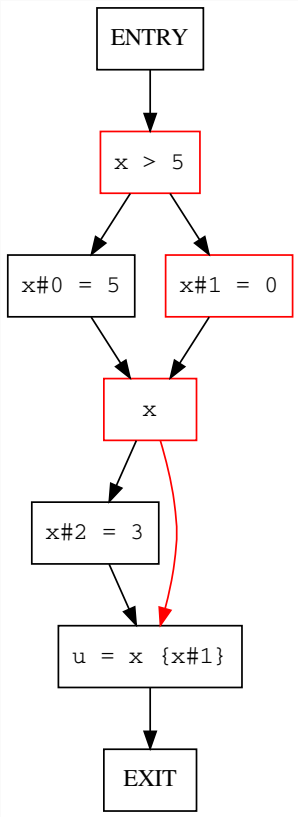
Meet over (all possible flow-graph) paths (MOP)

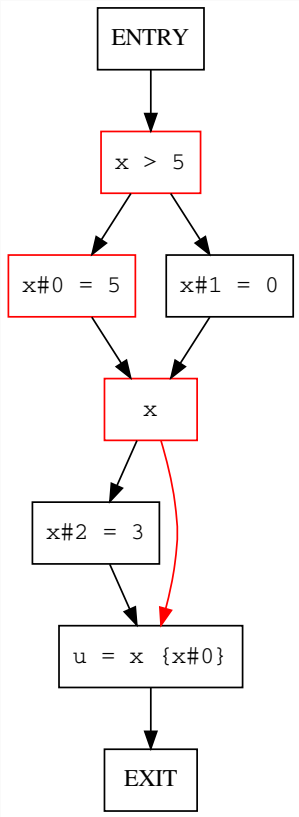
- IDEAL can be undecidable
  - "does this loop on the path to B_k terminate?"
- MOP approximates IDEAL
  - Only all paths in control flow graph are explored
- Calculate f over all paths in the CFG, one path at a time.
- Combine using $\wedge$, as in IDEAL

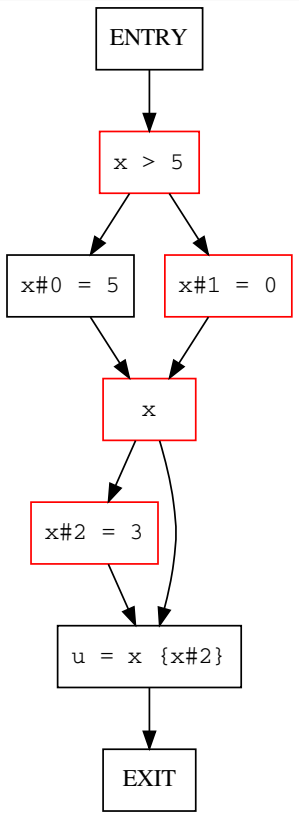


MOP solution









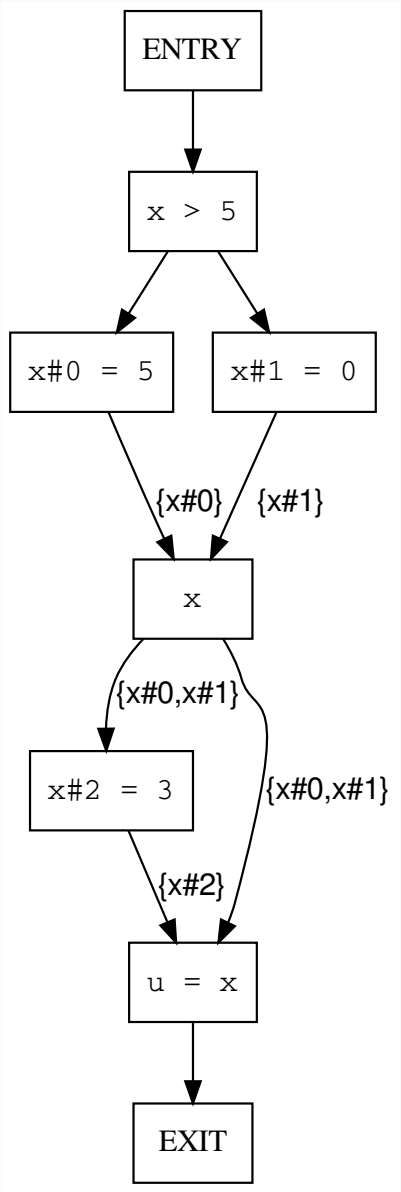
- The MOP solution is {x#0, x#1, x#2}
- Note IDEAL $\subset$ MOP

