CSC2/455 Software Analysis and Improvement
Type Inference - II

Sreepathi Pai

March 22, 2022

URCS

Outline

Type Systems for Realistic Languages

Type Checking ChocoPy

Postscript

Outline

Statements and Expressions

- The type system we studied in the last class was for a functional language
  - Programs consist of expressions
  - All well-formed expressions have a type
- Some languages, notably imperative languages (Python, C, etc.), differentiate between *statements* and *expressions*
  - Expressions have types
  - Statements do not have types
  - "Procedures" do not have return values

Additional judgements for imperative languages

- $\Gamma \vdash C$
    - C is a well-formed command (or statement) in Γ

In general, block-structured languages may have C be a block as well.

Subtyping

- Most languages have a subtype relation
  - C has *unsigned short* is a subtype of *unsigned int*
  - Subtypes can be used wherever the "supertype" is used
- Usually denoted by $A \leq B$
  - A is a subtype of B
  - $A \leq A$, obviously
- Type hierarchy, a tree of types
  - Usually `object` as the root
  - A edge from B to A if $A < B$

Outline

ChocoPy

- ChocoPy is simpler version of Python3
  - Developed as a "toy" language for compiler courses
- Fully-specified
  - Syntax (BNF)
  - Type System
  - Semantics (Operational Semantics)
- We will only look at its Type System

ChocoPy Types

- Basic types: `object`, `int`, `bool`, `str`
- List type: `[T]`, where `T` is another type
  - e.g. `[int]`
  - `T` be a list as well, this is an example of a *recursive type*
  - `[[int]]`
- Special types
  - `<None>`
  - `<Empty>`, indicating an empty list `[]`

ChocoPy Type Hierarchy

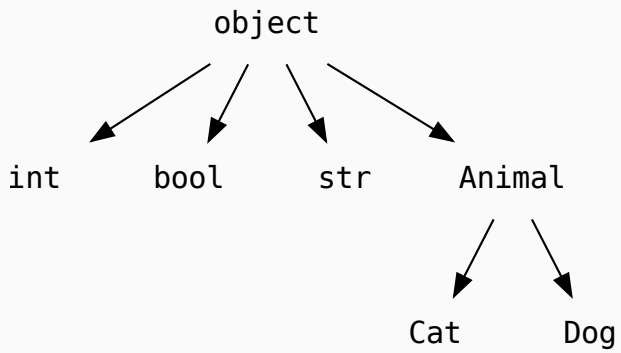
- `A` $\leq$ `A`, for all types `A`
- `C` $\leq$ `A` if `C` is a subclass of `A`
  - `int`, `bool`, `str` are all subclasses of `object`
- `[T]` $\leq$ `object`
  - but no relationship between two different list types
- `<None>` $\leq$ `object`
- `<Empty>` $\leq$ `object`

ChocoPy Assignment Compatibility

A type T_1 is assignment compatible ($\leq_a$) to T_2 iff values of type T_1 can be used wherever values of T_2 are expected.

- $T_1 \leq T_2$ (note: this is subtyping $\leq$)
- T_1 cannot be `<None>` if T_2 is `int`, `bool` or `str`
- T_1 can be `<Empty>` only if T_2 is `[T]`
- T_1 can be `[<None>]` if T_2 is `[T]`, where `<None>` $\leq_a$ `T`
  - Note that means `T` can only be `object` (*)

Join of types



The join $\sqcup$ of two types A and B denoted as $A \sqcup B$ is defined as:

- B if $A \leq_a B$
  - This is commutative, so $B \sqcup A$ is also B in this case
- C where C is the least common ancestor of A and B in the type hierarchy defined by $\leq$
  - So `int` $\sqcup$ `str` is `object`
  - Note this is the *least upper bound* or LUB (analogous to GLB in meet lattices)

