

# **CSC2/455 Software Analysis and Improvement**

## **Dominators and SSA Form - II**

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URCS

# Outline

Review

Dominance Frontiers and Dominator Trees

Emitting code for SSA form

The SSA Form and Functional Programming

Postscript

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# Dominators

- A node  $n$  in the CFG dominates a node  $m$  iff:
  - $n$  is on all paths from entry to  $m$
  - by definition, a node  $n$  always dominates itself
  - if  $n \neq m$ , then  $n$  *strictly* dominates  $m$
- Computed using a dataflow-style analysis
  - Each node annotated with a set of its dominators

# Static Single Assignment Form

- Simple algorithm to generate SSA form
  - Introduce  $\phi$  functions
  - Rename variables using Reaching Definitions
- Algorithm can generate excessive  $\phi$  functions
  - TODAY: Use *dominance frontiers* to place the minimal number of  $\phi$  functions
- Also today: Removing  $\phi$  functions
  - Machines don't support  $\phi$  functions, so we must emulate them

# Maximal SSA Form

- Insert  $\phi$  nodes for each definition at every join node
- Rename LHS
- Rename RHS using reaching definitions

## Reducing the number of $\phi$ nodes

- Why insert  $\phi$  nodes at only join nodes?
- Can we skip inserting  $\phi$  nodes for a definition at some join node?

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# Dominance Frontiers

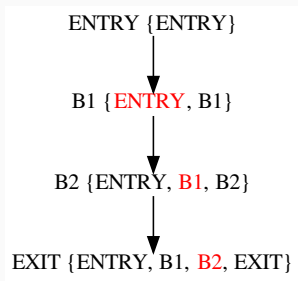
- The dominance frontier of a node  $n$  ( $DF(n)$ ) is a *set* of nodes
- A node  $m \in DF(n)$  iff:
  - $n$  does not strictly dominate  $m$
  - $n$  dominates  $q$  where  $q \in \text{pred}(m)$
- Note that dominance frontiers only contain *join* nodes
  - I.e. nodes with multiple predecessors
- Computing the dominance frontier of each node:
  - Iterative Data-flow analysis?

## Dominance Frontiers: Direct algorithm

Direct calculation of dominance frontiers using *dominator trees*.

# Immediate Dominators

- The *immediate* dominator of a node  $m$  ( $IDOM(m)$ ) is the node  $n$ :
  - such that  $n$  strictly dominates  $m$ , and
  - $n$  does not strictly dominate  $o$  where  $o \in (DOM(m) - \{m\})$
  - in some sense,  $n$  is the “closest” dominator in the CFG to  $m$ .
- By definition, ENTRY has no immediate dominator

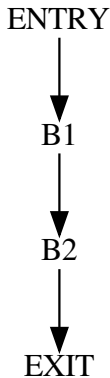


# Not Strictly Dominates

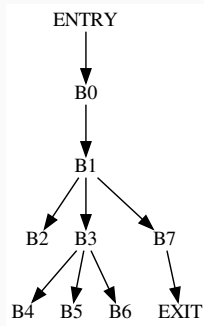
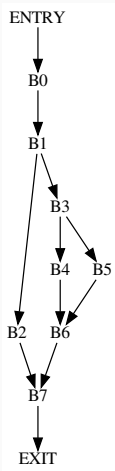
- $n$  strictly dominates  $m$ 
  - $SDOM(n, m) = n \in DOM(m) \wedge n \neq m$
- $n$  does not strictly dominate  $m$ 
  - $\neg SDOM(n, m) = n \notin DOM(m) \vee n = m$

# Dominator Tree

- Note that each node in the CFG can have only one immediate dominator
  - Can you see why?
- Create a graph  $G = (V, E)$ , where:
  - $V$  is the set of basic blocks
  - There is an edge  $(n, m)$  in  $E$  if  $n$  is the immediate dominator of  $m$  (i.e.  $IDOM(m) = n$ )



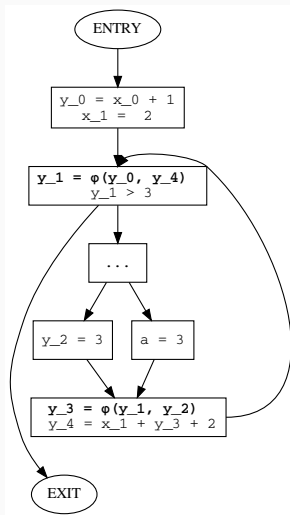
## Example: CFG and its dominator tree



## Computing the dominance frontier

- Find all join nodes in CFG, e.g.  $j$
- For all nodes  $n$  that dominate predecessors of  $j$  (in the CFG)
  - If  $n$  does not strictly dominate  $j$ , add  $j$  to  $DF(n)$
- This last step can be operationalized over all predecessors  $p$  of  $j$  in the CFG:
  - Start traversing the dominator tree at  $p$
  - If  $p$  is  $IDOM(j)$ , stop. Otherwise add  $j$  to  $DF(p)$
  - Repeat by moving up the dominator tree until you reach  $IDOM(j)$