Maximum Fixed Point (MFP)

- CFGs with cycles will have an unbounded number of paths
- Hence, our use of the iterative dataflow analysis framework
  - Compute function f at individual basic blocks (not on paths)
  - Use meet operator at join nodes (or "confluence" points)
  - Terminate when functions reach a fixpoint
  - (Ignore "maximum" for now)

MFP solution

- MFP solution is $\{$`x#0, x#1, x#2`$\}$
- In this case, $\mathrm{MFP} = \mathrm{MOP}$



Relationship between IDEAL, MOP and MFP

- For reaching definitions (and this example):
  - $\mathrm{IDEAL} \subset \mathrm{MOP} = \mathrm{MFP}$
- Precision also decreases from IDEAL to MFP
- Can we generalize?
  - $\mathrm{IDEAL} \subseteq \mathrm{MOP} \subseteq \mathrm{MFP}$
  - When would it be a proper subset ($\subset$)?
  - When would it be identical ($=$)?
  - Do our conclusions apply only to reaching definitions?

Outline

Recap: Building blocks of a Data Flow Analysis

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More formally

- A data flow analysis is a $(D, V, \wedge, F)$ where:
  - D is direction (backwards or forwards)
  - V is the domain of solutions (i.e. set) to the dataflow problem
  - $\wedge$ is the binary meet operator over V
  - $F : V \to V$ is the family of transfer functions
- V and $\wedge$ must define a *semilattice*

Semilattice

- A semilattice is a lattice with only a meet $\wedge$ operator (or a join [$\vee$] operator)
  - A lattice has both
- A semilattice $(V, \wedge)$ is a set V and an operator $\wedge$ such that:
  - $x \wedge x = x$ (idempotent)
  - $x \wedge y = y \wedge x$ (commutative)
  - $(x \wedge y) \wedge z = x \wedge (y \wedge z)$ (associative)
- Two elements of V are denoted as $\top$ (top) and $\bot$ (bottom) such that:
  - $\forall x \in V \top \wedge x = x$
  - $\forall x \in V \bot \wedge x = \bot$