Type Environment

- Judgements of the form:
  - $O, M, C, R \vdash ...$
- Where the type environment consists of:
  - O, "local" environment
  - M, method/attribute environment, i.e. for `A.x`-style code
  - Both are maps
- And:
  - C, the name of the current class, can be $\perp$ if outside class
  - R, the return type of the current function, can be $\perp$ if outside function

Function Type Notation in ChocoPy

```
def contains(items:[int], x:int) -> bool:
    i:int = 0
    while i < len(items):
        if items[i] == x:
            return True
        i = i + 1
    return False
```

The general type for a function f is $O(f) = \{T_1 \times T_2 \times ... \times T_n \rightarrow T_0; x_1, x_2, ..., x_n; v_1 : T'_1, v_2 : T'_2, ..., v'_m : T'_m\}$

- $T_1 \times T_2 \times ... \times T_n \rightarrow T_0$ is the type of arguments and the return type
  - Here, `[int]` $\times$ `int` $\rightarrow$ `bool`
- $x_1, x_2, ..., x_n$ are names of the arguments: `items`, `x`
- $v_1 : T'_1, ..., v_m : T'_m$ are types of variables/functions declared inside the function
  - Here `i` : `int`

Method/Attribute Type Notation in ChocoPy

```
class Point1D:
    x: int = 0

    # the quotes are used since Point1D is being defined ...
    def distance(self: "Point1D", o: "Point1D") -> int:
        ...
```

- Attributes are simply $M(C, a) = T$, where C is the class, a is the attribute, and T is its type
  - So, $M(\texttt{Point1D}, \texttt{x}) = \texttt{int}$
- And methods $M(C, m)$ are just like functions in the previous slide
  - So,
    $M(\texttt{Point1D}, \texttt{distance}) = \{\texttt{Point1D} \times \texttt{Point1D} \rightarrow \texttt{int}; ...\}$

Type Judgements in ChocoPy

- $O, M, C, R \vdash e : T$
  - Expression e has type T in the environment O, M, C, R
- $O, M, C, R \vdash b$
  - b is a well-formed (block) statement in O, M, C, R
  - can also be a single statement, obviously
- If reading the ChocoPy reference, make sure to distinguish the colon in type judgements $e : T$ from the colon in Python syntax
  - I'll use $:$ to indicate the latter, and : to indicate the former

Expressions

$$\frac{}{O, M, C, R \vdash \texttt{True} : \mathit{bool}}(\textsc{bool-true})$$

$$\frac{}{O, M, C, R \vdash \texttt{False} : \mathit{bool}}(\textsc{bool-false})$$

$$\frac{i \text{ is an integer literal}}{O, M, C, R \vdash i : \mathit{int}}(\textsc{int})$$

$$\frac{s \text{ is a string literal}}{O, M, C, R \vdash s : \mathit{str}}(\textsc{str})$$

$$\frac{O(id) = T \quad T \text{ is not a function type}}{O, M, C, R \vdash id : T}(\textsc{var-read})$$

Variable Definitions

$$\frac{O(id) = T \quad O, M, C, R \vdash e_1 : T_1 \quad T_1 \leq_a T}{O, M, C, R \vdash id : T = e_1} (\textsc{var-init})$$

Statements

$$\frac{\begin{array}{c} O, M, C, R \vdash s_1 \\ O, M, C, R \vdash s_2 \\ \vdots \\ O, M, C, R \vdash s_n \end{array}}{O, M, C, R \vdash s_1 \ \text{NEWLINE} \ s_2 \ldots s_n \ \text{NEWLINE}} (\textsc{stmt-def-list})$$