## Example: Non-redundant $\phi$ functions





# Placing $\phi$ functions

- For each definition  $d$  in basic block  $n$ :
  - Place a  $\phi$  function for  $d$  in all nodes  $m$  where  $m \in DF(n)$
  - Note that each  $\phi$  function is also a definition!
  - Repeat, until no more  $\phi$  functions need to be inserted
- This is the minimal number of  $\phi$  functions for a definition  $d$  structurally
  - Can we further reduce the overall number of  $\phi$  functions?
- (Figure 9.9 in Cooper and Turczon)

## Other optimizations

- Dead definitions
  - Definitions that are not read (i.e. overwritten) do not need  $\phi$  functions
- Two forms:
  - *Semi-pruned SSA* form, using “globals” names (those variables that are live in to a block)
  - *Pruned SSA* form, using `LIVEOUT` information

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# Renaming variables

- SSA form introduced “subscripts” for each variable
- Should we drop them when generating code?

```
a_0 = x_0 + y_0  
b_0 = a_0  
a_1 = 17  
c_0 = a_0
```

## Problem with dropping subscripts

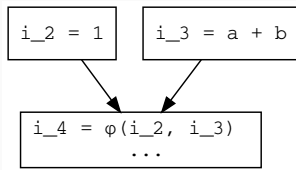
```
a = x + y  
b = a  
a = 17  
c = a    # WRONG!
```

# Handling subscripts

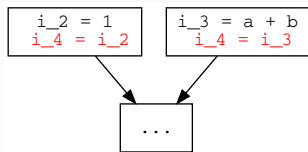
- Each definition becomes a new variable
  - I.e. Do NOT drop subscripts
- Preserves data dependences
  - Esp. important when we aggressively move code from basic blocks (e.g. very busy expressions, loop invariant code motion, etc.)

## Code for $\phi$ functions

- Introduce copies along each incoming edge to a join node



## Inserting appropriate copies along incoming edges

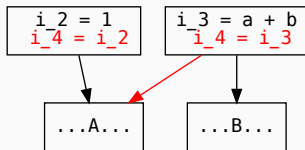




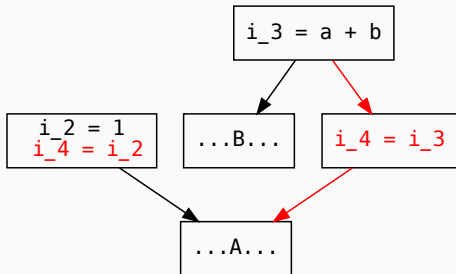
## Critical edges

- Executing  $\phi$  functions by inserting copies into predecessor blocks is not always correct
- If such a predecessor block has multiple successors, then the  $\phi$  function may execute when it shouldn't
  - This *may* be harmless, but not always
- Edges connecting such predecessors to the block containing the  $\phi$  function are called *critical* edges

## Critical Edges: Example



## Splitting critical edges



- Such edges need to be *split* by inserting a block on that edge
- See the discussion in Cooper and Turczon for more details and an example

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# Purely Functional Programs

- Everything is a value
- No “assignment”, just binding values to names
- No control flow such as jumps
  - Must be emulated using functions

## Example: Factorial

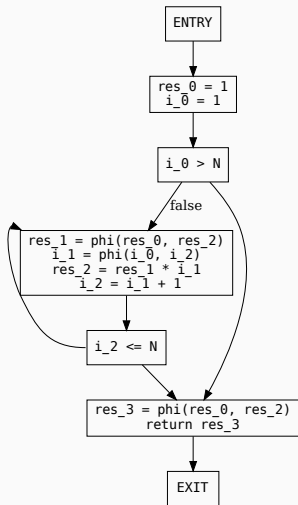
```
def fact(N):  
    res = 1  
    for i in range(1, N+1):  
        res *= i  
  
    return res  
  
def fac(N):  
    return 1 if N <= 1 else N * fac(N - 1)
```

## Factorial: 3 Address Code

```
def fact(N):  
    res = 1  
    i = 1  
    if i > N goto loop_end  
  
    loop_head:  
        res = res * i  
        i = i + 1  
        if i <= N goto loop_head  
  
    loop_end:  
        return res
```

# Factorial: SSA form

```
def fact(N):  
    res_0 = 1  
    i_0 = 1  
    if i_0 > N goto loop_end  
  
    loop_head:  
        res_1 = phi(res_0, res_2)  
        i_1 = phi(i_0, i_2)  
  
        res_2 = res_1 * i_1  
        i_2 = i_1 + 1  
        if i_2 <= N goto loop_head  
  
    loop_end:  
        res_3 = phi(res_0, res_2)  
        return res_3
```





## Factorial: Function Conversion

```
def fact(N):
    res_0 = 1
    i_0 = 1

    def loop_head(res_1, i_1):
        res_2 = res_1 * i_1
        i_2 = i_1 + 1
        return loop_head(res_2, i_2) if i_2 <= N else loop_end(res_2)

    def loop_end(res_3):
        return res_3

    return loop_end(res_0) if i_0 > N else loop_head(res_0, i_0)
```

- Each basic block is converted to a function
- Parameters to this function are the LHS of the  $\phi$  functions in that BB
- Arguments picked from arguments of  $\phi$  function depending on the path the BB was on.

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# References

- Chapter 9 of Cooper and Turczon
  - Section 9.2.1
  - Section 9.3
- Andrew W. Appel, SSA is functional programming \*
- Optional:
  - Various authors, The SSA book