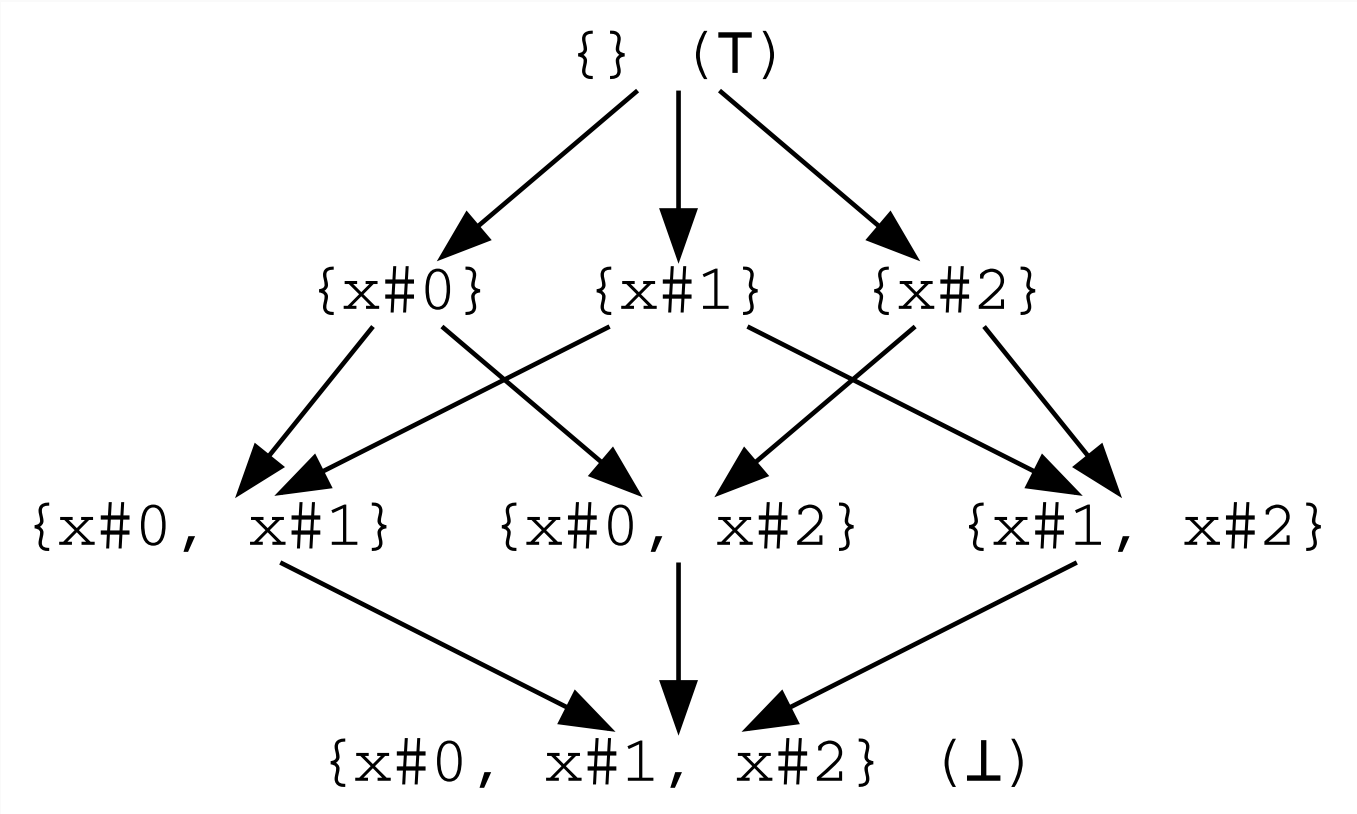
Reaching Definitions

- $D = ?$
- $V = ?$
- $\wedge = ?$
- $\top = ?$
- $\bot = ?$

Reaching Definitions

- $D =$ forward
- $V =$ powerset of definitions
- $\wedge = \cup$
- $\top = \emptyset$ (no definitions)
- $\bot = U$ (all definitions)

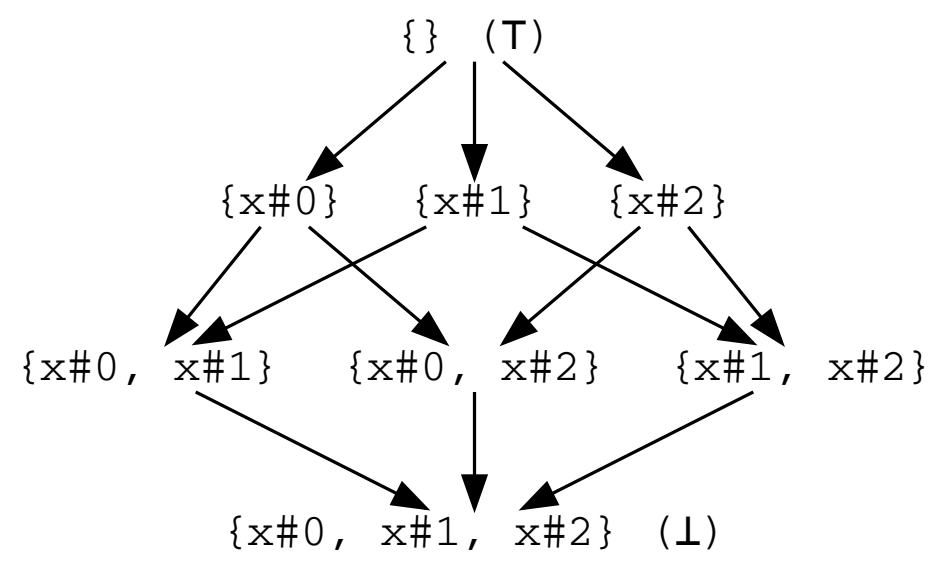
Semilattice for our Reaching Definitions Example