Operators

$$\frac{O, M, C, R \vdash e_1 : \mathit{int} \quad O, M, C, R \vdash e_2 : \mathit{int} \quad \mathit{op} \in \{+, -, *, //, \%\}}{O, M, C, R \vdash e_1 \ \mathit{op} \ e_2 : \mathit{int}} (\textsc{arith})$$

$$\frac{O, M, C, R \vdash e_1 : \mathit{int} \quad O, M, C, R \vdash e_2 : \mathit{int} \quad \bowtie \in \{<, <=, >, >=, ==, !=\}}{O, M, C, R \vdash e_1 \bowtie e_2 : \mathit{bool}} (\textsc{compare-int})$$

$$\frac{O, M, C, R \vdash e_0 : \mathit{bool} \quad O, M, C, R \vdash e_1 : T_1 \quad O, M, C, R \vdash e_2 : T_2}{O, M, C, R \vdash e_1 \ \texttt{if} \ e_0 \ \texttt{else} \ e_2 : T_1 \sqcup T_2} (\textsc{if-expr})$$

Return and Class Definitions

$$\frac{O, M, C, R \vdash e : T \quad T \leq_a R}{O, M, C, R \vdash \texttt{return } e}(\textsc{return-e})$$

$$\frac{O, M, C, R \vdash b}{O, M, \bot, R \vdash \texttt{class } C(S)\texttt{:}b}(\textsc{class-def})$$

Function Definitions

```
x: str = "hello, world"

def somefn(x: int, y: str):
   # what is the type of x here?
```

- Function argument names:
  - shadow names outside the function (like `x` here)
  - or are introduced in the definition (like `y`)
- The notation for shadowing or introducing a variable is:
  - $O[T/x]$ which is defined as $O[T/x](x) = T$ and $O[T/x](y) = O(y)$ where $y \neq x$
  - i.e., $O[T/x]$ is a new map with x mapped to T and the other mappings unchanged
  - repeated applications are possible: $O[T_1/x][T_2/y]$

Function Definition - II

$$T = \begin{cases} T_0, & \text{if -> is present,} \\ \texttt{<None>}, & \textit{otherwise.} \end{cases}$$

$$O(f) = \{T_1 \times \cdots \times T_n \to T;\ x_1, \ldots, x_n;\ v_1 : T'_1, \ldots, v_m : T'_m\}$$

$$n \geq 0 \qquad m \geq 0$$

$$\frac{O[T_1/x_1] \ldots [T_n/x_n][T'_1/v_1] \ldots [T'_m/v_m], M, C, T \vdash b}{O, M, C, R \vdash \texttt{def}\ f(x_1 : T_1, \ldots, x_n : T_n)\ [\![\texttt{->}\ T_0]\!]^? : b} \quad [\text{FUNC-DEF}]$$

Method Definition

$$T = \begin{cases} T_0, & \text{if -> is present,} \\ \texttt{<None>}, & \textit{otherwise.} \end{cases}$$

$$M(C, f) = \{T_1 \times \cdots \times T_n \to T;\ x_1, \ldots, x_n;\ v_1 : T'_1, \ldots, v_m : T'_m\}$$

$$n \geq 1 \qquad m \geq 0$$

$$C = T_1$$

$$\frac{O[T_1/x_1] \ldots [T_n/x_n][T'_1/v_1] \ldots [T'_m/v_m], M, C, T \vdash b}{O, M, C, R \vdash \texttt{def}\ f(x_1 : T_1, \ldots, x_n : T_n)\ [\![\texttt{->}\ T_0]\!]^? : b} \quad [\text{METHOD-DEF}]$$

Outline

References

- A self-contained introduction to type systems
  - Luca Cardelli, Type Systems, Handbook of Computer Science and Engineering, 2nd Ed
- An updated version (available only through the library)
  - Stephanie Weirich, Type Systems, Handbook of Computer Science and Engineering, 3rd Ed
- Rohan Padhye, Koushik Sen, Paul Hilfinger, ChocoPy v2.2: Language manual and reference
- Leandro TC Melo, Rodrigo G Riberio, Breno CF Guimaraes, Fernando Magno Quintao, Type Inference for C: Applications to the Static Analysis of Incomplete Programs, ACM TOPLAS
  - The psychec tool