Partial Orders

- A semilattice $(V, \wedge)$ also induces a partial order among elements of V
- A partial order $\leq$ for values $x, y, z \in V$ has the following properties:
  - $x \leq x$ (reflexive)
  - if $x \leq y$ and $y \leq x$, then $x = y$ (anti-symmetric)
  - if $x \leq y$ and $y \leq z$, then $x \leq z$ (transitive)
- A partial order $\leq$ for a semilattice is defined as:
  - $x \leq y$ if and only if $x \wedge y = x$
- The pair $(V, \leq)$ is known as a partially ordered set or *poset*

Partial Order Example



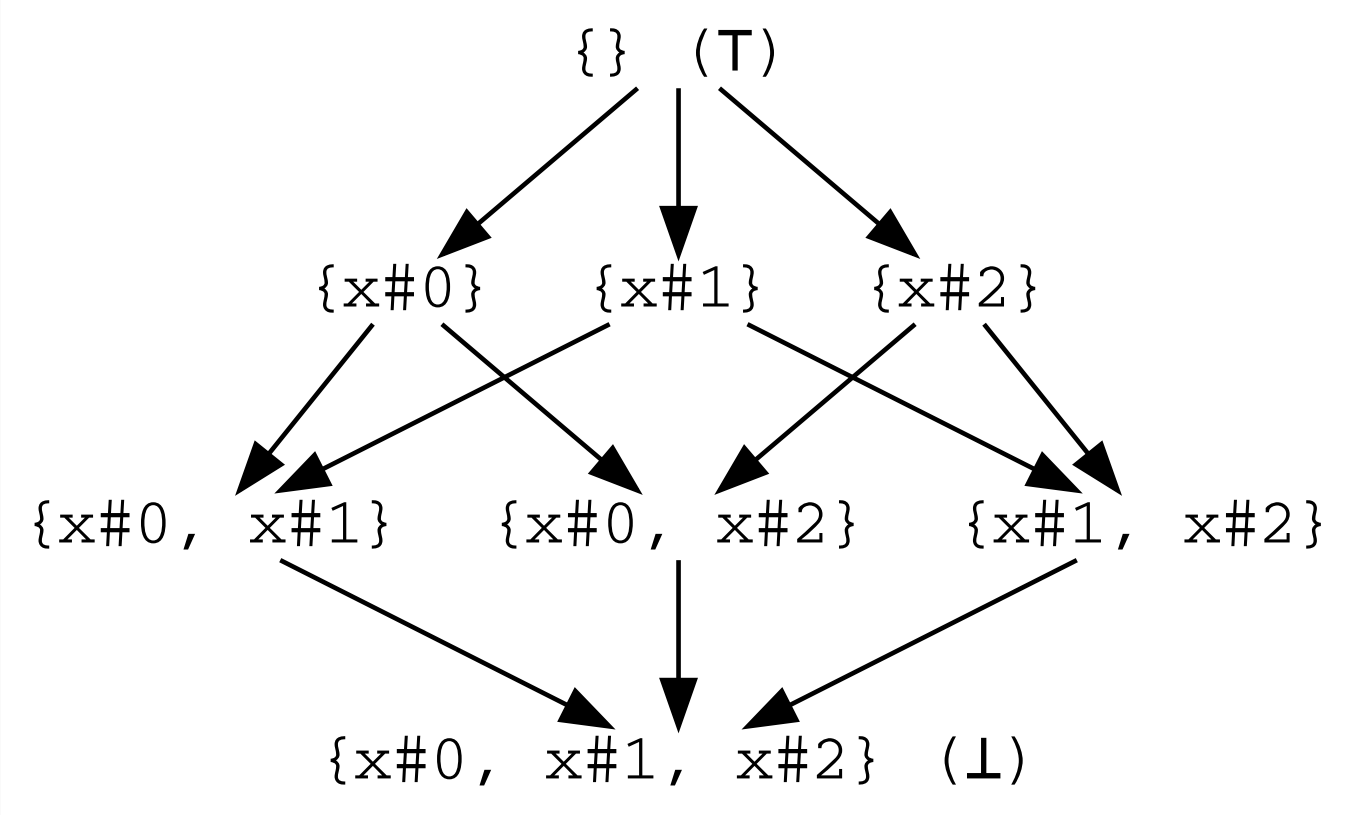
- What is the partial order ($x \leq y$ iff $x \wedge y = x$) between:
  - $\{\texttt{x\#0}\}$ and $\{\texttt{x\#0, x\#1}\}$?
  - $\{\texttt{x\#0}\}$ and $\{\texttt{x\#1}\}$?
- What is the name of partial order when:
  - $\wedge = \cup$? (i.e. $x \cup y = x$)
  - $\wedge = \cap$?

A relationship between $\wedge$ and $\leq$: the *glb*

- Informally, the greatest lower bound (*glb*) is the first element $\leq$ than both x and y
  - it can be x or y as well (note $=$ in $\leq$)
- The greatest lower bound g of x and $y \in V$ is
  - $g = x \wedge y$
- See the textbook for the definition and proof

Putting it all together

Think about how values at each basic block change we go about performing an iterative data flow analysis, especially how the values are related to each other in the lattice.



Transfer functions

- $F : V \to V$, F is a set of functions f
- $f(x) = G \cup (x - K)$
- F must satisfy two conditions:
  - must contain an identity function: $I(x) = x$
  - must be closed under composition $h(x) = f(g(x))$ is also in F
- E.g. reaching definitions transfer functions satisfy these conditions
  - See textbook for proof.

Monotone Framework

- A given $(D, V, \wedge, F)$ is monotone if for all $x, y \in V$, and $f \in F$:
  - $x \leq y \rightarrow f(x) \leq f(y)$
  - equivalently, $x \leq y \rightarrow f(x \wedge y) \leq f(x) \wedge f(y)$
- In addition, the framework is *distributive* if:
  - $f(x \wedge y) = f(x) \wedge f(y)$
- Note that these properties do not necessarily arise automatically, F must be designed to have these properties
  - And proofs must be written to show that F does.

General Iterative Algorithm

```
forwards(IN, OUT, meet, top, v_entry, f_transfer)
   OUT[entry] = v_entry

   for each basic block B except ENTRY:
      OUT[B] = top

   do {
      for each basic block B except ENTRY:
          # this calculates the meet over predecessors, /\p OUT[p]
          IN[B] = reduce(meet, [OUT[p] for p in B.predecessors])
          OUT[B] = f_transfer(IN[B])

   } while(some OUT changes value)
```

- Does this calculate the solution to the dataflow problem?
- Does this algorithm terminate?
- Does this algorithm calculate the *maximum* fixed point – i.e. the most precise solution admissible?

Next class

- Proofs that explore the three questions
- Relationships between IDEAL, MOP and MFP in terms of the framework
- Examples of:
  - a non-distributive framework (read Dragon 9.4, Constant Propagation)
  - lattices containing infinite values
  - possibly some proof writing exercises (read 9.3)

Outline

References

- Chapter 9 of the Dragon book
  - Section 9.3
  - 4xx students are expected to be able to solve exercises therein